TMAS Academy

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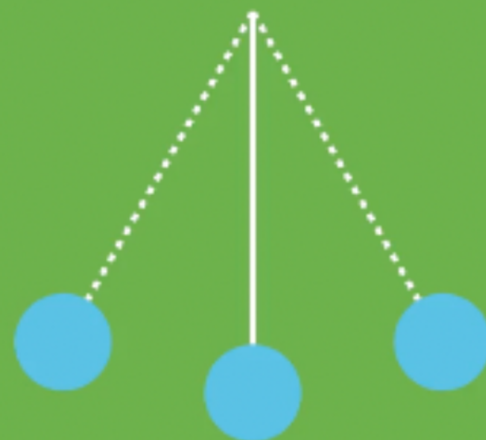
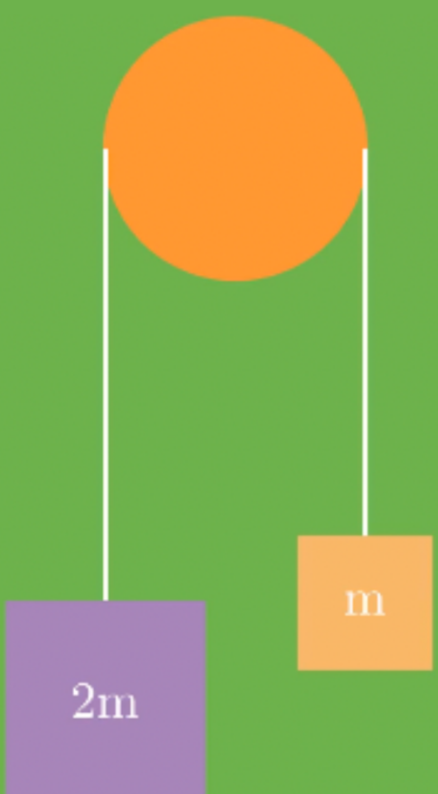


AP Physics 1

2024



- 100+ Problems
- All Topics
- Detailed Solutions

$$F_{net} = ma$$

Ritvik Rustagi

Contents

Important Information

If you found this book from any platform other than the website, then there's a high chance that you don't have the latest version with all the updates. Make sure to use the version from the official TMAS Academy website. If you are using the book from another platform, then you're at a risk of missing out on important content that is available on the new version which can always be found on the TMAS Academy website. In addition, important links in this book might not work on many platforms. However, they will work if you use the book from the official website.

About TMAS Academy

This book is brought to you by me (Ritvik Rustagi). TMAS Academy, previously known as Explore Math, was started by me in 2020. TMAS stands for The Math and Science. Currently, I have written five free books for students around the world. Those books include the *ACE The AMC 10/12*, *ACE AP Physics 1*, *ACE AP Calculus AB*, *ACE AP Physics C: Mechanics*, and *ACE AP Calculus BC*. All of the books have been designed to make preparing for these exams efficient and accessible for everyone.
You can find more info about this program on my website linked below.
Website: https://www.tmasacademy.com/

Opportunities For You To Contribute To TMAS Academy

Contributing to TMAS Academy is simple.
You can **join the team** by checking out the form below which can also be found on the website:
https://forms.gle/VXGvj27UvcZPGhiJ8

Donations: If you want to assist me in my monthly payments to run this program which includes website costs, Overleaf costs (the platform used to write such books), and filming/editing costs, then please consider donating! For those that are willing to contribute, I have listed a few ways below. **Don't forget to write a message so I know who you are which will allow me to send you a thank you note**.

- You can donate through PayPal to the email: ritvikrustagi7@gmail.com
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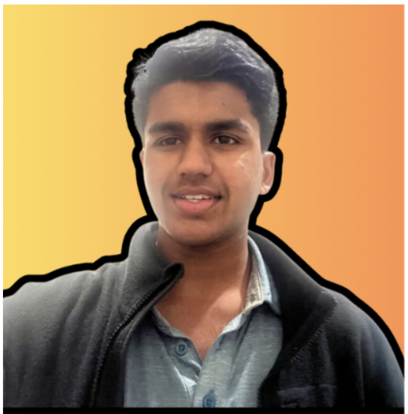
About The Author: Ritvik Rustagi

My name is Ritvik Rustagi, and I am a student at Prospect High School. Some information about me is that I enjoy doing math, physics, and programming.

Some of my qualifications include qualifying for USAJMO and USAMO (United States of America Mathematical Olympiad), qualifying for USAPHO (United States of America Physics Olympiad), achieving a gold medal in the finals round for MathCon in Chicago, and qualifying for the AIME several times.
During Covid, I discovered my passion for teaching math competition topics through my Youtube channel. It also allowed me to absorb these complicated topics more efficiently since teaching can help one improve their own skills. After that, I began my journey of writing various books for math competitions and AP courses. By October of 2023, I released my first major book for the AMC 10/12. In March 2024, I released several AP books which can all be found on the website.

This book has been written to help any student aiming to do well on the AP exam and the class itself. Due to the difficulty of this exam, a good guide is necessary with a rich problem set for students to practice with. This is what the book aims to do. Many students these days struggle to prepare for AP exams due to the vast amount of content. However, productive preparation can solve that problem. That is how I got 6 5s on the following AP exams in my sophomore year of high school: AP Physics 1, AP Calculus BC, AP Physics C: EM, AP Physics C: Mech, AP World History, and AP Statistics. Anyone can do it if they believe in themselves and choose the right resources to prepare with. Tons of problems are contained within this book with well written solutions. This will allow even the most inexperienced students to have a productive session of preparation while comprehending the problems and theory.



Benefits of Taking AP Exams

Preparing for AP exams such as the AP Calculus AB/BC, AP Physics 1, and AP Physics C is a great way to expand your knowledge. These exams go a step further to deepen your knowledge of subjects that you might have previously encountered. On top of that, you will learn many concepts that will be used throughout your life. It's a great learning experience and can give you the opportunity to enrich your journey. It also improves your problem solving skills which can serve as a life skill in many situations.

What if there is an error in the book?

There are possibilities for minor errors such as typos or a mistake in latex for some of the solutions to the problems. If that's the case, then please click on this link (`https://forms.gle/3mxZb4izUuBZLkmz5`) to report the mistake.

If you have any other questions or concerns, then please feel free to reach out to **ritvikrustagi7@gmail.com**

Credits

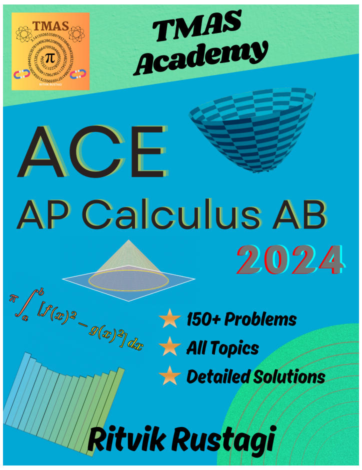
I would like to thank **The College Board** for their high quality problems that were used to teach concepts for this course.

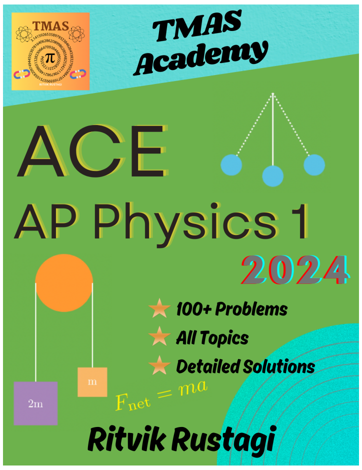
I would also like to thank **Evan Chen**, a PhD student at MIT (Massachusetts Institute of Technology), for his latex template which made it easy to format this book. I would also like to thank him for answering all of my questions regarding the format of this book.

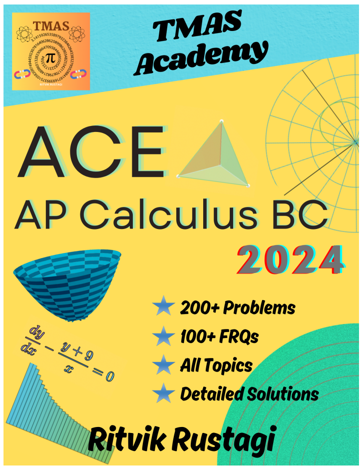
I would also like to thank my parents for supporting me with my goals and for everything else that they have done.

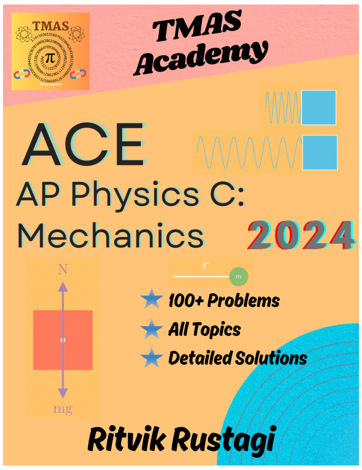
I would like to thank everyone else that has supported the content that I have made and encouraged me to continue to do so.

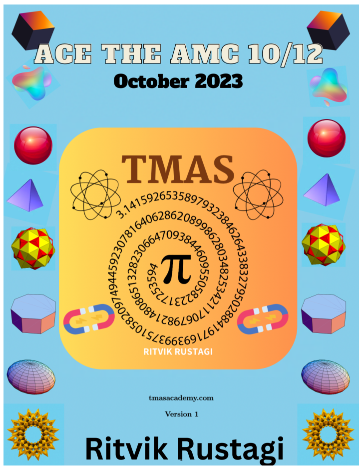
Other Important Resources











All five of these books were written by me. They can all be found on the TMAS Academy website. All five of the books are comprehensive and contain all the topics that you need to know. They have been designed to make your preparation productive through the vast number of official free response questions.

Make sure to check out the following playlists on the TMAS Academy youtube channel! These are important to learn all the topics that show up on the following AP exams: AP Physics 1, AP Calculus AB/BC, and AP Physics C: Mechanics.

AP Calculus AB/BC Playlist

AP Physics 1 Playlist

AP Physics C: Mechanics Playlist

Connect with the Author

Feel free to connect with me on Linkedin, Instagram, Discord, or through email!

I highly recommend joining the Discord server to access study and review sessions hosted in the server. You should also consider following the Instagram to access animations that will be posted there to allow you to learn.

Linkedin: https://www.linkedin.com/in/ritvik-638590210/

Instagram: https://www.instagram.com/ritvik_rustagi_tmas/

Discord: https://discord.gg/tmas-academy-1019082642794229870

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Unit 1 Kinematics

Kinematics is the study of motion of objects. We will deal with many variables in this unit such as velocity, time, displacement, distance, etc. After working with 1D kinematics, we will deal with 2D kinematics.

Note 1.0.1

Scalars vs Vectors
A scalar quantity is a quantity with no direction – it only has a magnitude. On the other hand, a vector quantity has a magnitude *and* a direction.

Scalar quantities include **distance** and **speed**.

Vector quantities include **displacement** and **velocity**.

Note 1.0.2

Scalar Quantities in Depth
Distance is defined as the amount of space traveled when moving from one point to another point. This is typically denoted by d.

Speed is defined as the rate at which the distance is traveled, and is typically denoted by s. That is, $s = \frac{d}{t}$.

Note 1.0.3

Vector Quantities in Depth
Displacement is defined as the change in position of an object in a given time. This is denoted by $\vec{d}$ or Δx.

Velocity is defined as the rate at which the object displaces, and is typically denoted by $\vec{v}$. That is, $\vec{v} = \frac{\vec{d}}{t}$ or $\frac{\Delta x}{t}$.

Acceleration is defined as the rate at which the velocity of the object changes, and is denoted by a. That is, $a = \frac{\Delta \vec{v}}{t}$.

In general, if you want to find the speed at a given time period and you're given velocity, then you can simply take the absolute value of velocity to find the speed. The reason is that speed has no direction, only a magnitude!

Problem 1.0.4 — Matt bikes from position $x = 50$ to position $x = 100$, and then to position $x = 55$. This occurs in 25 seconds. Assume that Mike travels at constant velocity.

a) What is the distance Matt travels?

b) What is Matt's speed?

c) What is Matt's velocity?

d) What is Matt's acceleration?

Solution to part a: He travels a distance of $(100 - 50) + (100 - 55) = \boxed{95}$ meters.

Solution to part b: From part (a), Matt travels 95 meters in 25 seconds, so his speed is $\frac{95}{25} = \boxed{\frac{19}{5}}\frac{\text{m}}{\text{s}}$

Solution to part c: Matt's displacement is 5, which occurs in 25 seconds. The reason is that he starts at a position of $x = 50$ and ends up at a position of $x = 55$. Thus, $\Delta x = 5$.

We can find that his velocity is $\frac{5}{25} = \boxed{\frac{1}{5}}\frac{\text{m}}{\text{s}}$

Solution to part d: Matt moves with constant velocity, so his acceleration is $\boxed{0}\frac{\text{m}}{\text{s}^2}$

Note 1.0.5

Average vs. Instantaneous values

Average velocity is simply the displacement divided by the total time. However, instantaneous velocity is the velocity of an object at a **specific** instant of time.

Similarly, this difference between average acceleration and instantaneous acceleration holds.

Note 1.0.6

Kinematics Equations

There are 4 kinematics equations that are the key to solve all 1D kinematics problems on the exam. Note that these equations only hold true in situations with constant acceleration. Let v_i, v_f be the initial and final velocities, t be the time taken in the journey, Δx be the displacement during the journey, and a be the acceleration. The kinematics equations are all shown below.

$$v_f = v_i + at \tag{1.1}$$

$$\Delta x = \frac{v_i + v_f}{2} \cdot t \tag{1.2}$$

$$\Delta x = v_i t + \frac{1}{2}at^2 \tag{1.3}$$

$$v_f^2 = v_i^2 + 2a\Delta x \tag{1.4}$$

Out of the 5 variables $t, v_i, v_f, a, \Delta x$, you must know 3 values to find the rest using the equations above.
Also, note that other forms of the variables can be used. For example, sometimes people label displacement as d. You can often see v_0 or v_o for the initial velocity.

Note 1.0.7

Note that the acceleration due to gravity is g. The exact value of g is around $9.81\frac{m}{s^2}$. In AP Physics 1, the answer choices might be in terms of g. Sometimes, you will have to use 9.81 instead. On top of that, sometimes the problem will allow you to round g and use $10\frac{m}{s^2}$

You also need to know how to read kinematics graphs.
In kinematics graphs, this is all you need to know:

When you have a velocity-time graph, the slope at any point is the acceleration at that specific time, and the area under the graph up to a specific time t is the displacement occurred up to that time. In simple words, the area under a velocity-time graph is the displacement.

When you have a position-time graph, the slope at any point is the velocity at that specific time.

In general, remember that d and Δx are both used to represent displacement.

Problem 1.0.8 — How long would it take a car, starting from rest and accelerating uniformly in a straight line at $5\frac{\text{m}}{\text{s}^2}$, to cover a distance of 200m?

Solution: From equation (1.3), we have

$$d = v_i t + \frac{1}{2}at^2$$

$$200 = 0 + \frac{1}{2}(5)t^2$$
$$\implies t = \sqrt{80} \approx \boxed{9} \text{ seconds.}$$

Problem 1.0.9 — A ball is dropped off a cliff and strikes the ground with an impact velocity of $30\frac{\text{m}}{\text{s}}$. How high was the cliff?

Solution: From equation (1.4), we have

$$v_f^2 = v_i^2 + 2ad$$
$$30^2 = 0^2 + 2gd$$
$$\implies d = \frac{30^2}{2g} = \boxed{45}\text{meters.}$$

Problem 1.0.10 — A stone is thrown vertically upward with an initial speed of $5\frac{\text{m}}{\text{s}}$. What is the velocity of the stone 3 seconds later?

Solution: From equation (1.1), we have

$$v_f = v_i + at$$
$$v_f = 5 + (-g)(3)$$
$$\implies v_f = 5 - 3g = \boxed{-25\frac{\text{m}}{\text{s}}}.$$

Problem 1.0.11 — A car traveling at a speed of v_i applies its brakes, skidding to a stop over a distance of x meters. Assuming that the deceleration due to the brakes is constant, what would be the skidding distance of the same car if it were traveling with twice the initial speed?

Solution: From equation (1.4),

$$v_f^2 = v_i^2 + 2ad.$$

Since v_f in this scenario is 0, we have

$$d = \frac{v_i^2}{2a}.$$

Note that doubling v_i leads to increasing d by a factor of 4. So the answer is

$$\boxed{4x} \text{ meters.}$$

Problem 1.0.12 — A rocket initially moves at constant velocity v at $t = 0$. At time t_1, it starts to accelerate upwards with acceleration a. If the rocket moves until time t_2, how much distance does it travel?

Solution: Until time t_1, the rocket moves with constant velocity, so the distance traveled during that time interval is vt_1. The rocket also accelerates for time $t_2 - t_1$, so by equation (1.3), during that time interval, it travels a distance

$$d = v_i t + \frac{1}{2}at^2 = v(t_2 - t_1) + \frac{1}{2}a(t_2 - t_1)^2.$$

The total distance traveled is

$$vt_1 + v(t_2 - t_1) + \frac{1}{2}a(t_2 - t_1)^2$$

$$= vt_1 + vt_2 - vt_1 + \frac{1}{2}a(t_2 - t_1)^2$$

$$= \boxed{vt_2^2 + \frac{1}{2}a(t_2 - t_1)^2} \text{meters.}$$

Remark: If you want a challenge ahead of the AP Physics 1 level, there's a simpler way to come up with the answer to this problem. Try to find it!

Also, please use the right subscripts in problems! For example, v_f typically denotes final velocity while v_i or v_0 typically denotes initial velocity. This is important since problems can involve multiple velocities. You need to know what numerical value represents what specifically. This will reduce the chances of making an error!

Now it's time for some **2D Kinematics**

Before I introduce some techniques, just remember that projectile motion/2D Kinematics problems is all about solving 2 1D kinematics problems at once. The reason is that you have motion occurring in two directions. However, even though there is motion occurring in both directions, you can work separately with quantities in the x and y direction!

Note 1.0.13

Projectile Motion Basics
Projectile motion happens in a parabolic manner. The path of the object looks like a parabola.

The best way to deal with such problems is to work separately with quantities in the horizontal direction and quantities in the vertical direction. Also, remember that gravity always points downward in the y direction. The acceleration in the y direction is equal to $g(9.8)$.

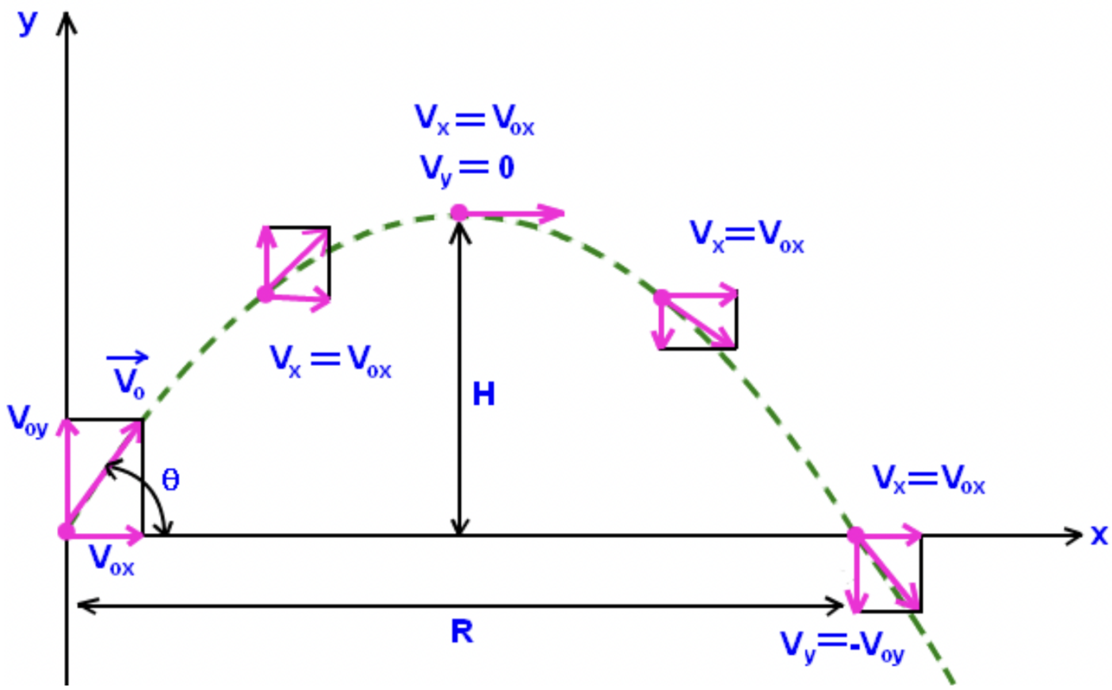
In projectile motion problems, one variable that is always the same in the x and y direction in AP Physics is t (time). Thus, solving for time is extremely important since it gives you information for a specific variable in both the x and y-direction.

Note 1.0.14

At the top of the projectile motion (at the very tip), the vertical velocity is 0. The horizontal velocity, however, remains the same always.

The horizontal velocity can change during the motion if we're dealing with rotated reference frames, but note that this is way beyond the scope of AP Physics, so you will never have to work with such a case.

Full credit to the physics and universe website for the image below.



Note 1.0.15

The image with the arc represents how a ball thrown in projectile motion would move. The arrow at the very top that is pointing to the right shows the velocity of the ball. There will only be one component in the horizontal direction, but no vertical velocity at all.

Also, note that in projectile motion, the object will be thrown at some angle θ with respect to the horizontal direction.

Using this information, we can write out our variables and see which ones we know (seen in the table below). Now we can calculate the time to reach the top point of the projectile motion and even find the displacement of it (seen in the equation after the table).

$v_{ox} = v_o \cos\theta$	$v_{oy} = v_o \sin(\theta)$
$v_x = v_o \cos\theta$	$v_y = 0$
$a_x = 0$	$a_y = g$
t	t
Δx	Δy

Note that v_{ox} and v_{oy} represent the initial velocity in the x and y-direction while v_x and v_y represent the velocity at the very top of the arc.

Now, we can try to find time t when we're only given the initial velocity v_o and the angle θ.
We will apply kinematics equations for the motion in the **vertical** direction to find time to reach the top of the arc.
Since $v_{oy} = v_o \sin\theta$, $v_y = 0$, and $a_y = g$, we can apply the kinematics equation shown

below.

$$v_f = v_i + at$$

However, we will rewrite this equation in terms of the variables we have. For example, our final velocity at the top in the y-direction is v_y, so we should use that instead of v_f.

$$v_y = v_{oy} + a_y t$$

Now, we can plug in our values to get

$$0 = v_o \sin(\theta) + gt$$

Just remember that in this problem, upwards direction is considered to be positive. This means that the acceleration in the y-direction is actually $-g$ (since it opposes the velocity that is upwards)

Now, we can solve for t to find that $\boxed{t = \frac{v_o \sin(\theta)}{g}}$

We can use another kinematics equation to find the vertical displacement.

$$v_y^2 = v_{oy}^2 + 2a\Delta y$$

$$0 = (v \sin\theta)^2 - 2g\Delta y$$

$$\boxed{\Delta y = \frac{v^2 \cdot (\sin\theta)^2}{2g}}$$

Note 1.0.16

Tips for Solving Projectile Motion Problems

1. Separate the variables in the x direction from the y direction. You are basically solving two separate kinematics problems, but it's part of one.

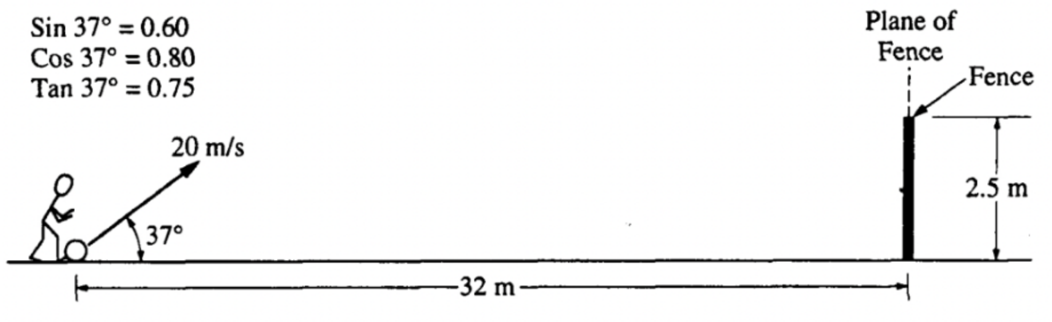
2. Write out all your variables as shown below. It's important to note that gravity is always in the y direction.

$v_{ox} = v\cos\theta$	$v_{oy} = v\sin\theta$
$v_x = v\cos\theta$	$v_y = 0$
$a_x = 0$	$a_y = g$
t	t
Δx	Δy

3. In most projectile motion problems, remember that at the very top of the motion, the velocity in the y direction is 0. Also remember that the horizontal component of velocity stays the same (initial = final) in 99.99 % of the projectile motion problems in AP Physics 1.

4. Remember that t (time) is the same for both the x and y axis.

Problem 1.0.17 — 1994 AP Physics B FRQ



A ball of mass 0.5 kilogram, initially at rest, is kicked directly toward a fence from a point 32 meters away, as shown above. The velocity of the ball as it leaves the kicker's foot is 20 meters per second at an angle of 37° above the horizontal. The top of the fence is 2.5 meters high. The ball hits nothing while in fight and air resistance is negligible.

(a) Determine the time it takes for the ball to reach the plane of the fence.

(b) Will the ball hit the fence? If so, how far below the top of the fence will it hit? If not, how far above the top of the fence will it pass?

Solution to part a: In this problem, we will first write out all the variables that we have.

$$\begin{array}{cc} v_{ox} = v_0 \cos\theta & v_{oy} = v_0 \sin\theta \\ v_x = v_{ox} & v_y = ? \\ a_x = 0 & a_y = g \\ \text{t} & \text{t} \\ \Delta x = 32 & \Delta y = ? \end{array}$$

In this problem, we don't know the final velocity in the y direction (v_y). The final velocity now occurs when the times hits the plane of the fence. It's no longer the situation when we considered the final velocity to be at the top of the arc (which was 0, or at the end of a perfectly symmetrical projectile problem).

However, we seem to have much more information about the variables in the x direction. Thus, let's investigate the motion in the x-direction.

We know that $v_x = v_{ox}$ because the velocity in the horizontal direction remains the same in projectile motion.

Also, v_0 is simply the initial velocity which is $20\frac{\text{m}}{\text{s}}$.

We can apply our kinematics equations to find t (time). We'll do this in the x direction.

Since velocity in the x direction is constant, acceleration in the x direction is 0. We can simply apply the equation $\Delta x = vt$ (which is true when the acceleration is 0).

Since Δx is 32 and $v = v_{ox} = 20\cos(37) = 16$, $t = \frac{32}{16} =$ **2 seconds**.

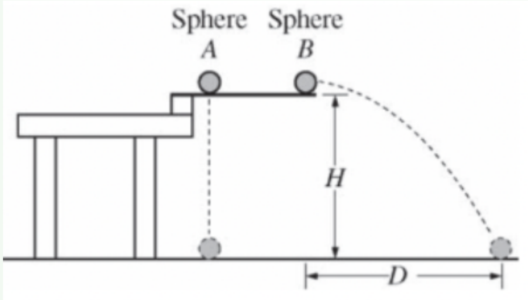
Solution to part b: Since we know the time of the motion, we will use kinematics in the y direction to see the position of the ball in the vertical direction after this time period.

$$\begin{array}{cc} v_{ox} = v_0 \cos\theta & v_{oy} = v_0 \sin\theta \\ v_x = v_{ox} & v_y = ? \\ a_x = 0 & a_y = g \\ t = 2 & t = 2 \\ \Delta x = 32 & \Delta y = ? \end{array}$$

Since $v_{oy} = v_0 \sin\theta = 20\sin(37) = 12.036$, $a_y = g = -9.8$, and $t = 2$, we can use the equation $y = v_{oy}t + \frac{1}{2}a_y t^2$.

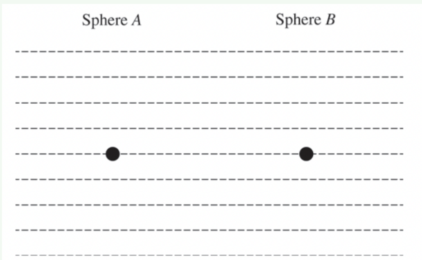
Plugging in our values gives $y = 12.036 \cdot 2 - \frac{9.8 \cdot 2^2}{2}$ which is 4.472 m. Since this is more than the height of the fence (2.5 m), the ball lands above the top. The distance above the top is $4.472 - 2.5 = \mathbf{1.972}$ **m**.

Problem 1.0.18 — 2016 AP Physics 1 FRQ

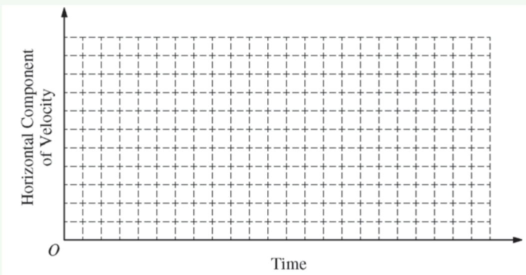


Two identical spheres are released from a device at time $t = 0$ from the same height H, as shown above. Sphere A has no initial velocity and falls straight down. Sphere B is given an initial horizontal velocity of magnitude v_0 and travels a horizontal distance D before it reaches the ground. The spheres reach the ground at the same time t_f, even though sphere B has more distance to cover before landing. Air resistance is negligible.

(a) The dots below represent spheres A and B. Draw a free-body diagram showing and labeling the forces (not components) exerted on each sphere at time $\frac{t_f}{2}$.

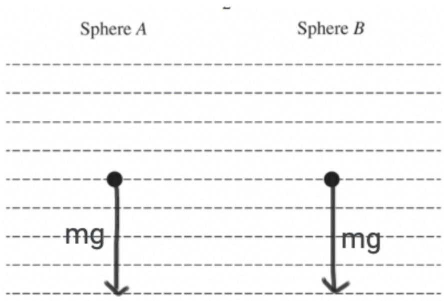


(b) On the axes below, sketch and label a graph of the horizontal component of the velocity of sphere A and of sphere B as a function of time.



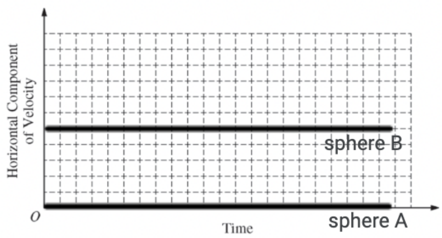
(c) In a clear, coherent, paragraph-length response, explain why the spheres reach the ground at the same time even though they travel different distances. Include references to your answers to parts (a) and (b).

Solution to part a: The only forces on each sphere are the gravitational force. The gravitational force on each sphere will be drawn with the same length since their masses are the same.



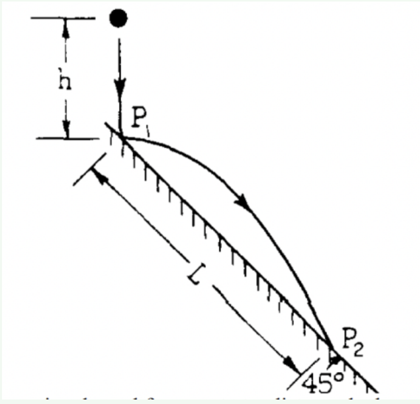
Solution to part b: Sphere A has no horizontal velocity. The reason is that it is simply in free fall. There is only vertical velocity.

On the other hand, Sphere B will have horizontal velocity. Since there is no acceleration in the x-direction, the horizontal velocity on sphere B will remain constant.



Solution to part c: We must know that the horizontal motion is independent from the vertical motion. If we consider motion in the vertical direction, then the initial velocity for both spheres is the same. The acceleration will be g for both. Since the vertical displacement for both is the same (h), the time it will take for both spheres to fall will also be the same. Both spheres reach the ground at the same time since they travel the same vertical displacement with the same initial vertical velocity.

Problem 1.0.19 — 1979 AP Physics B FRQ



A ball of mass m is released from rest at a distance h above a frictionless plane inclined at an angle of 45° to the horizontal as shown above. The ball bounces horizontally off the plane at point P_1 with the same speed with which it struck the plane and strikes the plane again at point P_2. In terms of g and h, determine each of the following quantities:

(a) The speed of the ball just after it first bounces off the plane at P_1.

(b) The time the ball is in flight between points P_1 and P_2.

(c) The distance L along the plane from P_1 to P_2.

(d) The speed of the ball just before it strikes the plane at P_2.

Solution to part a: For the motion towards point P_1, the ball starts with initial velocity v_i that is 0 (since it's simply dropped from rest).

The vertical displacement is h. The acceleration in this direction is g.
We can use the kinematics equation $v_f^2 = v_i^2 + 2a\Delta y$

We can plug in our variables to find that $v_f^2 = 0 + 2gh$
Now, we can take the square root of both sides to find $v_f = \sqrt{2gh}$

Solution to part b: The problem says that the ball bounces horizontally off the plane. This means that the initial vertical velocity is 0 for the motion from P_1 to P_2

The vertical displacement between point P_1 and P_2 is Δy which is $L\sin(45) = \frac{L\sqrt{2}}{2}$.
The acceleration in this direction is g.
We can now use the equation $\Delta y = v_i t + \frac{1}{2}at^2$

Plugging in our variables gives that $\frac{L\sqrt{2}}{2} = 0 + \frac{1}{2}gt^2$
We can isolate t^2 to find that $t^2 = \frac{L\sqrt{2}}{g}$
However, we need to replace L with something since our expression must be in terms of g and h.

Thus, we must consider motion in the horizontal direction. In the horizontal direction, the velocity will always be $\sqrt{2gh}$. The ball got this velocity due to its initial drop when it was released from a height of h. This velocity will stay the same (since in projectile motion, horizontal velocity is constant).

We can use the equation $\Delta x = v_x t$

We know that $\Delta x = L\cos(45) = \frac{L\sqrt{2}}{2}$

We can plug this in to find that $\frac{L\sqrt{2}}{2} = \sqrt{2gh}t$

Dividing both sides by $\sqrt{2gh}$ gives that

$$t = \frac{L\sqrt{2}}{2\sqrt{2gh}}$$

Since we already have an expression for t^2 from the vertical motion, we can square the time we found for the horizontal motion. Then, we can equate both expressions.

Squaring $t = \frac{L\sqrt{2}}{2\sqrt{2gh}}$ gives that $t^2 = \frac{L^2}{4gh}$
We can set this equal to the other expression which was $t^2 = \frac{L\sqrt{2}}{g}$
Equating both expressions gives that $\frac{L^2}{4gh} = \frac{L\sqrt{2}}{g}$

Now, we can cancel out some terms to find that $L = 4h\sqrt{2}$

Now, we can plug in $L = 4h\sqrt{2}$ into $t^2 = \frac{L^2}{4gh}$.

$$t^2 = \frac{(4h\sqrt{2})^2}{4gh} = \frac{8h}{g}$$

We can take the square root of both sides to find that $t = \sqrt{\frac{8h}{g}}$

Solution to part c: In part b, we already found that $L = 4h\sqrt{2}$. On the real AP exam, you probably won't have to find the answer for a later part in an earlier part. However, if you have to, it would be beneficial to show the work again to guarantee the points. For now, I will skip the solution to this part since it was already show in part b.

Solution to part d: To find the speed of the ball at point P_2, we can find the velocity in the x-direction and y-direction.

The reason is that the total speed will be $\sqrt{v_x^2 + v_y^2}$. We must account for both components of velocity!

In the x-direction, velocity will stay the same since there is no acceleration. We already found in part a that $v_x = \sqrt{2gh}$.

In the y-direction, we know that the acceleration is g. On top of that, the initial vertical velocity is 0. We also know from part b that the time is $t = \frac{8h}{g}$.
We can plug this into the equation $v_y = v_{iy} + at$
Plugging in our variables gives $v_y = 0 + g \cdot \sqrt{\frac{8h}{g}} = \sqrt{8gh}$

Since $v_x = \sqrt{2gh}$ and $v_y = \sqrt{8gh}$, we can plug this into $\sqrt{v_x^2 + v_y^2}$ to find the speed. Doing so gives that the speed is $\sqrt{(\sqrt{2gh})^2 + (\sqrt{8gh})^2} = \sqrt{10gh}$

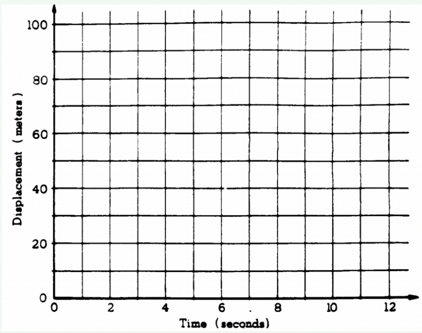
Problem 1.0.20 — 1982 AP Physics B FRQ
The first 10 meters of a 100-meter dash are covered in 2 seconds by a sprinter who starts from rest and accelerates with a constant acceleration. The remaining 90 meters are run with the same velocity the sprinter had after 2 seconds.

(a) Determine the sprinter's constant acceleration during the first 2 seconds.

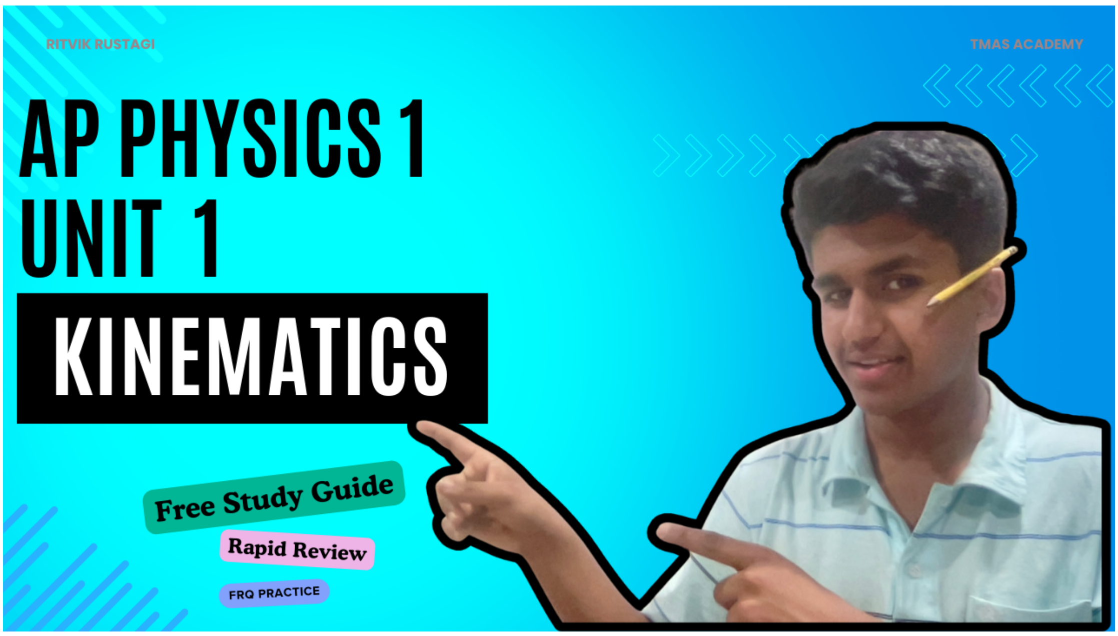
(b) Determine the sprinter's velocity after 2 seconds have elapsed.

(c) Determine the total time needed to run the full 100 meters.

(d) On the axes provided below, draw the displacement-time curve for the sprinter.



Solution: Video Solution



Overall, unit 1 doesn't have many FRQs to practice with. The reason is that many problems combine topics from other units. It is very unlikely to find an FRQ that only has kinematics concepts in it. You simply need to know your formulas really well and how to apply them.

Unit 2 Dynamics

Have you ever tried to push a shopping cart? Have you ever tried to push a car? Certainly, pushing the shopping cart is a lot easier. The reason is that it has less inertia. Inertia is the tendency of objects at rest to stay at rest. Of course a car will have a lot more inertia. It's much heavier when compared to a shopping cart.

In this unit, you will deal with such scenarios and think about the forces that are involved.

Newton's Laws of Motion are a crucial topic on the AP exam. There are three laws that you must be familiar with.

Note 2.0.1

Newton's First Law

Newton's First Law states that an object at rest always stays at rest unless an external force acts on it. It also states that an object at constant velocity continues to move at that same velocity unless acted by an external force.

If we logically think about Newton's First Law, then it should make sense. Of course an object at rest will always remain at rest unless a certain force acts on it. For example, a shopping cart at a store will always remain at rest. However, it will start moving if a shopper applies a force to push it.

Note 2.0.2

Newton's Second Law

Newton's Second Law states that a force acted on an object will lead to acceleration. This leads to a crucial formula:

$$F_{net} = ma$$

It states that net force = mass times acceleration.

To conceptually imagine what net force is, think about a tug of war competition. If there is one person on each side, and both are equally strong, then the rope will not move. The reason is that both people will apply the same force. However, if one person is stronger, then they will apply a stronger force. That is when the rope will start to accelerate as the stronger person pulls the rope towards them.

Problem 2.0.3 — If the force applied on a box is 18 N, and the mass of the box is 6 kg, what is the acceleration?

Solution: Since we know that $F = ma$ from Newton's Second Law, we can use that and plug in our given numbers.

$$18 = 6 \cdot a$$

Dividing both sides by 6 gives us an acceleration of 3 $\frac{m}{s^2}$

Note 2.0.4

Newton's Third Law

Newton's third law states that for every action, there is always a reaction. This means that if you apply a force on something, that object will apply the same exact force back on you.

A Newton's third law pair comprises of specifically two objects interacting, both exerting a force of equal magnitude on each other.

Problem 2.0.5 —

a. If Bob applies a force of 60 N towards the right onto a wall, what force does the wall apply onto Bob? Indicate magnitude and direction.

b. If Bob's mass is 30 kg, what is his acceleration?

Solution to part a: From Newton's Third Law, since Bob applies a force of 60N to the right, we know that the wall must apply that exact same force onto him of 60N. However, it is applied to the left because it is a reaction force.

Solution to b: Since we know that the force applied onto Bob is 60 N to the left (or -60 N since left is negative), we can use that and Newton's Second Law.

$$F_{net} = m \cdot a$$

$$-60 = 30 \cdot a$$

Dividing both sides by 2 gives $a = -2$ m/s^2

Now let's discuss friction.
Friction is resistance that an object might face from another object or a surface. It opposes the relative motion of two objects.

Note 2.0.6

Static friction is the friction force on an object that does not slide relative to a surface.

Kinetic friction is the friction force on an object that does slide relative to a surface.

Note 2.0.7

Friction Analyzed Conceptually

You might still be confused in differentiating these two types of frictions. This short paragraph should clear it up.

When you try to push an object in real life, it does not immediately move. You start by applying a force of 0 Newtons and increase that force. After a few seconds, when your force is high enough, the object will start to accelerate. However, why does it not accelerate until it hits that certain amount? The reason is that static friction has prevented you from doing so. The force that you must apply must overtake the maximum force of static friction. Once that happens, kinetic friction will be the force that opposes your motion.

Static friction force is represented as f_s.
$f_s \leq \mu_s N$ is the relationship that we must know. μ_s represents the coefficient of static friction while N is the normal force.

The maximum static friction is $\mu_s N$. This doesn't mean that the static friction has to have that same magnitude. It can indeed be less than the maximum value.

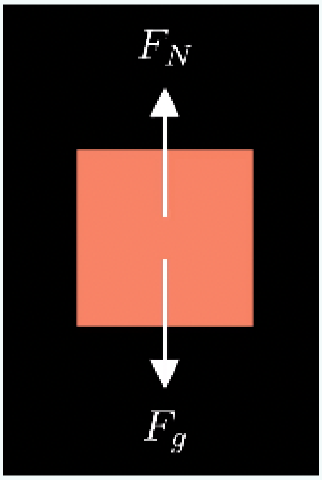
The numerical value for the kinetic friction force is calculated in the same way by multiplying the friction coefficient (kinetic friction coefficient, not static friction coefficient) by the normal force. In general, the kinetic friction coefficient will be smaller than the static friction coefficient!

To truly solve problems from this unit, you must do every single practice problem at the end of this unit. It's extremely important since that section will go through all the types of problems you can encounter on the AP exam.

Note 2.0.8

Free Body Diagrams
Free Body Diagrams are one of the most crucial part of this entire course. You must draw a free body diagram to be able to understand a problem and simplify it along with minimizing the number of errors that you make.

You draw a free body diagram by labelling all forces that act on your object and indicate the direction with an arrow. This is extremely important especially when you use Newton's Second Law to find the acceleration.



The above image is a good example of a free-body diagram. There is an object with normal force which points perpendicular to the surface. There is gravitational force which points downwards. You might still be confused, so the best way is to practice a few crucial problems that commonly show up for this unit.

Note 2.0.9

Atwood's Machines
Atwood's Machines are a type of device that commonly show up on the AP Physics C: Mechanics exam. In this unit, our Atwood's machines will have a **massless pulley**. There will be a lightweight string around it for which **the tension force remains constant throughout the entire string**. Often, the string connects two objects and we work from there using Newton's Laws to analyze the situation.

Important Tips for Solving Atwood's Machines
Since one object goes down while the other goes up, it's important to adapt the proper sign convention.
Sign convention can be something such as positive when a force makes the pulley move clockwise, but negative when it makes it counterclockwise.

We will also analyze each object attached on both sides of the pulley **separately**. We will write out separate equations for them using Newton's Second Law.

Problem 2.0.10 — Atwood's Machines

We have a pulley with two blocks of mass M and m attached. The pulley is frictionless and massless. Find an expression for the acceleration of the blocks once released? Now, find the acceleration when $M = 30$kg and $m = 20$kg.

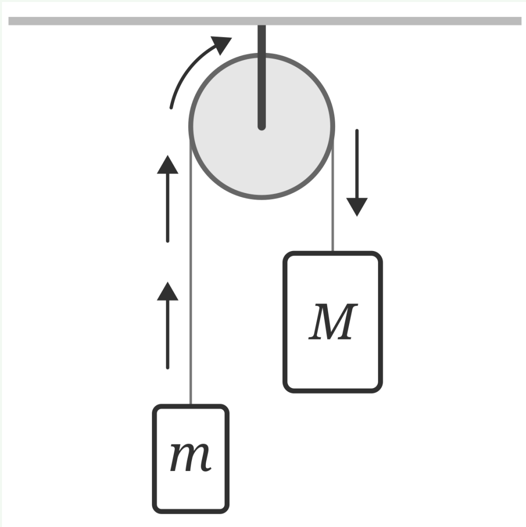
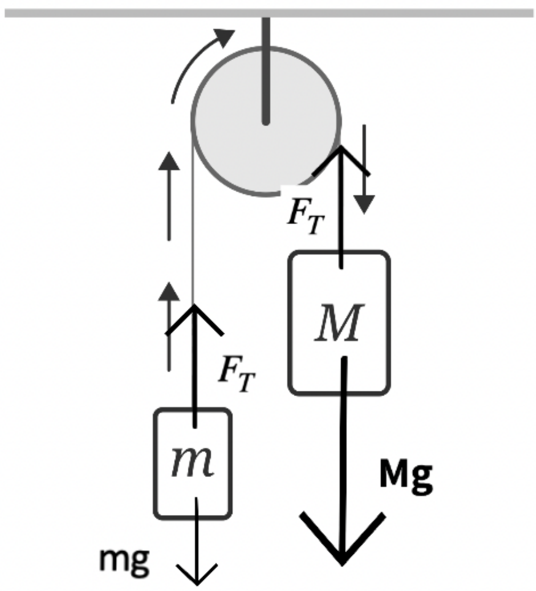


Image Credits: Phyley Website

Solution: In this problem, the first step will be to make a free-body diagram.



Now using our free body diagram, we can write out our two separate equations for each individual object.

Note, that F_T represents the tension force while M represents the mass of the heavier object and m represents the mass of the lighter object.

Newton's Second Law on heavy block: $Mg - T = Ma$

Newton's Second Law on light block: $T - mg = ma$

Now since the tension is a missing variable, we can add both equations since that eliminates the tension variable.
Adding both equations gives $Mg - mg = Ma + ma$

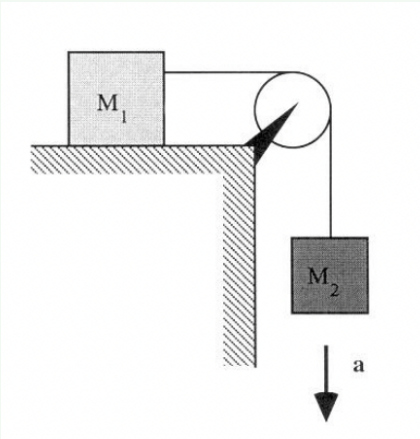
We can factor to get $g(M - m) = a(M + m)$

Now we simply divide both sides by $M + m$ to isolate a (acceleration) and this gives $a = \frac{g(M-m)}{M+m}$
Plugging in our values of M (30) and m (20) gives $\frac{10g}{50}$ which simplifies to $\frac{g}{5}$ which is 1.96 $\frac{\text{m}}{\text{s}^2}$

Bonus Question: What is the value of the tension force in the pulley above?

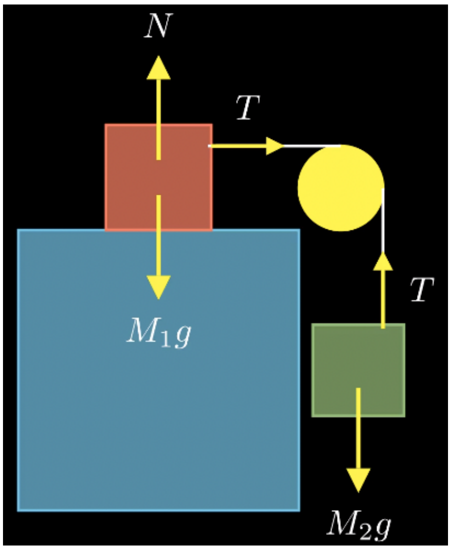
Solution: Using our value of the acceleration, we plug that back into one of the original equations such as $T - mg = ma$
We solve for T by adding mg to both sides and
$T = m(a + g)$
We plug in our values of m (20) and a (1.96) and g (9.8)
$T = 20(1.96 + 9.8) = 235.2$ N

Problem 2.0.11 — Pulley on a Table



Assuming that the table that block M_1 is sitting on is frictionless, and the pulley is massless and frictionless, what is the acceleration of the blocks as M_2 slides down?

Solution: In this problem, we will first draw our free body diagram like always. After that, we'll write out our equations using Newtons Laws. Then, we'll solve them.



We don't need to write an equation for the forces in the vertical direction on M_1 since the normal force balances out the gravitational force.

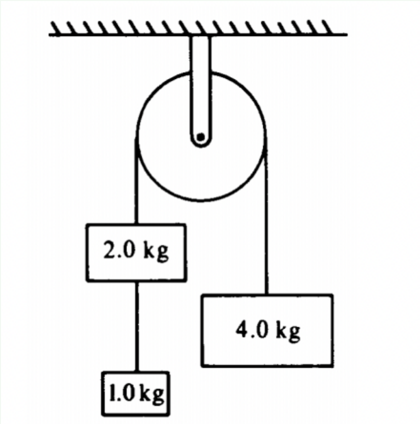
(Forces on M_2:) $M_2g - T = M_2a$
(Forces on M_1:) $T = M_1a$

We don't have our value of tension, so we can add both equations to eliminate tension.

$M_2g = M_2a + M_1a = a(M_2 + M_1)$

We can divide both sides by $M_2 + M_1$ to find acceleration. Doing so gives
a (acceleration) $= \frac{M_2g}{M_2+M_1}$

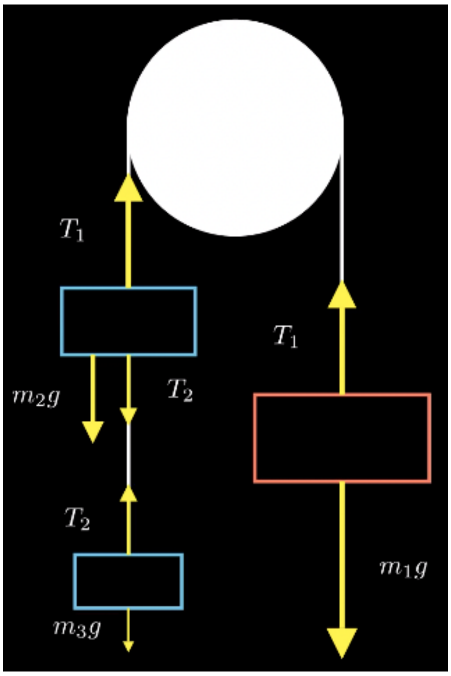
Problem 2.0.12 — Challenging Atwoods Machine



Three blocks of masses 1.0, 2.0, and 4.0 kilograms are connected by massless strings, one of which passes over a frictionless pulley of negligible mass, as shown above. Calculate each of the following.

a. The acceleration of the 4-kilogram block
b. The tension in the string supporting the 4-kilogram block
c. The tension in the string connected to the l-kilogram block

Solution to a: To find the acceleration for the 4kg block, we must draw a free body diagram again.



Obviously the pulley will cause the objects move to the right and downwards because the block of 4 kg is heavier than the other two blocks combined. Thus, we will assume that the clockwise direction is positive.

Before we write out equations using Newtons Laws, we can eliminate T_2 (tension force between the strings connecting the two lighter blocks) because it is an internal force if we consider the two blocks together (the two lighter blocks act like a larger block).

(Forces on m_1:) $m_1 g - T_1 = m_1 a$
(Forces on m_2 and m_3 as a system): $T_1 - (m_2 + m_3)g = (m_2 + m_3)a$

Since we don't know the value of T_1, we can simply add the two equations to cancel T_1 and get

$$(m_1 - m_2 - m_3)g = (m_2 + m_3)a$$

Dividing both sides by $m_2 + m_3$ gives

$$a = \frac{(m_1 - m_2 - m_3)g}{m_2 + m_3}$$

Substituting our masses ($m_1 = 4$, $m_2 = 2$, $m_3 = 1$) gives that

$$a = \frac{g}{5} = 1.96 \frac{\mathrm{m}}{\mathrm{s}^2}$$

Solution to b: The tension in the string supporting the 4-kg block is just T_1. We can substitute our value of acceleration into the equation we found in part a for the forces on m_1 which was $m_1 g - T_1 = m_1 a$

We can rearrange it to get $T_1 = m_1(g - a)$
Substituting 4 for m_1, 9.8 for g, and 1.96 for a gives 31.36 **N**

Solution to c: The tension in the string supporting the 1-kg block is T_2. To find this, we can't work with both blocks m_3 and m_2 as a system. We have to work separately with the system that only contains block m_3.

The forces on block m_3 are the gravitational force and the tension force.

(Forces on block m_3:) $T_2 - m_3 g = m_3 a$

$$T_2 = m_3(g + a)$$

Substituting 1 for m_3 and 1.96 for a gives that the tension force on the 1 kg block is $1(9.8 + 1.96)$ which is 11.76 N.

Note 2.0.13

Apparent Weight
Apparent weight is the weight that the person feels, but it often differs from their actual weight. In most problems, the normal force will be the apparent weight. The reason is that the normal force is the force that will be exerted on a person by something (such as the ground or an elevator), and that's the force that will be felt by the person, causing them to think that it's their actual weight when it's truly their apparent weight.

Most apparent weight problems involve an elevator that accelerates.

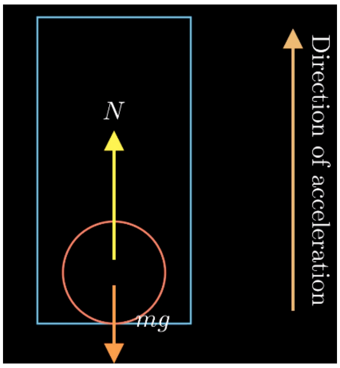
Problem 2.0.14 —
a. If an elevator accelerates **up** with a person standing on a scale, will the person's weight shown by the scale (apparent weight) be less than their actual weight or more?

b. If an elevator accelerates **down** with a person standing on a scale, will the person's weight shown by the scale (apparent weight) be less than their actual weight or more?

c. If an elevator moves at constant speed in any direction, will the person's weight shown by the scale (apparent weight) be less than their actual weight or more?

d. When the elevator is accelerating downwards, when will the person feel complete weightlessness?

Solution to a:

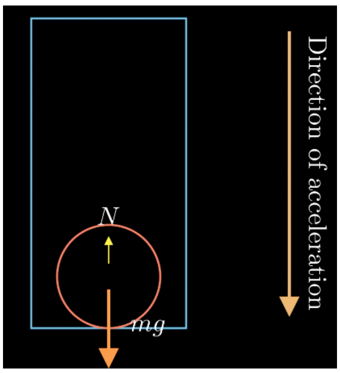


Using the free-body diagram above, we can write an equation for the person. The forces on it are the normal force from the scale (which represents the apparent weight) and the gravitational force. The normal force has a larger magnitude than the gravitational force since the elevator is accelerating up.

$$N - mg = ma$$
$$N = m(g + a)$$

Clearly from the normal force we found, the apparent weight when the elevator accelerates up is greater than the actual weight since the actual weight would be mg, but apparent weight is $m(g + a)$ which is also equal to $mg + ma$.

Solution to b:

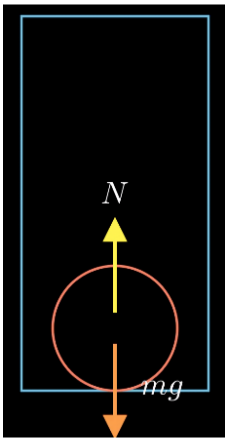


In this problem, we will again write out equation using Newton's Second Law and find the value of N (our normal force which is also the apparent weight). In this situation, the normal force will have a lower magnitude than gravitational force since the elevator is accelerating downwards.

$$mg - N = ma$$
$$N = m(g - a)$$

Clearly for the normal force in this case, the normal force (apparent weight) is less than the actual weight which is mg since we subtract the value of ma from it. Thus, when the elevator accelerates down, the apparent weight is less than the actual weight.

Solution to c:



In this case, we will again write our equation using Newton's Second Law.

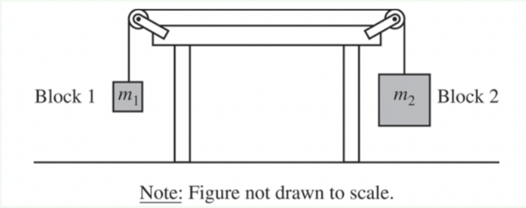
$$mg - N = ma$$

$$\text{Since acceleration is } 0, mg - N = 0$$

Simplifying gives $N = mg$ which means that the normal force (or apparent weight) is equivalent to the actual weight. Thus, when the elevator does not accelerate, the apparent weight is the actual weight.

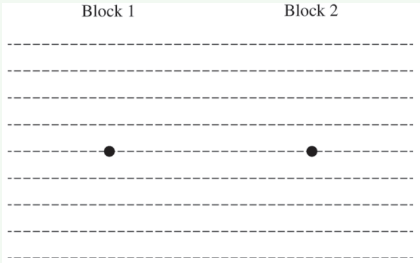
Solution to d:
Weightlessness is felt when the normal force is 0. In that case, we will now again use the equation $mg - N = ma$.
Plugging in 0 for N gives $mg = ma$ which simplifies to $a = g$.
This means that the elevator must accelerate downwards at g (9.8) which means that it must be in **freefall.** Thus, for a person in an elevator to feel weightlessness, the elevator must be in freefall.

Problem 2.0.15 — 2015 AP Physics 1



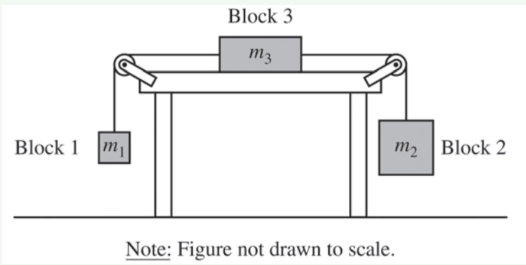
Two blocks are connected by a string of negligible mass that passes over massless pulleys that turn with negligible friction, as shown in the figure above. The mass m_2 of block 2 is greater than the mass m_1 of block 1. The blocks are released from rest.

(a) The dots below represent the two blocks. Draw free-body diagrams showing and labeling the forces (not components) exerted on each block. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces.



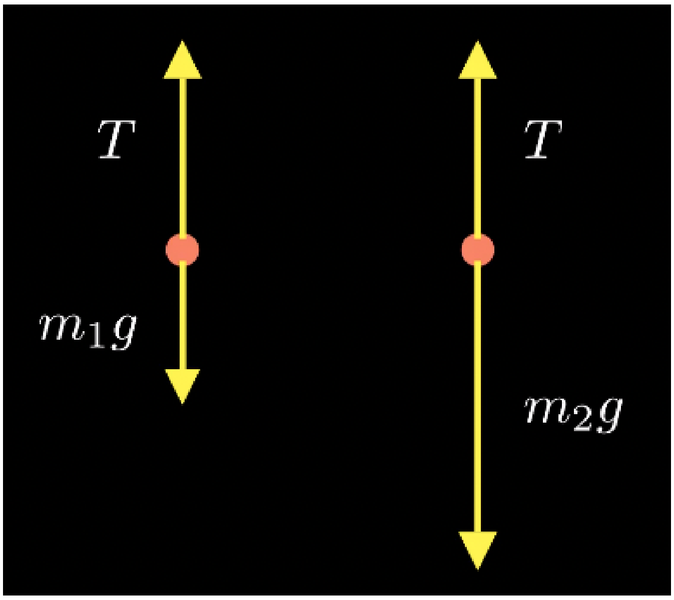
(b) Derive the magnitude of the acceleration of block 2. Express your answer in terms of m_1, m_2, and g.

Block 3 of mass m_3 is added to the system, as shown below. There is no friction between block 3 and the table.



(c) Indicate whether the magnitude of the acceleration of block 2 is now larger, smaller, or the same as in the original two-block system. Explain how you arrived at your answer.

Solution to part a: Since m_2 is greater than m_1 in mass, the gravitational force vector for m_2 will be longer than the force vector for m_1. On the other hand, the tension force for both masses will be pointing upwards and will have the same length since tension in a string is the same.



Solution to part b: We can write out an equation using Newton's Second Law ($F = ma$) for each block.

The two blocks will slide towards the direction where the heavier block is.
The equation for mass m_2 is $m_2g - T = m_2a$
Similarly, the equation for block 1 with mass m_1 is $T - m_1g = m_1a$

Now, we can add both of these equations to get

$$g(m_2 - m_1) = (m_1 + m_2)a$$

If we divide both sides by $m_1 + m_2$, then we get that $a = \frac{g(m_2-m_1)}{m_1+m_2}$

Solution to part c: Now, there are technically 2 ropes. One is connecting block 2 and block 3 while the other connects block 1 and block 2.

Now, we can simply consider the forces on the 3 block system as a whole. Tension in both strings will now be an internal force. This means that the net force is simply $g(m_2 - m_1)$
Since we know F_{net}, we simply need to find the mass of this entire system to find acceleration.
The mass of all three blocks combined is simply $m_1 + m_2 + m_3$

$$\text{This means that the acceleration of the 3 block system is } \frac{g(m_2 - m_1)}{m_1 + m_2 + m_3}$$

Since there is an increase of mass in the denominator, the acceleration will now be less on block 2.

Problem 2.0.16 — 1996 AP Physics B FRQ

A 300-kg box rests on a platform attached to a forklift, shown above. Starting from rest at time = 0, the box is lowered with a downward acceleration of 1.5m/s^2

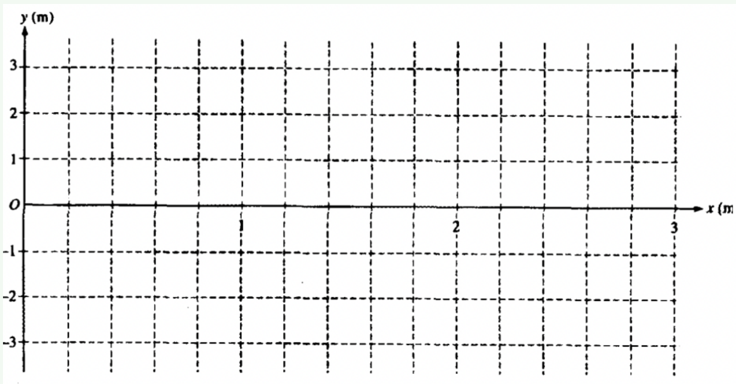
(a) Determine the upward force exerted by the horizontal platform on the box as it is lowered. At time $t = 0$, the forklift also begins to move forward with an acceleration of 2m/s^2 while lowering the box as described above. The box does not slip or tip over.

(b) Determine the frictional force on the box.

(c) Given that the box does not slip, determine the minimum possible coefficient of friction between the box and the platform.

(d) Determine an equation for the path of the box that expresses y as a function of x (and not of t), assuming that, at time $t = 0$, the box has a horizontal position $x = 0$ and a vertical position $y = 2$ m above the ground, with zero velocity.

e. On the axes below sketch the path taken by the box



Solution to part a: The upward force exerted by the horizontal platform is the normal force N.
The only two forces on the box are the gravitational force and normal force. The gravitational force has a larger magnitude since the box accelerates down.

The Newton's Second Law equation is $F_{net} = ma$
We can use this to find $F_{net} = mg - N = ma$
This means that $N = mg - ma = m(g - a)$

In this problem, the downwards direction is taken to be positive.
Since $m = 300$ kg and $a = 1.5\frac{m}{s^2}$, we can find that $N = 300(9.8 - 1.5) = 2490$ N.

Solution to part b: Friction is the only horizontal force on the block. Thus, it must be the force causing it to accelerate with respect to the ground.

This means that $F_f = ma_x$
Since $m = 300$ kg and $a_x = 2\frac{m}{s}$, we know that $F_f = 300 \cdot 2 = 600$ N.

Solution to part c: Since the box doesn't slip, we will use the maximum possible value of static friction. We know that $F_s \leq uN$

We know that $F_s = 600$ from part b.
We can plug in $F_s = 600$ into $F_s = uN$ to find that $uN = 600$

From part a, we already know that $N = 2490$

We can plug this in to find that $u = \frac{600}{2490}$ which is 0.241

Solution to part d: We will first find an equation for position in the x-direction and position in the y-direction with respect to t.
The initial x position is 0. In the x direction, the acceleration is $2\frac{m}{s^2}$ and the initial velocity is 0.

We can use the equation $\Delta x = v_{ix}t + \frac{1}{2}at^2$
We can plug in our variables to find that $\Delta x = \frac{1}{2} \cdot 2 \cdot t^2 = t^2$
We also know that $\Delta x = x_f - x_i$. We already know that $x_i = 0$.
This means $x_f = t^2$
We can take the square root of both sides to find $t = \sqrt{x_f}$

Now we can do something similar for motion in the y-direction.
The acceleration in the y-direction for the block is $-1.5\frac{m}{s^2}$. The initial velocity in the y-direction v_{iy} is 0. The time is $t = \sqrt{x_f}$

We can use the equation $\Delta y = v_{iy}t + \frac{1}{2}at^2$

Plugging in our variables gives $\Delta y = \frac{1}{2} \cdot -1.5 \cdot (\sqrt{x_f})^2$
Solving this gives that $\Delta y = -0.75x_f$
We also know that $\Delta y = y_f - y_i$

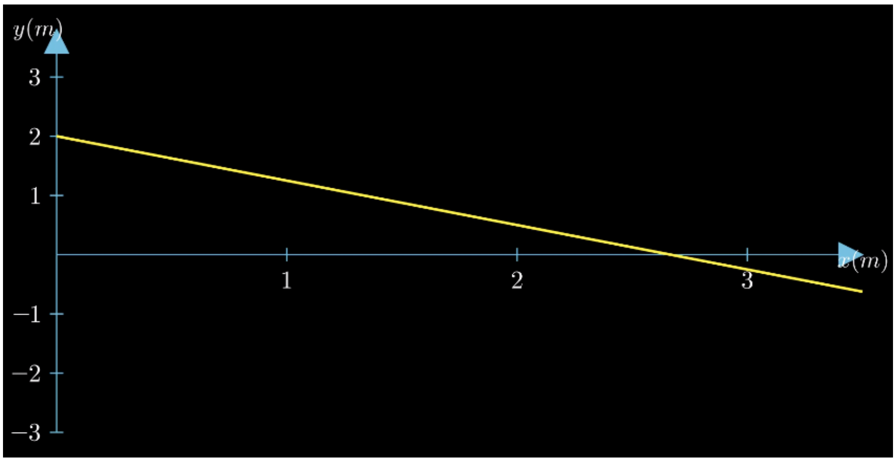
Since $y_i = 2$, we know that $\Delta y = y_f - 2$
We can equate this to $-0.75x_f$ (since we found that $\Delta y = -0.75x_f$ to find that $y_f - 2 = -0.75x_f$

We can add 2 to both sides to get $y_f = -0.75x_f + 2$

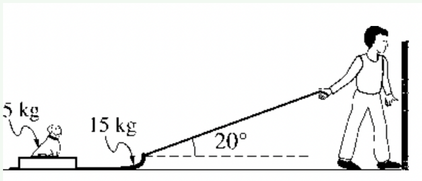
This means the equation of the path of the box in terms of y and x is $y = -0.75x + 2$

Solution to part e: We simply graph $y = -0.75x + 2$ to graph the path.

$y = -0.75x + 2$ is a linear line with y-intercept of 2. The line will have a slope of -0.75

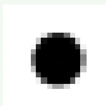


Problem 2.0.17 — 2007 AP Physics 1 FRQ



A child pulls a 15 kg sled containing a 5.0 kg dog along a straight path on a horizontal surface. He exerts a force of 55 N on the sled at an angle of 20° above the horizontal, as shown in the figure. The coefficient of friction between the sled and the surface is 0.22.

a. On the dot below that represents the sled-dog system, draw and label a free-body diagram for the system as it is pulled along the surface.

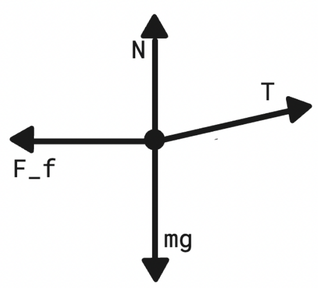


b. Calculate the normal force of the surface on the system.

c. Calculate the acceleration of the system.

d. At some later time, the dog rolls off the side of the sled. The child continues to pull with the same force. On the axes below, sketch a graph of speed v versus time t for the sled. Include both the sled's travel with and without the dog on the sled. Clearly indicate with the symbol t_r the time at which the dog rolls off.

Solution to part a: The forces on the sled-dog system are the tension force, normal force, gravitational force, and friction force.



Solution to part b: We will write Newton's Second Law equations for the forces in the x direction and y direction.

In the y direction, the forces are the normal force, gravitational force, and vertical component of tension.
We must use the equation $F_{net} = ma$ and find the net force.
We get $T\sin(\theta) + N - mg = ma$.
Since acceleration is 0, we get that $T\sin(\theta) + N - mg = 0$
We can rearrange this equation to get that $N = mg - T\sin(\theta)$

We know that $m = 15 + 5 = 20$ kg (since we must sum up the mass of the sled and dog). We also know that $\theta = 20°$ and $T = 55$ N since that is the force the student exerts.
We can plug this in to find that $N = 20 \cdot 9.8 - 55\sin(20) = 177.19$ N.

Solution to part c: There is obviously no acceleration in the y-direction. We must consider the forces in the x-direction to find the acceleration.

In the x direction, the forces are the horizontal component of tension and friction.
Using $F_{net} = ma$, we find that $T\cos(\theta) - \mu N = ma$

We already know that $\theta = 20°$ and $m = 20$ kg. On top of that, we know that the normal force n is 177.19 N from part b. We also know that $\mu = 0.22$. The tension force is 55 N.

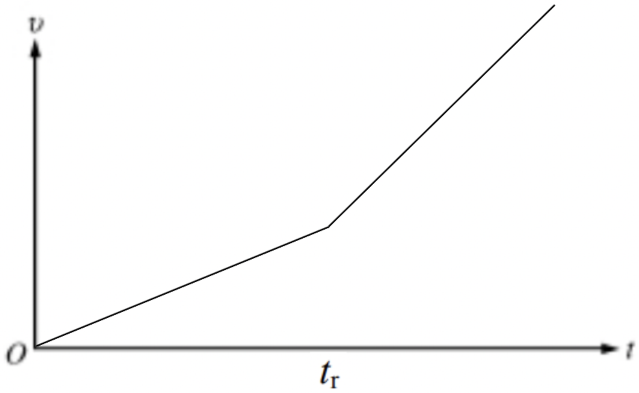
We can plug these variables in to find

$$55\cos(20) - 0.22 \cdot 177.19 = 20 \cdot a$$

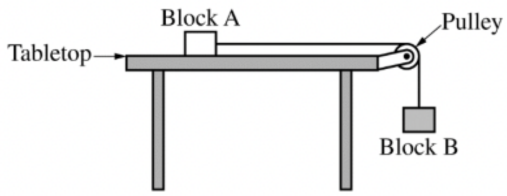
We can solve this equation for a to find that $a = 0.635\frac{m}{s^2}$

Solution to part d: Once the dog rolls off, the acceleration will increase. The reason is that the mass will now be a lot less. Before time t_r, the mass included the dog's mass and sled's mass. However, now it will just include the sled's mass.

The acceleration will rapidly increase as soon as the dog rolls off. It will immediately go up from a lower value. Thus, until time t_r, we will have a linear line. After t_r, we will still have a linear line. However, the linear line will now have a larger slope due to the larger constant acceleration.



Problem 2.0.18 — 2019 AP Physics 1



This problem explores how the relative masses of two blocks affect the acceleration of the blocks. Block A, of mass m_A, rests on a horizontal tabletop. There is negligible friction between block A and the tabletop. Block B, of mass m_B, hangs from a light string that runs over a pulley and attaches to block A, as shown above. The pulley has negligible mass and spins with negligible friction about its axle. The blocks are released from rest.

(a)

i. Suppose the mass of block A is much greater than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.
Briefly explain your reasoning without deriving or using equations.

ii. Suppose the mass of block A is much less than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.
Briefly explain your reasoning without deriving or using equations.

(b) Now suppose neither block's mass is much greater than the other, but that they are not necessarily equal. The dots below represent block A and block B, as indicated by the labels. On each dot, draw and label the forces (not components) exerted on that block after release. Represent each force by a distinct arrow starting on, and pointing away from, the dot.

(c) Derive an equation for the acceleration of the blocks after release in terms of m_A, m_B, and physical constants, as appropriate. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

(d) Consider the scenario from part (a)(ii), where the mass of block A is much less than the mass of block B. Does your equation for the acceleration of the blocks from part (c) agree with your reasoning in part (a)(ii) ?

Yes or **No**

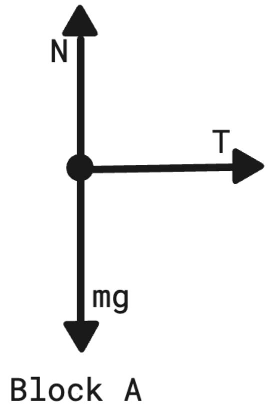
Briefly explain your reasoning by addressing why, according to your equation, the acceleration becomes (or approaches) a certain value when m_A is much less than m_B.

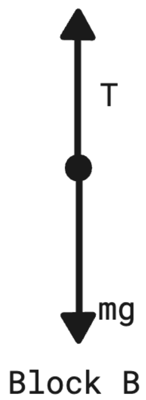
Solution to part a i: The acceleration will be close to 0. The reason is that block B is a lot lighter. It won't be able to pull such a heavy block down due to the fact that block

A has a much greater mass.

Solution to part a ii: The acceleration will be close to g. The reason is that block A is a lot heavier. It will easily be able to pull block B down since block B has a much lower mass.

Solution to part b: The only forces on block B will be tension force and gravitational force. The forces on block A will be tension force, gravitational force, and normal force. Remember that there is **no frictional force** since the problem says that friction is negligible.
Also, the length of the force vectors don't really matter for this problem. We are not told which block might be heavier, so it won't be possible to draw the lengths properly. In such cases, just make the lengths of all force vectors equal.





Solution to part c: We will write an equation using Newton's Second Law for each block.
For block B, the equation is $m_b g - T = ma$
For block A, the equation is $T = m_a a$
Note, for block A we don't need to write an equation for the forces in the y-direction. The reason is that there is no acceleration for block A in that direction when it's on the tabletop. That means the normal force will balance out with block A's weight.

Now, since we know that $m_b g - T = ma$ and $T = m_a a$, we can add both equations to cancel our tension force.
Doing so gives $m_b g = (m_a + m_b)a$
We can divide both sides by $m_a + m_b$ to get that

$$a = \frac{m_b g}{m_a + m_b}$$

Solution to part d: In part a ii, we said that the acceleration would be close to g when block A's mass would be much less.
Now, we will verify that statement using our equation for acceleration: $a = \frac{m_b g}{m_a + m_b}$

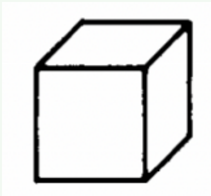
Since block m_a has a very small mass, we can basically say that it is negligible and close to 0.
Plugging in $m_a = 0$ gives $a = \frac{m_b g}{m_b} = g$

This means our answer is **yes**. The equation from part c supports our reasoning for part a ii.

Problem 2.0.19 — 1988 AP Physics B FRQ

A helicopter holding a 70-kilogram package suspended from a rope 5.0 meters long accelerates upward at a rate of 5.2m/s^2. Neglect air resistance on the package.

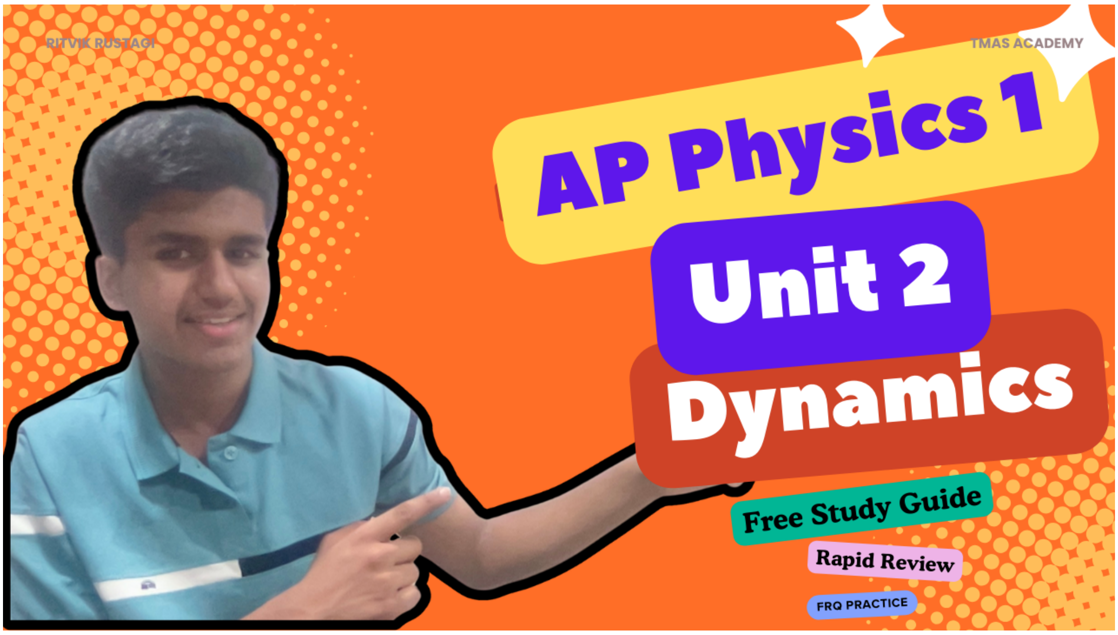
(a) On the diagram below, draw and label all of the forces acting on the package.



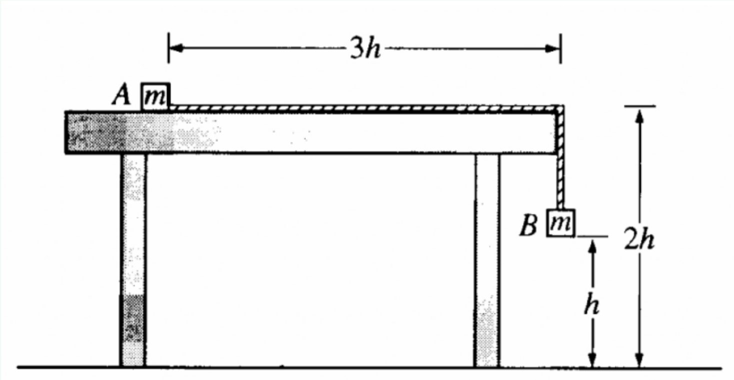
(b) Determine the tension in the rope.

(c) When the upward velocity of the helicopter is 30 meters per second, the rope is cut and the helicopter continues to accelerate upward at 5.2m/s^2. Determine the distance between the helicopter and the package 2.0 seconds after the rope is cut.

Solution: Video Solution



Problem 2.0.20 — 1998 AP Physics B FRQ



Two small blocks, each of mass m, are connected by a string of constant length $4h$ and negligible mass. Block A is placed on a smooth tabletop as shown above, and block B hangs over the edge of the table. The tabletop is a distance $2h$ above the floor. Block B is then released from rest at a distance h above the floor at time $t = 0$. Express all algebraic answers in terms of h, m, and g.

(a) Determine the acceleration of block B as it descends.

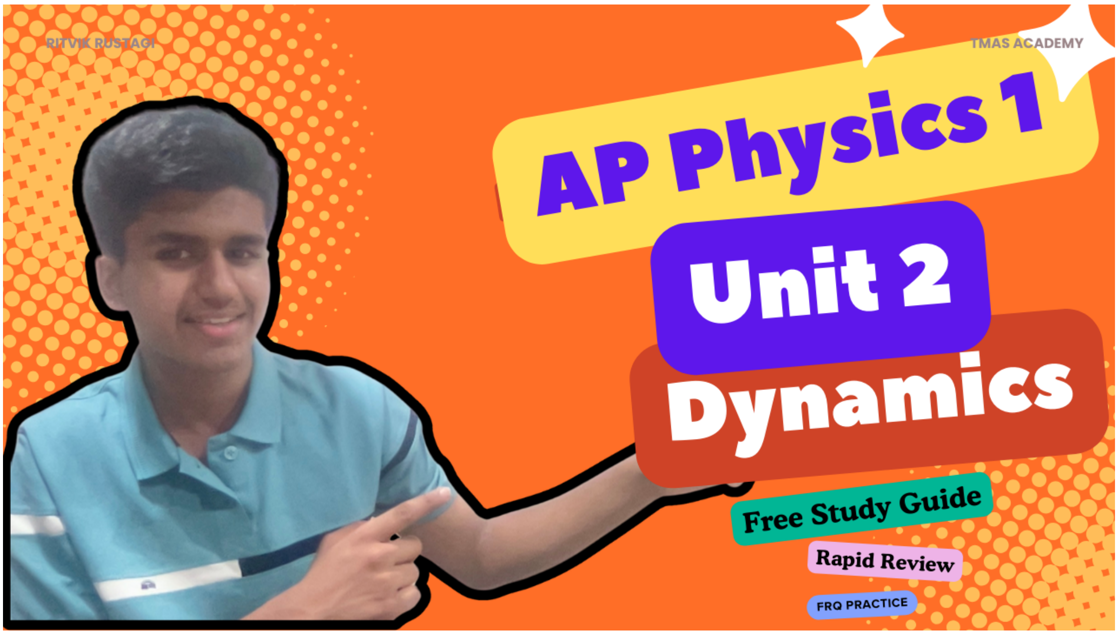
(b) Block B strikes the floor and does not bounce. Determine the time $t = t_1$ at which block B strikes the floor.

(c) Describe the motion of block A from time $t = 0$ to the time when block B strikes the floor.

(d) Describe the motion of block A from the time block B strikes the floor to the time block A leaves the table.

(e) Determine the distance between the landing points of the two blocks

Solution: Video Solution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Unit 3 Circular Motion and Gravitation

Note 3.0.1

Circular Motion

Uniform circular motion happens in a circular path. The object that undergoes uniform circular motion (UCM) must do it at constant speed. The velocity of that object is tangent to the circle it makes. The acceleration (called centripetal acceleration) always points towards the center of the circle, which just changes the direction of the object, NOT the speed.

The formula for centripetal acceleration is $a_c = \frac{v^2}{r}$.

Another way to write this is by plugging in $v = \frac{2\pi r}{T}$.

$$\text{For those that don't know, velocity can be represented as } v = \frac{2\pi r}{T}$$

$$\text{for an object undergoing circular motion.}$$

$$\text{Plugging that in gives } a_c = \left(\frac{2\pi r}{T}\right)^2 \cdot \frac{1}{r} = \frac{4\pi^2 r}{T^2}$$

Also, remember that centripetal acceleration doesn't change the speed nor velocity in uniform circular motion. However, centripetal acceleration changes the direction of an object and allows it to move in a circle.

Note 3.0.2

Centripetal Force

The centripetal acceleration always points inward, as stated before. Thus, there must be a force inwards that is causing this inward acceleration. That force is called the centripetal force, and for an object in circular motion with mass m, velocity v, and radius r, the force is

$$F = \frac{mv^2}{r}.$$

Problem 3.0.3 — A ball of mass 5 kilograms is swung in a horizontal circle with a constant speed of $v = 15$ meters per second, and an inward force of 4500N is applied. What is the radius of the circle?

We have

$$F = \frac{mv^2}{r}$$

$$4500 = \frac{5 \times 15^2}{r}$$

$$\implies r = \boxed{4 \text{ m}}$$

Problem 3.0.4 — When a road is dry, a particular car can safely navigate a turn with a 50m radius of curvature at 20m/s without slipping. What is the coefficient of friction if this is the fastest speed the car can take this turn?

Surprisingly, we don't need to know the mass of the car for this problem. We have

$$F = \frac{mv^2}{r}$$

$$\mu mg = \frac{mv^2}{r}$$

$$\mu g = \frac{v^2}{r} = \frac{20^2}{50} = 8$$

$$\implies \mu = \frac{8}{g} = \boxed{0.8}.$$

Note 3.0.5

Is Centripetal Force an Actual Force?

Never label an arrow with centripetal force on any free body diagram. It is not an actual force. Centripetal force is the net force that leads to circular motion. It is a special name that we have given to the net force for an object undergoing circular motion.

To avoid making errors due to an improper sign convention, if a force is directed towards the center in circular motion problems, define that force as positive. However, if it points away from the center, then define it as negative.

Problem 3.0.6 — A stone of mass 3 kg is wrapped around a rope and is swung in a *vertical* circle with radius 6 meters and with a constant velocity of 10 meters per second. What is the tension in the rope at the highest point in the motion? At the lowest point?

Solution: We will still start with

$$F = \frac{mv^2}{r}.$$

But now, notice that at the highest point, $F_{net} = T + mg$. That means that gravity is also contributing to the centripetal force here, not just the tension of the rope.

$$T + mg = \frac{mv^2}{r} = 50$$

$$\implies T = 50 - mg = \boxed{20 \text{ N}}.$$

Now, if we were instead asked the tension at the lowest point, we would use $F_{net} = T - mg$, since mg is opposing the tension force.

To find the speed at the lowest point, you will need to use conservation of energy. Don't worry about finding the speed at the lowest point for this problem right now.

Problem 3.0.7 — You are riding a roller coaster going around a vertical loop, on the inside of the loop. If the loop has a radius of 50m, how fast must the cart be moving in order for you to feel three times as heavy at the top of the loop?

Solution: For the person to feel three times as heavy, we need the normal force from the loop to be 3 times the weight of the person. That is $N = 3mg$. Now, we have

$$F = \frac{mv^2}{r}$$

$$N + mg = \frac{mv^2}{r}$$

$$4mg = \frac{mv^2}{r}$$

$$4g = \frac{v^2}{50}$$

$$\implies v = \boxed{45 \text{ m/s}}$$

Now before we practice more, I want to clarify a few things. There is a common misconception when it comes to uniform circular motion.

As of now, you should know that circular motion can occur both horizontally and vertically. An example of vertical circular motion includes a roller coaster making a 360° turn.

Note 3.0.8

Vertical circular motion is not uniform circular motion. The reason is that uniform circular motion occurs at constant velocity. However, in vertical circular motion, the velocity is constantly changing due to the force of gravity.

Note 3.0.9

Gravitation
Until now, we've only covered the gravitation force for an object on Earth. But there is actually a gravitational force between any two particles, which is defined by

$$F = \frac{Gm_1m_2}{R^2},$$

where m_1, m_2 are the masses of the objects and R is the distance between them.

Note that the G here is not equal to the g, which is the acceleration due to Earth's gravity. G is more universal, which means that it applies to all objects, not just the Earth. But you won't need the exact value of it on the AP Physics exams.

Most problems will just be plug-and-chug, where you will be given three of F, m_1, m_2, R, and will be asked to find the other variable.

In some experiment problems, you might be asked to find the gravitational field. One possible way is to find is to measure the gravitational force on a test object. Then, we can divide the test object's mass to find the gravitational field.

Assuming m_1 is the mass of the test object and since $F = \frac{Gm_1m_2}{R^2}$, we can find that

$$g = \frac{F}{m_1} = \frac{Gm_2}{R^2}$$

This means that the gravitational field does not depend on the test object's mass because the variable m_1 isn't in our formula for the gravitational field.

Problem 3.0.10 — A certain planet has three times the radius of Earth and nine times the mass. How does the acceleration of gravity at the surface of this planet compare to the acceleration at the surface of Earth?

Solution: We know that $F = \frac{Gm_1m_2}{R^2}$. If the test object's mass is m_1, then the gravitational field will be

$$g\frac{F}{m_1} = \frac{Gm_2}{R^2}.$$

For the new planet, m_1 is multiplied by 9, but R^2 is also multiplied by $3^2 = 9$. Thus, the acceleration stays the same because $\frac{9}{9}$ is 1.

Note 3.0.11

Escape and Orbital velocities
There are two more important things in gravitation: the escape and orbital velocities of an object.

The Orbital velocity is the required velocity of an object in order to stay in the orbit around a planet. This velocity is

$$v_{\text{orbital}} = \sqrt{\frac{GM}{R}},$$

where M is the mass of the planet, not the revolving object.

The Escape velocity is the minimum velocity that will take the object out of its current orbit around a planet. This velocity is

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}},$$

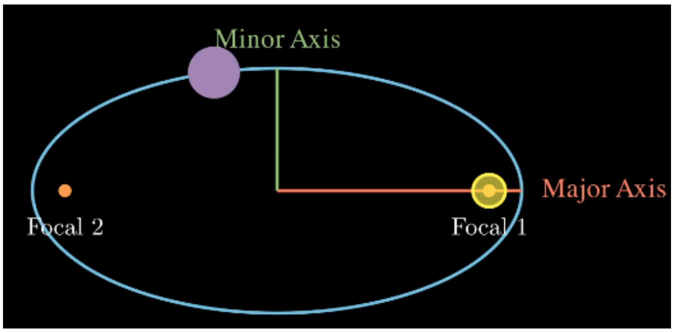
where again, M is the mass of the planet.

To derive the orbital and escape velocity, you will need to know energy conservation. I recommend trying it out on your own once you learn unit 4.

Note 3.0.12

Kepler's First Law
Kepler's First Law says that all planets move in an elliptical orbit around the Sun. Please note the word elliptical; it's different from a circular orbit!

The sun will be located at one of the focal points of the ellipse.

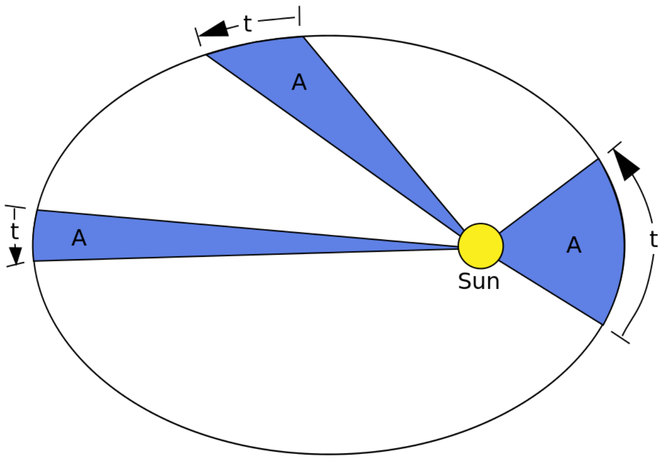


The above represents an elliptical orbit around the sun. The purple circle represents a planet that is moving in an elliptical orbit.
Remember that energy will still be conserved throughout the orbit.

Note 3.0.13

Kepler's Second Law
The second law describes the changes in velocity as the planet moves around the sun in the elliptical orbit.

As the planet moves closer to the sun, its velocity increases. However, as it moves further away from the sun, its velocity decreases. The reason is that when the planet is closer to the Sun, gravitational potential energy will be less. This causes kinetic energy to be greater; hence the planet is moving faster at points closer to the Sun.



The second law also says that the planet sweeps out equal areas in time intervals of equal length. This idea can be seen in the image above.

Note 3.0.14

Kepler's Third Law
The third law says that the square of the orbital period of a planet around something like the sun is proportional to the cube of the semi-major axis.
Note that the semi-major axis has half the length of the entire major axis.

$$\frac{T_1^2}{R_1^3} = \frac{T_2^2}{R_2^3}$$

The above shows the relationship that the third law highlights.

It's important to understand how this relationship is derived. By now we know that the gravitational force is what drives such motion.
Let's say that m_1 is the mass of the planet and m_2 is the mass of the Sun. Then, we can find the gravitational force and equate it to $\frac{m_1 v^2}{R}$.

$$\frac{Gm_1 m_2}{R^2} = \frac{m_1 v^2}{R}$$

$$\text{Now, we can cancel out like terms to get } \frac{Gm_2}{R} = v^2$$

Now, we can use the fact that $v = \frac{2\pi R}{T}$. We substitute that to get

$$\frac{Gm_2}{R} = \left(\frac{2\pi R}{T}\right)^2 = \frac{4\pi^2 R^2}{T^2}$$

We can rearrange this to get $\dfrac{T^2}{R^3} = \dfrac{4\pi^2}{Gm_2}$

Clearly, we can see that $\frac{T^2}{R^3}$ equals to a constant. The reason is that m_2 is a constant, since it's just the mass of the Sun. Thus, this proves that $\frac{T^2}{R^3}$ is constant.

Also, another key takeaway from this should be the substitution $v = \frac{2\pi R}{T}$. The reason regarding why this is true is that $2\pi R$ represents the circumference, the length of the path. Then, you can divide it by the period to find the velocity.

Now, it's time for you to learn about curved roads (in a circular shape) and banked curves. They both are common applications for circular motion concepts.

If a car is travelling around a flat curve, friction force on the tires will be causing the centripetal acceleration. The person present in the car will also have a force that points inwards towards the center of the circle. That force can come from either the seat or even the side door.

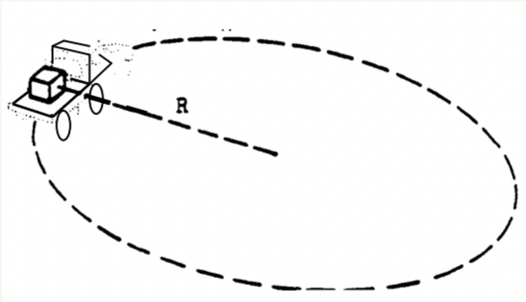
The force that turns the car around the car specifically is **static friction**.

Note 3.0.15

Banked curves are a little more complicated. Instead of travelling in a flat circle, the car or any other object will now travel in a circular curve that is at an angle.

The best way to learn how to solve a problem involving banked curves is to see an example. One will be shown soon.

Problem 3.0.16 — 1977 AP Physics B FRQ



A box of mass M, held in place by friction, rides on the flatbed of a truck which is traveling with constant speed v. The truck is on an unbanked circular roadway having radius of curvature R.

a. On the diagram provided above, indicate and clearly label all the force vectors acting on the box.

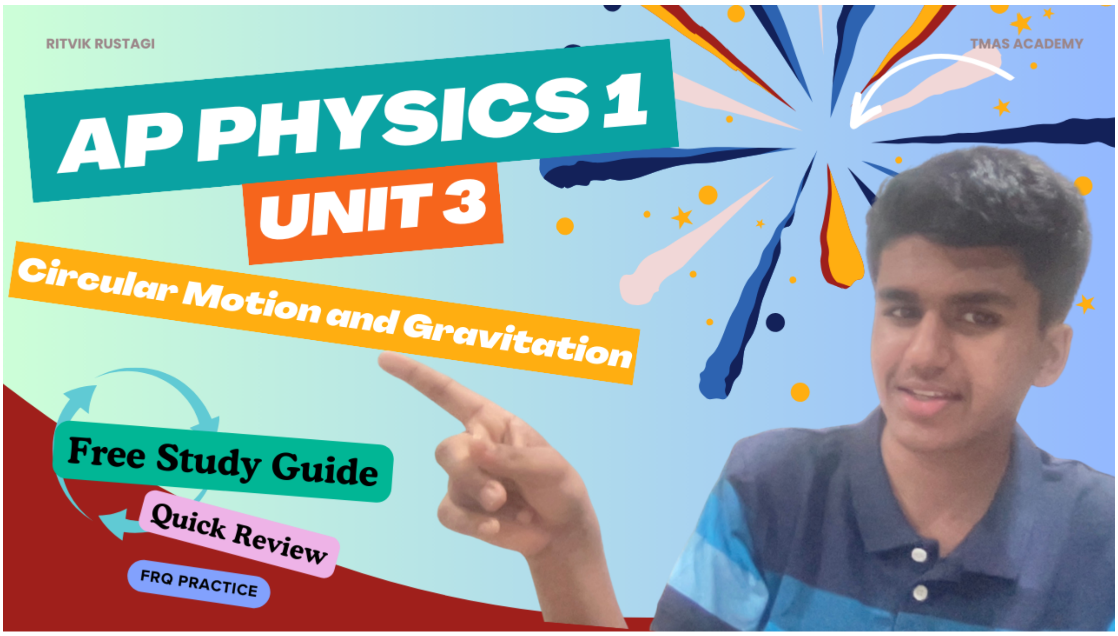
b. Find what condition must be satisfied by the coefficient of static friction μ between the box and the truck bed. Express your answer in terms of v, R, and g.

If the roadway is properly banked, the box will still remain in place on the truck for the same speed v even when the truck bed is frictionless.

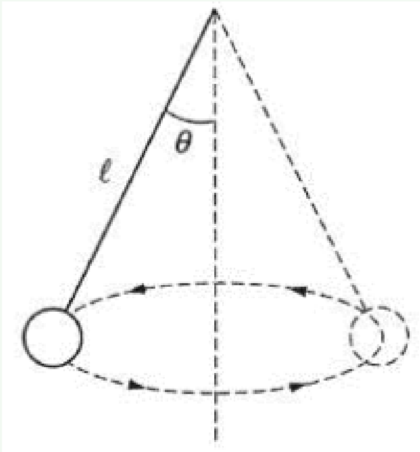
c. On the diagram above indicate and clearly label the two forces acting on the box under these conditions

d. Which, if either, of the two forces acting on the box is greater in magnitude?

Solution: Video Solution

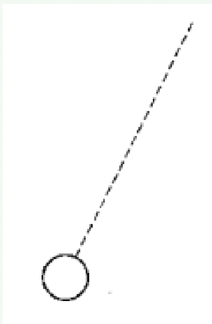


Problem 3.0.17 — 2002 AP Physics B FRQ



A ball attached to a string of length l swings in a horizontal circle, as shown above, with a constant speed. The string makes an angle θ with the vertical, and T is the magnitude of the tension in the string. Express your answers to the following in terms of the given quantities and fundamental constants.

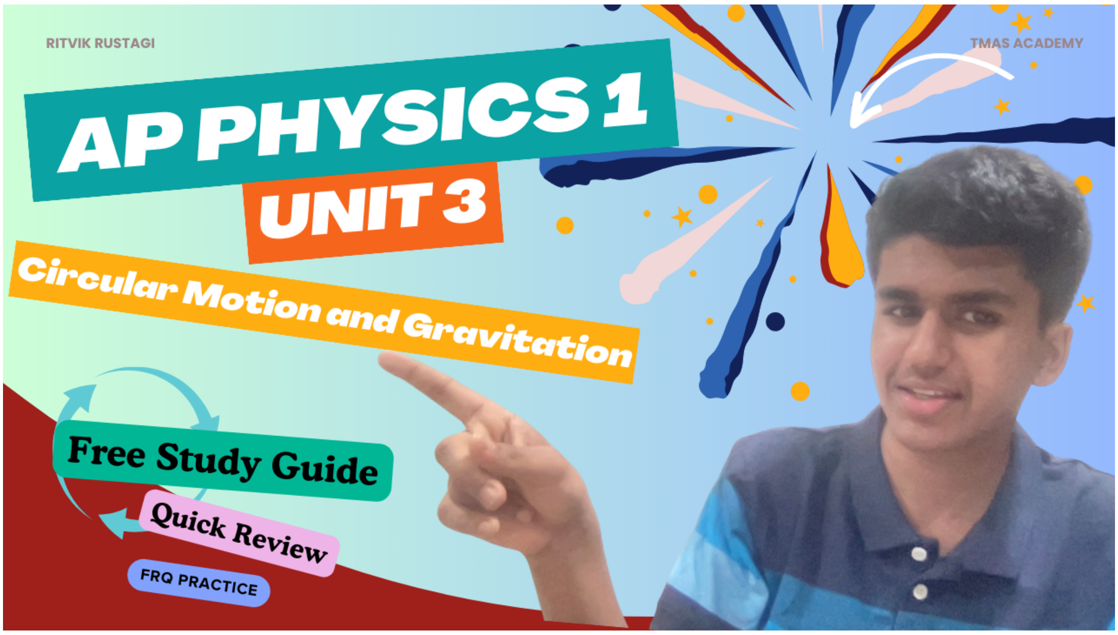
a. On the figure below, draw and label vectors to represent all the forces acting on the ball when it is at the position shown in the diagram. The lengths of the vectors should be consistent with the relative magnitudes of the forces.



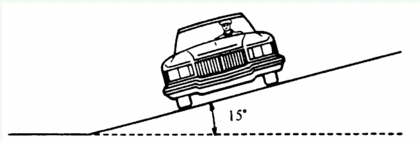
b. Determine the mass of the ball.

c. Determine the speed of the ball.

Solution: Video Solution



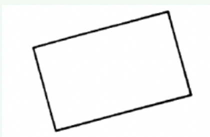
Problem 3.0.18 — 1988 AP Physics 1 FRQ



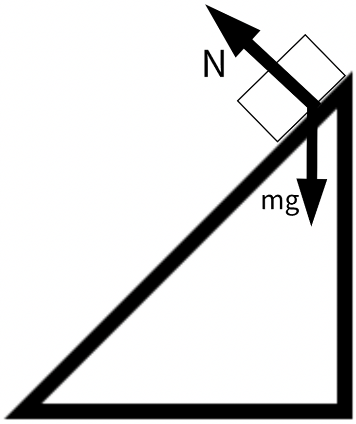
A highway curve that has a radius of curvature of 100 meters is banked at an angle of 15° as shown above.

(a) Determine the vehicle speed for which this curve is appropriate if there is no friction between the road and the tires of the vehicle.

On a dry day when friction is present, an automobile successfully negotiates the curve at a speed of 25m/s.



Solution to part a: To solve such a problem, draw the banked curve from the perspective of an observer staring at it from the front. This will give us a clean right triangle and allow us to see the front of the car. this might be a bit hard to understand since it requires a lot of visualization.



We can tell that the vertical component of normal force must balance the gravitational force out. This will cause the car to remain at the same height.

The horizontal component of normal force will cause the centripetal motion. That means $N\sin\theta = ma = \frac{mv^2}{r}$.
We also know that $N\cos\theta = mg$

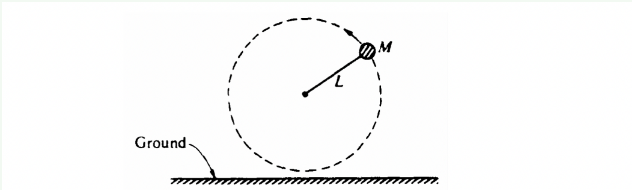
We can manipulate $N\cos\theta = mg$ to find that $N = \frac{mg}{\cos\theta}$
We plug that into $N\sin\theta = \frac{mv^2}{r}$ to find that $mg\tan\theta = \frac{mv^2}{r}$.

We can isolate v to find that $v^2 = gr\tan\theta$ which means $v = \sqrt{gr\tan\theta}$.

Don't mess up in plugging in the values. Even though the radius of curvature is 100 m, that is not the radius for the centripetal motion! The horizontal distance to the center is $100\cos(15)$

We can plug this in to find that $v = \sqrt{9.8 \cdot 100\cos(15) \cdot \tan(15)} = 15.93\text{m/s}$.

Problem 3.0.19 — 1984 AP Physics B FRQ



A ball of mass M attached to a string of length L moves in a circle in a vertical plane as shown above. At the top of the circular path, the tension in the string is twice the weight of the ball. At the bottom, the ball just clears the ground. Air resistance is negligible. Express all answers in terms of M, L, and g.

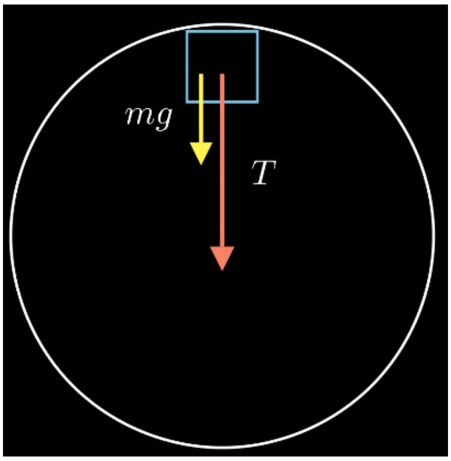
(a) Determine the magnitude and direction of the net force on the ball when it is at the top.

(b) Determine the speed v_o of the ball at the top.

The string is then cut when the ball is at the top.
(c) Determine the time it takes the ball to reach the ground.

(d) Determine the horizontal distance the ball travels before hitting the ground.

Solution to part a: At the top, the net force of the ball points downward since that is also where the centripetal acceleration points (towards the center).
The magnitude of it can be found by drawing a free body diagram first to show all the forces on that ball.



Using the free body diagram, we know that the net force is $T + mg$. Since it's given that the tension at the top is twice the weight, it means $T = 2mg$. We can plug that in to get that the net force is $3mg$ (downwards towards the center).

Solution to part b: We know that the net force pointing towards the center will be causing the centripetal motion.

Since we know our net force as $3mg$, we can equate that to $\frac{mv^2}{r}$

$$3mg = \frac{mv^2}{r}(m \text{ cancels out})$$

$$3g = \frac{v^2}{L}(\text{our radius is equal to } L, \text{ the length of the string})$$

$$3gl = v^2$$

$$v = \sqrt{3gl}$$

Solution to part c: Once the string is cut at the top, the ball will undergo projectile motion. There is an initial horizontal velocity which remains constant for the whole time, and the initial vertical velocity is 0. We can write out all of our variables.

$v_{ox} = \sqrt{3gl}$	$v_{oy} = 0$
$v_x = \sqrt{3gl}$	$v_{oy} =?$
$a_x = 0$	$a_y = g$
$t = ?$	$t= ?$
$\Delta x = ?$	$\Delta y = 2L$

To find the time, we can simply use our variables in the y direction. We will use our kinematics equation that is

$$\Delta y = v_{oy}t + \frac{a_y t^2}{2}$$

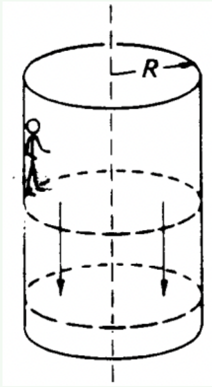
We manipulate this equation to isolate t, and this gives us $t^2 = \frac{2\Delta y}{a_y}$

$$\text{This simplifies to } t^2 = \frac{2 \cdot 2L}{g} \text{ which means } t \text{ is } \sqrt{\frac{4L}{g}}$$

Solution to part d: Since now we know the time and the fact that the horizontal velocity is constant (just like how it is in projectile motion problems that we've solved), we simply multiply the horizontal velocity to the time we found in part C.

$$d = \sqrt{\frac{4L}{g}} \cdot \sqrt{3gL} = 2\sqrt{3}L$$

Problem 3.0.20 — 1984 AP Physics C: Mechanics FRQ



An amusement park ride consists of a rotating vertical cylinder with rough canvas walls. The floor is initially about halfway up the cylinder wall as shown above. After the rider has entered and the cylinder is rotating sufficiently fast, the floor is dropped down, yet the rider does not slide down. The rider has mass of 50 kilograms, the radius R of the cylinder is 5 meters, the angular velocity of the cylinder when rotating is 2 radians per second, and the coefficient of static friction between the rider and the wall of the cylinder is 0.6.

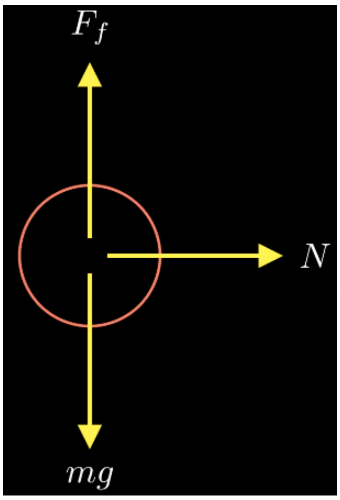
(a) Draw and identify the forces on the rider when the system is rotating and the floor has dropped down.

(b) Calculate the centripetal force on the rider when the cylinder is rotating and state what provides that force.

(c) Calculate the upward force that keeps the rider from falling when the floor is dropped down and state what provides that force.

(d) At the same rotational speed, would a rider of twice the mass slide down the wall? Explain your answer.

Solution to a part a: The forces present are N (normal force), F_f (frictional force), and F_g or mg (gravitational force)



Solution to part b: The centripetal force is clearly caused by the normal force. It is the only force towards the center.

Since we know that the centripetal force equals to $\frac{mv^2}{r}$, we can directly find that value by plugging in our variables.

For that, we need to find v. This one step might be confusing for many of you. It involves a topic that we will learn in a future unit. The relationship in this problem between velocity and angular velocity is $v = \omega r$. Thus, since $\omega = 2$ and $r = 5$, we can find that $v = 2 \cdot 5 = 10$.

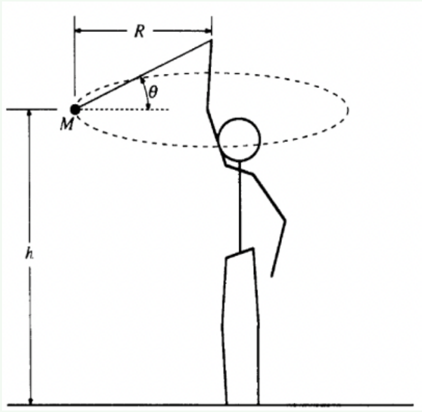
Now we can plug this into $\frac{mv^2}{r}$ to get $\frac{50 \cdot 10^2}{5} = 1000$ N.

Solution to part c: The upward force that prevents the rider from falling is friction force. We know from our free body diagram that $F_f - mg = 0$ since the net force is 0 in the y direction.
We know that $mg = 50 \cdot 9.8 = 490$. This means that the upward friction force will also be 490 N.

Solution to part d: Even if the mass would double, that would mean that the centripetal force would double. The reason is that the radius and rotational speed are both the same. Thus, the centripetal force would simply double in magnitude.

We already found that the force causing the centripetal force would be the normal force since it points towards the center. Since the centripetal force doubles, we know that the normal force doubles. Since friction is proportional to the normal force (because $F_f = \mu N$), the maximum value of friction would also double along with the gravitational force exerted on this heavier mass. However, the mas will cancel out and we're left with the same scenario. Thus, the rider would not slide down even if the mass would double.

Problem 3.0.21 — 1989 AP Physics B FRQ



An object of mass M on a string is whirled with increasing speed in a horizontal circle, as shown above. When the string breaks, the object has speed v_0, and the circular path has radius R and is a height h above the ground. Neglect air friction.

a. Determine the following, expressing all answers in terms of h, v_o, and g.

i. The time required for the object to hit the ground after the string breaks

ii. The horizontal distance the object travels from the time the string breaks until it hits the ground

iii. The speed of the object just before it hits the ground

b. On the figure below, draw and label all the forces acting on the object when it is in the position shown in the diagram above.

c. Determine the tension in the string just before the string breaks. Express your answer in terms of M, R, v_o, and g.

Solution to part a i: After the string breaks, v_{oy} (initial velocity in y-direction) is 0. Also, the distance travelled is simply h (in the y-direction), and the acceleration is g.

We can apply the equation $y = v_{oy}t + \frac{1}{2}a_y t^2$. Plugging in our values gives
$h = \frac{gt^2}{2}$
Solving for t gives $\sqrt{\frac{2h}{g}}$

Solution to part a ii: When the string breaks, the object has speed v_o. This means that is our initial velocity in the x direction. There is also no acceleration in the x-direction which means that this velocity is constant in the x-direction.

Thus, we can just apply the equation $d = vt$ which can also be written as $\Delta x = v_x t$
Plugging in our values gives $\Delta x = v_o t$.

We can substitute our value of t found from part a i.

$$\text{This gives that the horizontal distance travelled is } v_o\sqrt{\frac{2h}{g}}$$

Solution to part a iii: The total velocity of any object is $\sqrt{v_x^2 + v_y^2}$. As long as we know both components of velocity, we can find the total velocity.

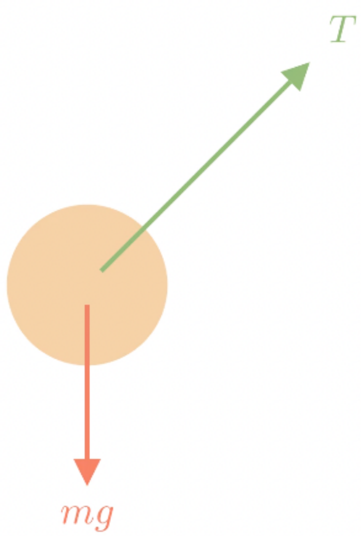
We already know that before the ball hits the ground its velocity in the x direction will simply be v_o.

We can find velocity in the y direction through our kinematics equations.
Since v_{oy} is 0, $a_y = g$, and $t = \sqrt{\frac{2h}{g}}$, we can apply the equation $v_y = v_{oy} + a_y t$

This gives that $v_y = 0 + g\sqrt{\frac{2h}{g}}$ which can be simplified to $v_y = \sqrt{2gh}$.

Since $v_x = v_o$ and $v_y = \sqrt{2gh}$ (our two components of velocity before the object hits the ground), we can plug this into $v = \sqrt{v_x^2 + v_y^2}$ to get that $v = \sqrt{v_o^2 + 2gh}$.

Solution to part b: When the object is in the position shown, the only forces on it are tension and gravity.



Tension simply points along the rope and gravity points downward.

Solution to part c: Right before the string breaks, the object is moving in circular motion.
We can write out the forces in the x and y direction.

The only force in the x direction is the horizontal component of the tension force. This component of tension is what provides the centripetal force. In the y direction, we have the force of gravity and the vertical component of tension. Both equate to each other as there is no motion in the y-direction.

$$F_x : T\cos\theta = Ma = \frac{Mv_o^2}{R}$$

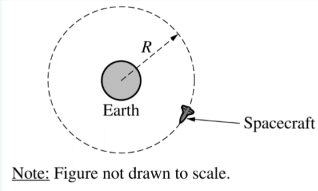
$$F_y : T\sin\theta - Mg = Ma = 0$$

Clearly from the summation of forces in the y-direction, $T\sin\theta = Mg$.
Since we have an expression for both the horizontal and vertical component of the tension force, the total tension force (T) $= \sqrt{(T\cos\theta)^2 + (T\sin\theta)^2}$.

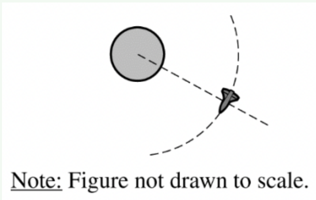
$$\text{Plugging in our expressions gives that tension is } \sqrt{\frac{M^2v_o^4}{R^2} + M^2g^2}$$

$$\text{We can factor out } M^2 \text{ to get that the tension force is } M\sqrt{\frac{v_o^4}{R^2} + g^2}$$

Problem 3.0.22 — 2018 AP Physics 1 FRQ



A spacecraft of mass m is in a clockwise circular orbit of radius R around Earth, as shown in the figure above. The mass of Earth is M_E. (a) In the figure below, draw and label the forces (not components) that act on the spacecraft. Each force must be represented by a distinct arrow starting on, and pointing away from, the spacecraft.



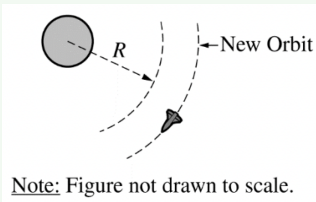
(b) i. Derive an equation for the orbital period T of the spacecraft in terms of m, M_E, R, and physical constants, as appropriate. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

ii. A second spacecraft of mass $2m$ is placed in a circular orbit with the same radius R. Is the orbital period of the second spacecraft greater than, less than, or equal to the orbital period of the first spacecraft?

_____ Greater than _____ Les than _____ Equal to

Briefly explain your reasoning.

(c) The first spacecraft is moved into a new circular orbit that has a radius greater than R, as shown in the figure below.

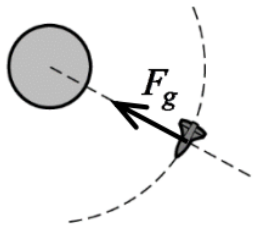


Is the speed of the spacecraft in the new orbit greater than, less than, or equal to the original speed?

_____ Greater than _____ Les than _____ Equal to

Briefly explain your reasoning.

Solution to part a: The only force is the gravitational force that occurs between the two masses. The gravitational force will point towards Earth.



Solution to part b i: In general we know that $v = \frac{2\pi R}{T}$ for the spaceship. This is the formula for its velocity. The reason is that it travels in a circle of radius R. Using that, we were able to find the distance it travels to make one orbit around Earth. Then, we divided that by T which is the period.

We also know that the force on the spaceship is $\frac{GmM_E}{R^2}$. Since the spaceship moves in a circular orbit, there will be a centripetal force on it.

$$F = \frac{GmM_E}{R^2} = \frac{mv^2}{R}$$

We can now plug in our expression for v which is $v = \frac{2\pi R}{T}$.

$$\frac{GmM_E}{R^2} = \frac{m}{R}(\frac{2\pi R}{T})^2$$

$\implies \frac{GmM_E}{R^2} = \frac{m}{R} \cdot \frac{4\pi^2 R^2}{T^2}$ We can cancel out like terms from both sides and isolate T^2.

$$T^2 = \frac{4\pi^2 R^3}{GM_E}$$

We can take the square root of both sides to find that $T = 2\pi\sqrt{\frac{R^3}{GM_E}}$

Solution to part b ii: Our equation for period doesn't have the variable m. This means that the period isn't affected by the mass of the spaceship. Thus, changing the mass to $2m$ won't change the orbital period. Thus, the answer is "equal to."

Solution to part c: We know that changing the orbit's radius will effect the gravitational force.
We know that the gravitational force will cause the centripetal force.

$$F = \frac{GmM_E}{R^2} = \frac{mv^2}{R}$$

We can cancel out like terms to get $\frac{GM_E}{R} = v^2$
This means that $v = \sqrt{\frac{GM_E}{R}}$ for an arbitrary radius R.
From the formula, we can tell that R is in the denominator. Thus, increasing the orbital radius will cause the speed of the spaceship to go down. Thus, the answer is "less than."

Unit 4 Energy

Have you ever rode a bicycle down a mountain or a steep road? Have you noticed that the bike speeds up as you move down. Why does this happen? Well the answer to this will be found in this unit. You will learn about different forms of energy to understand how energy is transferred into different forms.

Note 4.0.1

Energy
Energy can be defined as the capacity to do work. There are two ways energy can change: within a system, or between a system and the external world.

Note 4.0.2

Types of Energy
1. Kinetic Energy

Kinetic Energy can be thought of as the energy of motion. The equation includes mass (m) and velocity (v).

$$K = \frac{1}{2}mv^2 \tag{4.1}$$

2. Potential Energy

Potential energy can be thought of as stored energy due to position. In AP Physics, there are two types of potential energies: gravitational and spring.

The gravitational potential energy equation includes mass (m), the gravitational acceleration constant (g), and h (vertical displacement).

$$U_g = mgh \tag{4.2}$$

The spring potential energy equation includes k (spring constant) and x, the distance the spring is stretched from equilibrium.

$$U_s = \frac{1}{2}kx^2 \tag{4.3}$$

In general, objects want to move to a point where there is less potential energy. That is the natural tendency of every object.
For example, an object at a certain height will always want to move down.

Note 4.0.3

Work

Work is defined as a force applied over a displacement. Work done on an object can transfer energy. The unit of work is J (Joules). Energy also has that unit.

Examples:

1. When a force is applied in the direction of displacement,

$$W = Fd.$$

2. When a force is applied in a direction that is an angle θ from the direction of displacement,

$$W = Fd\cos\theta.$$

Note that work is a scalar quantity, not a vector.

Note that only the component of force that is parallel to displacement can do work. Thus, if the force is perpendicular to the displacement vector, then 0 work is done. There is one EXTREMELY important example of this. To learn about it, check out the AP Physics 1 Unit 4 Rapid Review video on the TMAS Academy youtube channel.

Now, let's derive the work energy theorem by using $W_{net} = F_{net}d$. You won't know the work-energy theorem right now. It will be covered soon. Before we cover it, let's derive it.

We know that d is the displacement in this situation. Thus, we can use the kinematics equation $v_f^2 = v_i^2 + 2ad$
We can solve for d to find that $d = \frac{v_f^2 - v_i^2}{2a}$
We also know that $F_{net} = ma$ (from Newton's Second Law).

We can plug this into $W_{net} = F_{net}d$ to find that $W_{net} = ma \cdot \frac{v_f^2 - v_i^2}{2a} = \frac{mv_f^2}{2} - \frac{mv_i^2}{2}$
Since KE $= \frac{1}{2}mv^2$, we know that $K_i = \frac{mv_i^2}{2}$ (initial kinetic energy) and $K_f = \frac{mv_f^2}{2}$ (final kinetic energy).

This means that $W_{net} = \Delta K$ (net work done is the change in kinetic energy)

Work can also be found using calculus (extremely common on the AP exam).
Instead of $W = Fd$, you can also use $W = \int_a^b \vec{F}(r) \cdot d\vec{r}$

Don't forget that work is a **scalar** quantity. It also has a sign (it can be negative, positive, or zero).

Also, if we are given a force-time graph, then the area under the curve is equivalent to the work done by that specific force.

Note 4.0.4

Work-Energy Theorem
The net-work done on an object is equivalent to the change in kinetic energy.

$$W_{net} = \Delta K$$

Now, it's important to relate potential energy to force.
We already know how kinetic energy relates to force. It relates through the work-energy theorem.

Note 4.0.5

Conservative vs Non-conservative
When finding work done by a conservative force, only the initial and final points matter. The way you approach the final point doesn't matter (the work done is independent to the path). You only need to consider the final and initial position. For example, in a complete closed path, the work done by a conservative force is zero.

On the other hand, the work done by a non-conservative force depends on the path. The most common example of a non-conservative force is friction.

Now, you should also know the **change in gravitational potential energy**. This is seen all the time on the AP Exam.
The change in gravitational potential energy can be written as $\Delta U_g = mg\Delta h$. Δh represents the change in height.

In general, the change in gravitational potential energy provides much more useful information than just gravitational potential energy. The reason is that gravitational potential energy can be defined by you based on your reference level. For example, if there's an object 2 m above the ground, then you can assume the reference level to be something like the floor or the object's location. Then, if you find $U = mgh$ using both reference levels, you will get a different value for the potential energy

HOWEVER, the change in gravitational potential energy will be constant despite what reference level you use.

Note 4.0.6

Ideal Spring
An ideal spring is extremely important to know in depth. The force caused by an ideal spring is a conservative force. That means we only care about the initial and final positions when finding the change in potential energy for the spring.

By Hooke's Law, we know that the force on an ideal spring is $F = -k\Delta x$ where Δx is the extension length.

The potential energy can be represented as $U_s = \frac{1}{2}k(\Delta x)^2$

On top of what you just learned, remember that the gravitational potential energy for an Earth-object system can be written as $U_g = -\frac{Gm_1m_2}{r}$ where r is the distance between the two masses. Just remember that if the two masses are an infinite distance apart, then the potential energy is 0.

Note 4.0.7

Conservation of energy states that mechanical energy is conserved if there are no external forces on a system.

$$K_i + U_i = K_f + U_f$$

Remember that potential energy includes both gravitational potential energy and spring potential energy.

When a system has **non-conservative forces**, then $W_{nc} = \Delta ME$. This means that the work done by the non-conservative force is equivalent to the change in mechanical energy.

$$W_{nc} = \Delta ME = \Delta K + \Delta U$$

Note 4.0.8

Total Energy **(E)**

E can be defined as the sum of all different types of energy in the system. Here, we must consider kinetic energy, potential energy, and thermal energy. If a problem asks us to find the total energy, we must account for other forms of energy other than potential and kinetic. However, if we are asked to find the total mechanical energy, then we only include potential (gravitational and spring) and kinetic energy.

Note 4.0.9

Power

Power is denoted by P and is defined as the rate of change of work. Assuming constant power, that means

$$P = \frac{W}{t},$$

where W is the work done in a time t.

When an object is moving at constant velocity, the power is

$$P = \frac{W}{\Delta t} = Fv$$

If the angle between the force and velocity vectors is θ, then $P = Fv\cos\theta$

Problem 4.0.10 — A bodybuilder is in the midst of a an intense training session. He is currently bench pressing a bar with a mass of 250kg. If he does six reps of this mass and his arms are 0.75m long, how much work has been done on the bar between the time the bar was removed from its rack and placed back on the rack?

Solution: This is a tricky problem. We know that the work done is equivalent to the product of the force and displacement. The *net* displacement of the bar 0, so the net work on it is $\boxed{0J}$. The reason is that the bodybuilder brings the bar back to the location where it started. That is what causes the displacement to be 0.

Problem 4.0.11 — A semi-truck carrying a trailer has a total mass of 1500kg. If it is traveling up a slope of 5 degrees to the horizontal at a constant rate of 20m/s, how much power is the truck exerting?

Solution: Note that the force exerted on the truck is equal to the gravitational force parallel to the slope, which is $1500g\cos(5°)$. Since the truck moves at constant velocity, we know that

$$P = Fv = (1500g\cos(5°))(20) = \boxed{26000W}$$

Problem 4.0.12 — An upward force is applied to lift a 20 kg bag a to a height of 5m. The bag is lifted at a constant speed. What is the work done on the bag?

Solution: The work done is equal to the change in potential energy, ΔU, which is equal to

$$mgh = (20)(g)(5) = \boxed{1000\text{ J}}$$

Problem 4.0.13 — Juri is tugging her wagon behind her. She has a trek ahead of her–five kilometers–and she's pulling with a force of 200 newtons. If she's pulling at an angle of 35 degrees to the horizontal, what work will be exerted on the wagon to get to the repair shop?

Solution:

$$W = Fd\cos\theta = (200)(5)(\cos 35°) = \boxed{819.152\text{ J}}$$

Problem 4.0.14 — What is the work done in increasing the speed of a 2 kg block from 4 m/s to 9 m/s?

Solution: This is an example of the work-energy theorem. The work done will be equivalent to the change in kinetic energy.

$$W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(2)(9)^2 - \frac{1}{2}(2)(4)^2 = \boxed{65\text{ J}}$$

Problem 4.0.15 — A 2.5 kg block moving at 5.6 m/s hits a spring with a spring constant of 50 N/m. How much is the spring compressed from its equilibrium position?

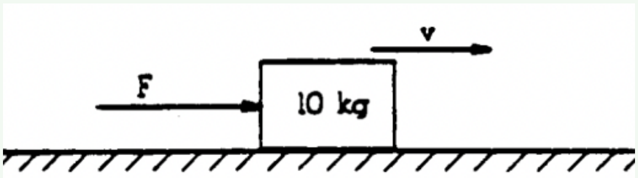
Solution: In this problem, the initial kinetic energy will completely convert into spring potential energy.

$$\frac{1}{2}mv_i^2 = \frac{1}{2}kx^2$$

We can rearrange the equation to find an expression for x.

$$x = v_i\sqrt{\frac{m}{k}} = 5.6\sqrt{\frac{2.5}{50}} = \boxed{1.252 \text{ m}}$$

Problem 4.0.16 — 1981 AP Physics B FRQ



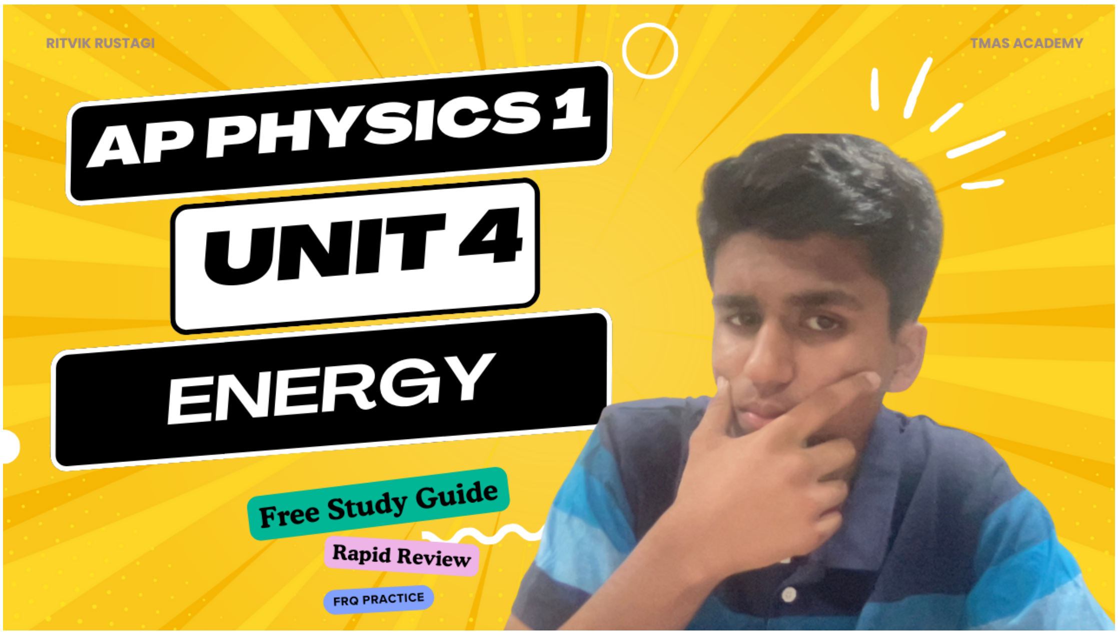
A 10-kilogram block is pushed along a rough horizontal surface by a constant horizontal force F as shown above. At time $t = 0$, the velocity v of the block is 6.0 meters per second in the same direction as the force. The coefficient of sliding friction is 0.2. Assume $g = 10$ meters per second squared.

(a) Calculate the force F necessary to keep the velocity constant.

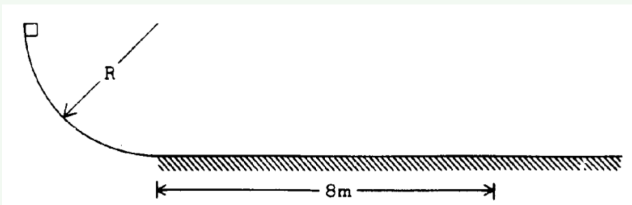
The force is now changed to a larger constant value F'. The block accelerates so that its kinetic energy increases by 60 joules while it slides a distance of 4.0 meters.
(b) Calculate the force F'.

(c) Calculate the acceleration of the block.

Solution: Video Solution



Problem 4.0.17 — 1975 AP Physics B FRQ



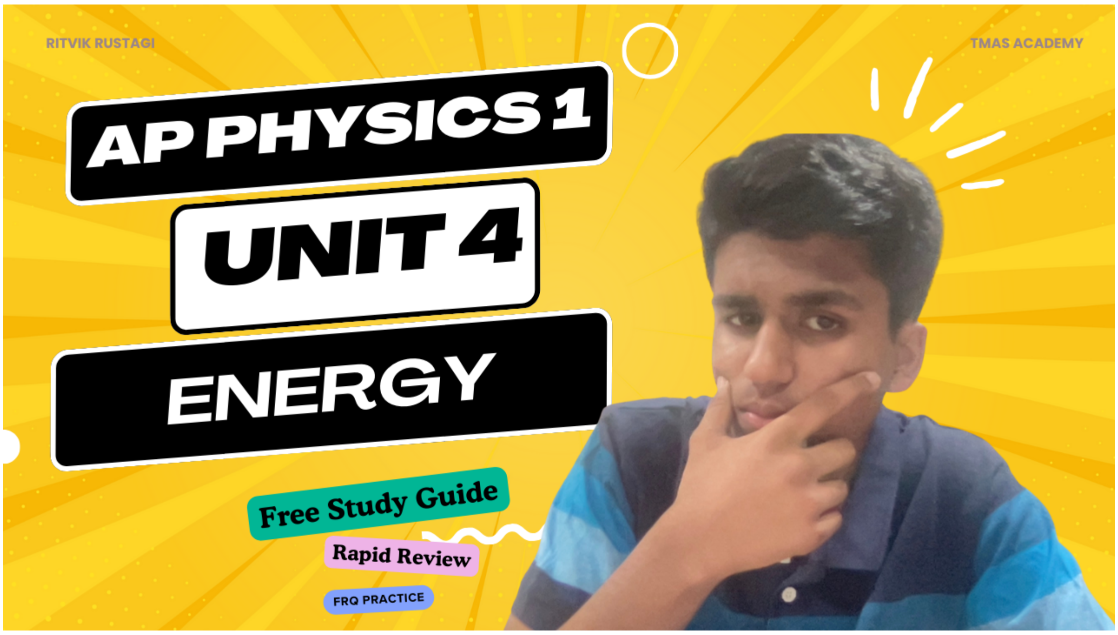
A 2-kilogram block is released from rest at the top of a curved incline in the shape of a quarter of a circle of radius R. The block then slides onto a horizontal plane where it finally comes to rest 8 meters from the beginning of the plane. The curved incline is frictionless, but there is an 8-newton force of friction on the block while it slides horizontally. Assume $g = 10$ meters per second^2.

a. Determine the magnitude of the acceleration of the block while it slides along the horizontal plane.

b. How much time elapses while the block is sliding horizontally?

c. Calculate the radius of the incline in meters.

Solution:



Solution to part a: The problem says that there is a 8 N force of friction on the horizontal surface. This force will point leftwards, so the force is -8 N. It also says that the block weighs 2 kg.
We simply apply $F = ma$ and rearrange it to get $a = \frac{F}{m}$. We plug in $F = -8$ and $m = 2$ to find that the acceleration is $-4\frac{m}{s^2}$

Solution to part b: When the block slides horizontally, it covers a distance of 8m with an acceleration of $-4\frac{m}{s^2}$
We also know that v_f (the final velocity) is 0 since friction slows it down.

We can apply the equation $v_f^2 = v_i^2 + 2ad$
We can plug in our known values to get $0^2 = v_i^2 + 2 \cdot -4 \cdot 8$
Simplifying it gives that $v_i = 8\frac{m}{s}$. This is the velocity as soon as the block enters the horizontal part of the track.

Since we know that $v_i = 8$, $v_f = 0$, and $a = -4$, we can apply the equation $v_f = v_i + at$

We can rearrange that equation to get $t = \frac{v_f - v_i}{a}$. We can plug in our variables into this equation to find that $t = 2$ s

Solution to part c: We can find the radius by conserving energy.
We know that $K_i + U_i = K_f + U_f$
Our initial point is the point where the block is released from at rest. The final point we consider is right when the block hits the horizontal part.

The initial kinetic energy is 0. We can assume that the horizontal part of the track is our reference level which means it has $U_f = 0$
This means our equation simplifies to $U_i = K_f$. All the gravitational potential energy at the top will be converted to kinetic energy.

Gravitational potential energy at the top is mgR since R is the height the block is above the horizontal part of the track.

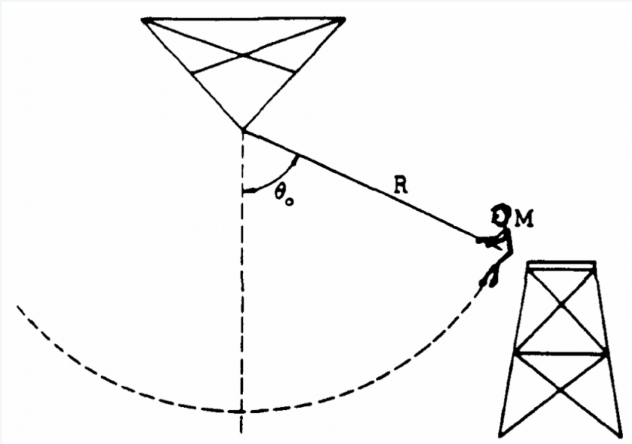
This means $mgR = \frac{1}{2}mv^2$

We can cancel m on both sides to get $gR = \frac{v^2}{2}$.
Now, we divide both sides by g to get $R = \frac{v^2}{2g}$

The velocity that we use is the velocity at the beginning of the horizontal part which was already found to be $8\frac{m}{s}$.

We can plug this in to find that $R = \frac{8^2}{g} = \frac{64}{2g} = 3.265$ m.

Problem 4.0.18 — 1982 AP Physics B FRQ



A child of mass M holds onto a rope and steps off a platform. Assume that the initial speed of the child is zero. The rope has length R and negligible mass. The initial angle of the rope with the vertical is θ_o, as shown in the drawing above.

(a) Using the principle of conservation of energy, develop an expression for the speed of the child at the lowest point in the swing in terms of g, R, and $\cos(\theta_o)$

(b) The tension in the rope at the lowest point is 1.5 times the weight of the child. Determine the value of $\cos(\theta_o)$.

Solution to part a: This problem is an energy conservation problem.
For all energy conservation problems, we should write out our equation.
$K_i + U_i = K_f + U_f$
The initial kinetic energy at the top is 0. We can say that the reference level for gravitational potential energy is the lowest point that the child goes to. This means that $U_f = 0$.

We can simplify the equation to $U_i = K_f$. This means that the initial gravitational potential energy converts to kinetic energy.
The initial gravitational potential energy can be found by first finding the height above the lowest point. Using some trigonometry, it's obvious that the child is a distance $R - R\cos(\theta_o)$ above the lowest point.

This means that the initial gravitational potential energy is $mg(R - R\cos(\theta_o))$
Now, we equate this to the final kinetic energy which can be represented as $\frac{1}{2}mv^2$

After equating both, we get $mg(R - R\cos(\theta_o)) = \frac{1}{2}mv^2$

We can divide both sides by m and then multiply both sides by 2 to get $v^2 = 2g(R - R\cos(\theta_o))$
We can simplify the equation to get $v = \sqrt{2g(R - R\cos(\theta_o))}$

Solution to part b: The two forces on the child are tension force and gravitational force. Both of these forces cause centripetal acceleration.

We can write out our equation as $T - mg = \frac{mv^2}{R}$

Since tension is 1.5 times the weight, we can write this out mathematically to get $T = 1.5mg$
Plugging this into the equation gives $\frac{mg}{2} = \frac{mv^2}{R}$

Now, we can divide both sides by m to get $\frac{g}{2} = \frac{v^2}{R}$

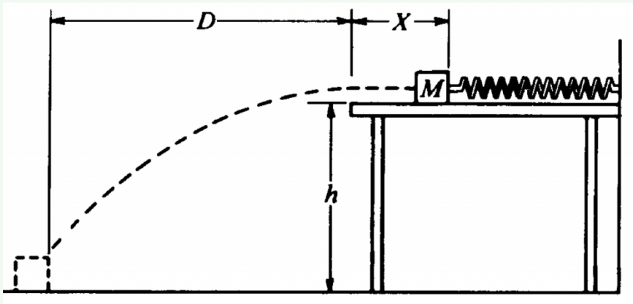
We can plug in our expression for v since we found it in part a ($v = \sqrt{2g(R - R\cos(\theta_o))}$)

$$\text{Plugging it in turns the equation to } \frac{g}{2} = \frac{2g(R - R\cos(\theta_o))}{R} = 2g(1 - \cos(\theta_o))$$

Now, we can divide g from both sides to get $\frac{1}{2} = 2 - 2\cos(\theta_o)$
Multiplying both sides by 2 gives $1 = 4 - 4\cos(\theta_o)$

We can solve the equation to find that $\cos(\theta_o) = \frac{3}{4}$

Problem 4.0.19 — 1986 AP Physics B FRQ



One end of a spring is attached to a solid wall while the other end just reaches to the edge of a horizontal, frictionless tabletop, which is a distance h above the floor. A block of mass M is placed against the end of the spring and pushed toward the wall until the spring has been compressed a distance X, as shown above. The block is released, follows the trajectory shown, and strikes the floor a horizontal distance D from the edge of the table. Air resistance is negligible. Determine expressions for the following quantities in terms of M, X, D, h, and g. Note that these symbols do not include the spring constant.

(a) The time elapsed from the instant the block leaves the table to the instant it strikes the floor

(b) The horizontal component of the velocity of the block just before it hits the floor

(c) The work done on the block by the spring

(d) The spring constant

Solution to part a: The time elapsed from the moment it leaves the table can be found by considering the motion in the y-direction.

The initial velocity in y-direction ($v_o y$) is 0. The acceleration in the y-direction is $a_y = g$. The displacement is $\Delta y = h$

Now, we can use the equation $d = v_o t + \frac{1}{2}at^2$
We can plug in our variables to get $h = \frac{1}{2} \cdot g \cdot t^2$

We can multiply both sides by $\frac{2}{g}$ to get $t^2 = \frac{2h}{g}$.
We can simplify the equation to get $t = \sqrt{\frac{2h}{g}}$.

Solution to part b: There is no acceleration in the x-direction. We should remember this from our projectile motion section in Unit 1.
The initial velocity in x-direction is the same as the final velocity in x-direction.
Since there is no acceleration, we know that $D = vt$ which means $v = \frac{D}{t}$.

We can plug in our expression for t into this to find that $v = D\sqrt{\frac{g}{2h}}$

Solution to part c: The work done on the block by the spring is what caused the block

to gain kinetic energy.
Instead of finding the work done, we can simply find the kinetic energy right at the edge of the table.

We know that the velocity at that point is $v = D\sqrt{\frac{g}{2h}}$.
We can plug this into $K = \frac{1}{2}mv^2$ to get $\frac{MD^2g}{4h}$
The work done is $\frac{MD^2g}{4h}$

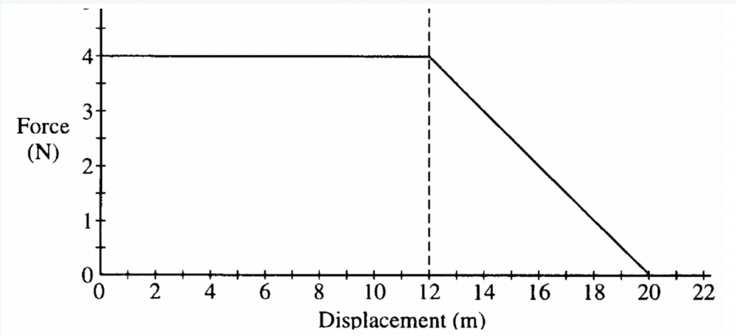
Solution to part d: We know that the work done is equivalent to the kinetic energy. We also know that the kinetic energy was caused by the spring potential energy due to compression.

This means $U_s = K$
We know that $U_s = \frac{1}{2}kx^2$. Since the distance compressed in our case is X, we can plug that in to get $\frac{1}{2}kX^2$
We also know that our kinetic energy is $\frac{MD^2g}{4h}$

We can equate both equations: $\frac{1}{2}kX^2 = \frac{MD^2g}{4h}$

$$\text{We can isolate } k \text{ (spring constant) on one side to find that } k = \frac{MD^2g}{2hX^2}$$

Problem 4.0.20 — 1997 AP Physics B FRQ



A 0.20 kg object moves along a straight line. The net force acting on the object varies with the object's displacement as shown in the graph above. The object starts from rest at displacement $x = 0$ and $timet = 0$ and is displaced a distance of 20 m. Determine each of the following.

(a) The acceleration of the particle when its displacement x is 6 m.
(b) The time taken for the object to be displaced the first 12 m.
(c) The amount of work done by the net force in displacing the object the first 12 m.
(d) The speed of the object at displacement $x = 12$ m.
(e) The final speed of the object at displacement $x = 20$ m.

Solution to part a: When the displacement is 6, the force is 4 N. This can be observed from the graph.

We also know that the mass is 0.2 kg. We can plug in our variables into the equation for Newton's Second Law: $F = ma$.
Doing so gives $4 = 0.2 \cdot a$
This means $a = 20\frac{m}{s^2}$

Solution to part b: Since the force is constant for the first 12 m, we can simply use our kinematic equations because acceleration is constant.
The acceleration for the first 12 m is $20\frac{m}{s^2}$ (we found this in the last part).

We also know that $v_o = 0$ (initial velocity is 0).
We can use the kinematics equation $\Delta x = v_o t + \frac{1}{2}at^2$

We can plug our variables into the equation to get $12 = \frac{1}{2} \cdot 20 \cdot t^2$
This simplifies to $t^2 = 1.2$
We can simplify this to get that $t = 1.095$ s.

Solution to part c: The work done is the area under a force displacement graph.
In this case, we can find the work done for the first 12 m by finding the area under the curve.
it is simply a rectangle with area 48. This means that the work done is 48 N·kg

Solution to part d: To find the speed at displacement $x = 12$ m, we need to know that $W = \Delta K$

The work done is simply the change in kinetic energy.
This means that $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{m}{2}(v_f^2 - v_i^2)$

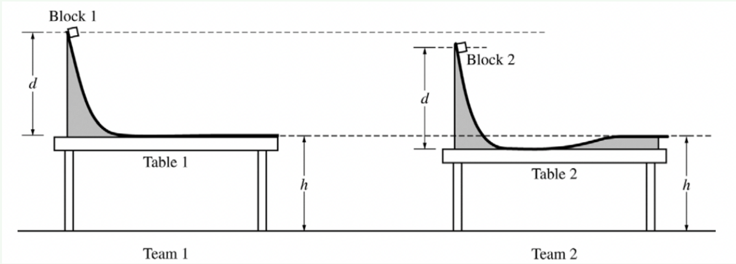
v_i (initial velocity) is just 0. We also know that $W = 48$ (from part c).

We can plug this in along with the mass to find that $48 = \frac{0.2}{2} \cdot v^2$
We can simplify this equation to find that the speed is $21.91\frac{m}{s}$

Solution to part e: Now, we will find the area under the entire curve. We already know that the rectangle has an area of 48. Now, there's a triangle on the right of it with an area of 16.
The total work done is $48 + 16$ which is 64.

We know that the work done is the change in kinetic energy.
This means that $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{m}{2}(v_f^2 - v_i^2)$
v_i (initial velocity) is just 0. We also know that $W = 64$
We can plug this along with the mass to find that $64 = \frac{0.2}{2} \cdot v^2$
Simplifying this equation gives that $v = 25.3$ m/s.

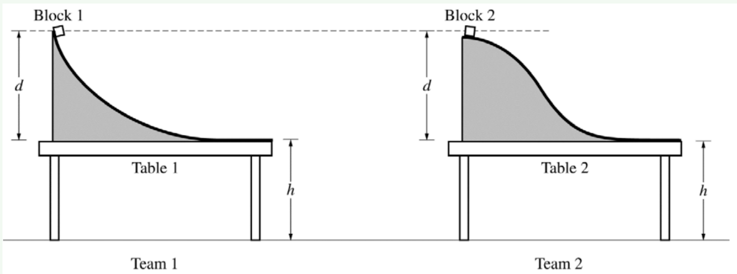
Problem 4.0.21 — 2017 AP Physics 1 FRQ



A physics class is asked to design a low-friction slide that will launch a block horizontally from the top of a lab table. Teams 1 and 2 assemble the slides shown above and use identical blocks 1 and 2, respectively. Both slides start at the same height d above the tabletop. However, team 2's table is lower than team 1's table. To compensate for the lower table, team 2 constructs the right end of the slide to rise above the tabletop so that the block leaves the slide horizontally at the same height h above the floor as does team 1's block (see figure above).

(a) Both blocks are released from rest at the top of their respective slides. Do block 1 and block 2 land the same distance from their respective tables?
_____ Yes _____ No
Justify your answer.

In another experiment, teams 1 and 2 use tables and low-friction slides with the same height. However, the two slides have different shapes, as shown below.



(b) Both blocks are released from rest at the top of their respective slides at the same time.
i. Which block, if either, lands farther from its respective table?
_____ Block 1 _____ Block 2 _____ The two blocks land the same distance from their respective tables.
Briefly explain your reasoning without manipulating equations.
ii. Which block, if either, hits the floor first?
_____ Block 1 _____ Block 2 _____ The two blocks hit the floor at the same time.
Briefly explain your reasoning without manipulating equations.

Solution to part a: The slide is the same distance above the ground. This means that both blocks will take the same amount of time from the end of the slide to reach the ground.

We know that time is constant for that part of the motion. However, we must now find the velocity in the x-direction when both leave the slide.

Block 1 moves down a distance d before reaching the end of its slide. However, block 2 travels a distance that is a little less than d before reaching the end of its slide.

This means that block 1 has a higher change in gravitational potential energy due to this larger distance. This higher change in gravitational potential energy will become kinetic energy due to conservation of energy. Block 1 will ultimately have more kinetic energy at the end of its slide. Higher kinetic energy corresponds to a higher speed at the end of the slide.

Since block 1 has a higher speed than block 2 at the end of the slide and both blocks take the same amount of time to reach the ground, block 1 will travel a greater distance which can be found through the formula $d = vt$ (and v is larger for block 1). Thus, the answer is "No" since the blocks will not land the same distance from the table.

Solution to part b i: At the end of the slide, both blocks will have the same speed. The reason is that they both go down the same height of d. All of the potential energy that will be lost will become kinetic energy. Since both blocks are identical and have the same mass, we know that they will have the same speed after.

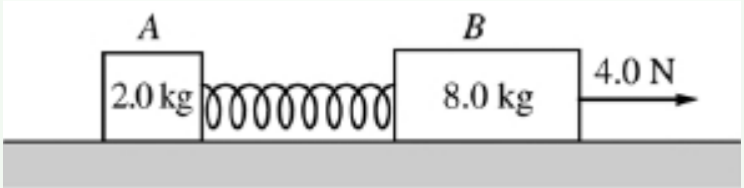
Both of the blocks will also spend the same amount of time in the air since they both fall a distance h from the end of the slide to the ground.

Since the time of the motion and horizontal speed are both constant for blocks, they both will end up travelling the same horizontal displacement from the table. Thus, the answer is "The two blocks land the same distance from their respective tables."

Solution to part b ii: This problem require us to infer a bit by looking at the shape of the slides. The slide for block 1 is a lot steeper initially. This will cause it to speed up a lot faster. This means that block 1 will reach a higher velocity quickly leading to an increase in the average speed. On the other hand, block 2 will take longer to reach a higher speed since the slide isn't as steep. Although both block 1 and 2 will have the same speed at the end of the slide, block 1 will have a higher average speed due to the steepness of its slide.

Block 1's high average speed will cause it to hit the floor first.

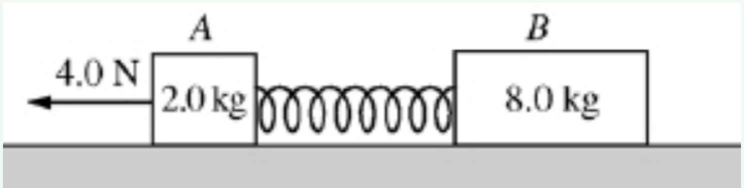
Problem 4.0.22 — 2008 AP Physics B FRQ



Block A of mass 2.0 kg and block B of mass 8.0 kg are connected as shown above by a spring of spring constant 80 N/m and negligible mass. The system is being pulled to the right across a horizontal frictionless surface by a horizontal force of 4.0 N, as shown, with both blocks experiencing equal constant acceleration.

(a) Calculate the force that the spring exerts on the 2.0 kg block.
(b) Calculate the extension of the spring.

The system is now pulled to the left, as shown below, with both blocks again experiencing equal constant acceleration.



(c) Is the magnitude of the acceleration greater than, less than, or the same as before?
______ Greater ______ Less ______ The same
Justify your answer.

(d) Is the amount the spring has stretched greater than, less than, or the same as before? ______ Greater ______ Less ______ The same
Justify your answer.

(e) In a new situation, the blocks and spring are moving together at a constant speed of 0.50 m/s to the left. Block A then hits and sticks to a wall. Calculate the maximum compression of the spring.

Solution to part a: Let's denote the spring force as F_s.
Since the spring force acts in the opposite direction for block B, we can write its Newton's Second Law Equation as $F - F_s = m_b a$ (F is the applied force on block B)
For block A, the spring force causes its acceleration and pulls it to the right. The Newton's Second Law Equation for block A is $F_s = m_a a$

We can add both of the equations we found to get $F = (m_b + m_a)a$
We can substitute our known values to get $4 = (8 + 2)a$
We can solve it to find that $a = 0.4\frac{m}{s^2}$

Now, we can substitute this value of acceleration to the equate for block A which was $F_s = m_a a$. Since we know that $m_a = 2$ kg, we can find that $F_s = 2 \cdot 0.4 = 0.8$ N.

Solution to part b: Since we know that the spring force is 0.8 N, we can use the equation $F_s = k{\cdot}\Delta x$ to find the extension.
We can plug in $F_s = 0.8$ and $k = 80$ to find that $x = 0.01$ m.

Solution to part c: The magnitude of acceleration will be the **same**.
The reason is that the net force on the two block system will still be the same. Since the force i the same and mass is also constant, acceleration will be the same.

Solution to part d: The spring will be stretched **more**.
The reason is that the spring force is causing block B to accelerate. Since block B has a larger mass than block A, it will need a larger force to be able to accelerate at the same rate.
For the spring force to be larger, the distance stretched must be greater for this case.

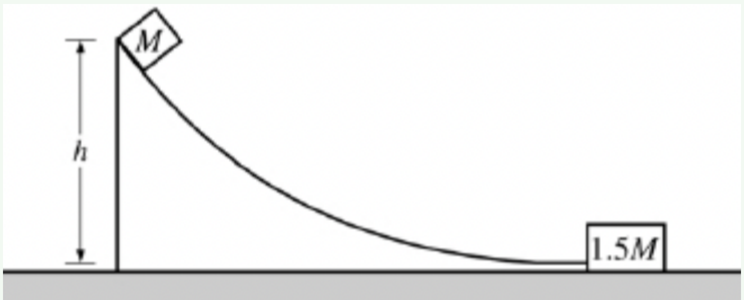
Solution to part e: Right after block A hits and sticks to the wall, block B will continue to move and now compress the spring.
Its kinetic energy will be transferred to spring potential energy.
We can write the equation $K = U_s$ which means $\frac{1}{2}m_b v^2 = \frac{1}{2}kx^2$

We can plug in $m_b = 8$, $v = 0.5$, and $k = 200$ into the equation.

$$\text{Plugging values in gives } \frac{1}{2} \cdot 8 \cdot 0.5^2 = \frac{1}{2} \cdot 200 \cdot x^2$$

We can solve for x (the distance the spring compresses) and find that it equals 0.16 m.

Problem 4.0.23 — 2006 AP Physics B FRQ



A small block of mass M is released from rest at the top of the curved frictionless ramp shown above. The block slides down the ramp and is moving with a speed $3.5v_o$ when it collides with a larger block of mass $1.5M$ at rest at the bottom of the incline. The larger block moves to the right at a speed $2v_o$ immediately after the collision.
Express your answers to the following questions in terms of the given quantities and fundamental constants.

(a) Determine the height h of the ramp from which the small block was released.

(b) The larger block slides a distance D before coming to rest. Determine the value of the coefficient of kinetic friction μ between the larger block and the surface on which it slides.

Solution to part a: We can find the height from which the block was released by using the conservation of energy theorem.
$K_i + U_i = K_f + U_f$
We'll find the initial kinetic energy at the top and the initial potential energy at the top. Then, for the final energy variables we'll use the point right before collision.

Initially, there is no kinetic energy since the block is at rest. Also, we can say that the bottom of the incline is our reference level when finding gravitational potential energy. That means the initial gravitational potential energy is mgh (since it's a distance h above our reference level.

Since $K_i = 0$ and $U_f = 0$, our equation is $U_i = K_f$
This means that the initial gravitational potential energy for the lighter block will convert to kinetic energy.

We know that the light block has a speed $3.5v_o$ at the bottom. That means the final kinetic energy is $\frac{1}{2}M(3.5v_o)^2$

We can now equate our initial gravitational potential energy to the final kinetic energy.

$Mgh = \frac{1}{2}M(3.5v_o)^2$
Cancelling M from both sides gives $gh = \frac{(3.5v_o)^2}{2}$

$$\text{We can isolate } h \text{ in the equation to find that } h = \frac{6.125 \cdot v_o^2}{g}$$

Solution to part b: We know that $W = \Delta K$
This means that the work done is the change in kinetic energy.

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The final speed of the large block is 0 since friction slows it down. The initial speed as already stated in the problem is $2v_o$. The mass of the block is $1.5M$

Also, the work done by the friction force can be found by using the equation $W = F \cdot \Delta x$. This means that the work done is the force times displacement.

We know that the force is $-\mu mg$ (since friction force is the frictional coefficient times normal force). Δx is D since that's the distance the block slides. This means that the work done on the block by the friction force is $-\mu mgD$

Now, we can plug our expression for the work done into $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$$-\mu mgD = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Although we could plug in $1.5M$ for the mass, we don't need to do that since the variable for mass cancels out in this equation.

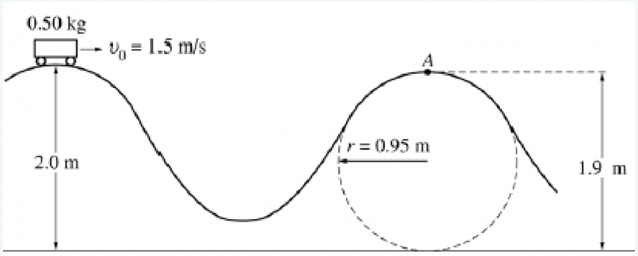
$$\text{It simplifies to } -\mu gD = \frac{v_f^2}{2} - \frac{v_i^2}{2}$$

Now we can plug in $v_i = 2v_o$ and $v_f = 0$ to get

$$-\mu g D = 0 - 2v_o^2$$

We can simplify this equation to find $\mu = \dfrac{2v_o^2}{gD}$

Problem 4.0.24 — 2004 AP Physics B FRQ



A designer is working on a new roller coaster, and she begins by making a scale model. On this model, a car of total mass 0.50 kg moves with negligible friction along the track shown in the figure above. The car is given an initial speed $v_0 = 1.5\,\text{m/s}$ at the top of the first hill of height 2.0 m. Point A is located at a height of 1.9 m at the top of the second hill, the upper part of which is a circular arc of radius 0.95 m.

(a) Calculate the speed of the car at point A.

(b) On the figure of the car below, draw and label vectors to represent the forces on the car at point A.

(c) Calculate the magnitude of the force of the track on the car at point A.

(d) In order to stop the car at point A, some friction must be introduced. Calculate the work that must be done by the friction force in order to stop the car at point A.

(e) Explain how to modify the track design to cause the car to lose contact with the track at point A before descending down the track. Justify your answer.

Solution to part a: The speed of the car at point A can be found by conserving energy.

$$K_i + U_i = K_f + U_f$$

To go from the starting point to point A, we know that the car will lose gravitational potential energy. This energy will convert to additional kinetic energy.

Using $\Delta U = mgh$, we can find that $h = 0.1$ m since the starting point is 0.1 m above point A.

Since $K = \frac{1}{2}mv^2$, we can substitute our known values into the equation for conservation of energy. Remember that there is kinetic energy both initially and after.

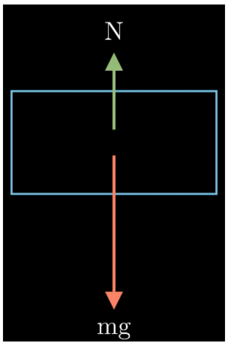
$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2$$

$$\text{We can cancel out } m \text{ from the equation to get } \frac{v_i^2}{2} + gh = \frac{v_f^2}{2}$$

$$\text{We can plug in our known variables to get: } \frac{1.5^2}{2} + 9.8 \cdot 0.1 = \frac{v_f^2}{2}$$

We can solve the equation to find that $v_f = 2.052$ m/s

Solution to part b: The gravitational force will have a greater magnitude than the normal force. The reason is that the cart is undergoing circular motion. Thus, there must be a net force pointing downwards towards the center; this net force will be causing the centripetal motion.



Solution to part c: The force of the track on the car will come from the normal force. We can write a Newton's Second Law equation as $mg - N = ma$
$a = \frac{v^2}{r}$ since the cart will be in circular motion at that point.

We can plug in $a = \frac{v^2}{r}$ to get $mg - N = \frac{mv^2}{r}$
We can rearrange the equation to get $N = m(g - \frac{v^2}{r})$

$$\text{We can plug in our known variables to get } N = 0.5(9.8 - \frac{2.052^2}{0.95}$$

We can do some arithmetic to find that the normal force $N = 2.684$ N.

Solution to part d: The work done by friction will equal to the change in kinetic energy.

$$W_f = \Delta K = K_f - K_i$$

The final kinetic energy will be 0 since the cart will be at rest after friction does work on it. The initial kinetic energy can be found by using the equation $\frac{1}{2}mv^2$

$$K_i = \frac{1}{2} \cdot 0.5 \cdot 2.052^2 = 1.053$$

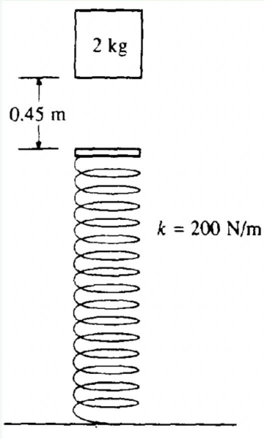
This means the work done by friction is $W_f = 0 - 1.053 = -1.053$ J

Solution to part e: The cart will lose contact with the track when there is no normal force. We can use the expression for normal force that we found in part c and figure out when this happens.

We know that $N = m(g - \frac{v^2}{r})$
We need to increase $\frac{v^2}{r}$ to make the normal force as close to 0. This means we can either increase v at point A or decrease r (the radius).

We can increase v by simply making the height of the initial hill higher. This will cause the change in gravitational potential energy to be greater leading to greater kinetic energy at point A.

Problem 4.0.25 — 1989 AP Physics C: Mechanics FRQ



A 2-kilogram block is dropped from a height of 0.45 meter above an uncompressed spring, as shown above. The spring has an elastic constant of 200 newtons per meter and negligible mass. The block strikes the end of the spring and sticks to it.

(a) Determine the speed of the block at the instant it hits the end of the spring
(b) Determine the force in the spring when the block reaches the equilibrium position
(c) Determine the distance that the spring is compressed at the equilibrium position
(d) Determine the speed of the block at the equilibrium position
(e) Determine the resulting amplitude of the oscillation that ensues
(f) Is the speed of the block a maximum at the equilibrium position, explain.
(g) Determine the period of the simple harmonic motion that ensues

Solution to part a: We can use conservation of energy to find the speed of the block when it hits the end of the spring.
$K_i + U_i = K_f + U_f$
By the time the block hits the end of the spring, it has moved down a distance of h. That means it loses gravitational potential energy, and the change in gravitational potential energy can be represented as mgh where $h = 0.45$ m.

The initial kinetic energy will be 0 since it is dropped at rest.
This means our equation becomes $\Delta U = K_f$ since the gravitational potentia energy is converted to kinetic energy

We can write this as $mgh = \frac{1}{2}mv^2$

We can cancel out mass m and simplify more to find that $v = \sqrt{2gh}$

$$\text{Since } h = 0.45, \text{ we know that } v = \sqrt{2 \cdot 9.8 \cdot 0.45} = 2.97\frac{m}{s}$$

Solution to part b: At equilibrium, the spring force balances the gravitational force.

That means we can just find the gravitational force on the block since that will equal to the magnitude of the spring force.
$F_s = mg = 2 \cdot 9.8 = 19.6$ N

Solution to part c: We can find that distance the spring is compressed by using the equation $F_s = k\Delta x$
Since we know that $F_s = mg$, we also know that $k\Delta x = mg$.

We can rearrange the equation to find that $\Delta x = \frac{mg}{k}$.

$$\text{Since } mg = 19.6 \text{ and } k = 200, \text{ we can plug this in to find that } x = \frac{19.6}{200} = 0.098 \text{ m.}$$

Solution to part d: Now, our new reference level will be the equilibrium point.
We will use the point from where we drop the 2 kg block.

The height of the point where we drop the block from is $h = 0.45 + 0.098 = 0.548$ m above the equilibrium point. The reason is that we must account for both the distance h and the distance that the spring is compressed.

Now, our conservation equation will no longer just be $K_i + U_i = K_f + U_f$
The reason is that now we also have spring potential energy. It will be

$$K_i + U_i + U_{si} = K_f + U_f + U_{sf}$$

Some of the forms of energy will be 0, so we can simplify the equation.

We can use the equation $mgh = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ to find the speed at the equilibrium point. The reason is that at the point where the ball is released initially, it only has gravitational potential energy. However, it will convert to kinetic and spring potential energy by the time it reaches the equilibrium point.

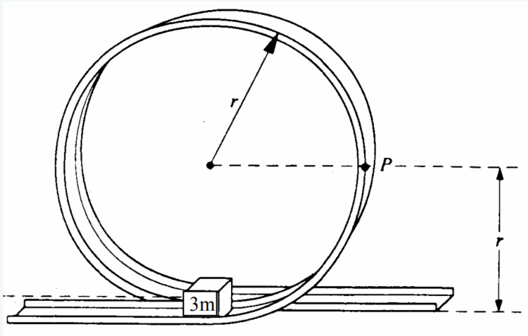
We already know that $h = 0.548$ and $x = 0.098$ (since that's the distance the spring has been compressed)
We can plug in our variables into the equation to get:

$$2 \cdot 9.8 \cdot 0.548 = \frac{1}{2} \cdot 2 \cdot v^2 + \frac{1}{2} \cdot 200 \cdot 0.098^2$$

After evaluating our expression, we can find that $v = 3.13$ m/s.

Solution to part e: Yes. The reason is that this is the point where the gravitational force and spring force balance each other. This means that the acceleration is 0. At any point above or below the equilibrium point, acceleration will oppose the motion causing the object to decelerate (speed will decrease).

Problem 4.0.26 — 1991 AP Physics C: Mechanics FRQ



A small block of mass $3m$ moving at speed $v_o/3$ enters the bottom of the circular, vertical loop-the-loop shown above, which has a radius r. The surface contact between the block and the loop is frictionless. Determine each of the following in terms of m, v_o, r, and g.

(a) The kinetic energy of the block and bullet when they reach point P on the loop

(b) The speed v_{min} of the block at the top of the loop to remain in contact with track at all times

(c) The new required entry speed v_o at the bottom of the loop such that the conditions in part b apply

Solution to part a: We can conserve energy to find the kinetic energy at point P.
$K_i + U_i = K_f + U_f$
We can set our reference level to be the bottom of the loop. This will cause initial potential energy (U_i) to be 0.

We know that kinetic energy $K = \frac{1}{2}mv^2$
The initial kinetic energy at the bottom of the loop is $\frac{1}{2} \cdot 3m \cdot (\frac{v_o}{3})^2$ which is $\frac{mv_o^2}{6}$
We also know that potential energy is mgh. This means that $U_f = 3mgr$ since point P is a distance r above the bottom of the loop and the mass is $3m$.

We can plug these expressions into our conservation of energy formula to get

$$\frac{mv_o^2}{6} = K_f + 3mgr$$

Basically, when the block enters the bottom of the cylinder, it will have kinetic energy. Some of that kinetic energy will be lost when it reaches point P since it will turn into gravitational potential energy.

Subtracting $3mgr$ from both sides gives that $K_f = \frac{mv^2}{6} - 3mgr$

Solution to part b: The block will remain in contact with the loop when a normal force exists. We don't want the normal force to be 0. To find the minimum velocity, we'll set the normal force to 0 (since it allows us to find the "limiting" value).

At the top, the two forces on the block are normal force and gravitational force. Both are pointing downwards.
Using Newton's Second Law, we can write the equation $mg + N = \frac{mv^2}{r}$
We can plug in $3m$ for the mass to get $3mg + N = \frac{3mv^2}{r}$.

Now, we substitute $N = 0$ to get $3mg = \frac{3mv^2}{r}$

We can solve this to find that $v = \sqrt{rg}$

This means that $v_{min} = \sqrt{rg}$

Solution to part c: We will use conservation of energy to find the new required entry speed v_o'

We know that $K_i + U_i = K_f + U_f$. U_i is simply 0 since the bottom of the circular track is our reference level.
This means that $K_i = K_f + U_f$
The final point that we are considering is the top of the loop.

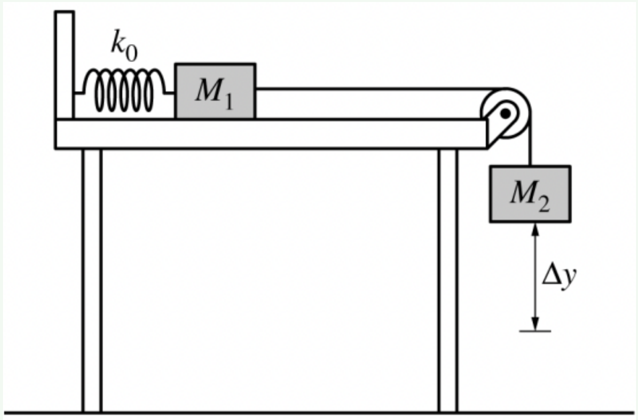
The initial kinetic energy is $\frac{3mv_o'^2}{2}$. The final kinetic energy at the top is $\frac{1}{2} \cdot 3m \cdot (\sqrt{rg})^2$ which is $\frac{3mrg}{2}$
The final potential energy can be found by using the equation mgh. Since the mass is $3m$ and the top of the loop is $2r$ above the bottom of the loop, $U_f = 6mrg$

We can plug in our expressions for the energies to get

$$\frac{3mv_o'^2}{2} = \frac{3mrg}{2} + 6mrg = \frac{15mrg}{2}$$

We can cancel $\frac{3m}{2}$ from both sides to get $v_o'^2 = 5rg$
We can take the square root of both sides to get $v_o' = \sqrt{5rg}$

Problem 4.0.27 — 2022 AP Physics 1 FRQ



Two blocks are connected by a string that passes over a pulley, as shown above. Block 1 is on a horizontal surface and is attached to a spring that is at its unstretched length. Frictional forces are negligible in the pulley's axle and between the block and the surface. Block 2 is released from rest and moves downward before momentarily coming to rest.
k_0 is the spring constant of the spring
M_1 is the mass of block 1
M_2 is the mass of block 2
Δy is the distance block 2 moves before momentarily coming to rest
(a)

i. Block 2 starts from rest and speeds up, then it slows down and momentarily comes to rest at a position below its initial position. In terms of only the forces directly exerted on block 2, explain why block 2 initially speeds up and explain why it slows down to a momentary stop.

ii. Derive an expression for the distance Δy that block 2 travels before momentarily coming to rest. Express your answers in terms of k_0, M_1, M_2, and physical constants, as appropriate.

(b) Indicate whether the total mechanical energy of the blocks-spring-Earth system changes as block 2 moves downward.
______ Changes ______ Does not change
Briefly explain your reasoning.

Solution to part a i: Block 2 initially speeds up since the gravitational force is stronger than tension. However, it slows down because the magnitude of tension force becomes larger than gravitational force.

Solution to part a ii: We can conserve energy in this problem.
Since energy is conserved, we know $K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}$
There are two potential energies on each side since one is spring potential energy while the other is gravitational potential energy.

The initial and final kinetic energies for each block is 0. The reason is that they both start at rest and momentarily end at rest.

This means the equation can be simplified to $U_{gi} + U_{si} = U_{gf} + U_{sf}$

In this problem, there will be no spring potential energy initially. The reason is that the spring isn't stretched. Eventually, the system of the two blocks will slide causing Block 2 to move down. Thus, gravitational potential energy will be lost. However, spring potential energy will increase since the spring will become stretched. This means that the change in gravitational potential energy will transform to spring potential energy.

Using this, we can know that $\Delta U_g = U_s$

$$\text{Since } M_2 \text{ moves down } \Delta y, U_g = M_2 g \Delta y$$

The spring also gets stretched by a distance Δy since that's the distance block 2 moves down. This means that the spring potential energy at the end is $\frac{1}{2}k_0\Delta y^2$

$$\text{We can substitute this to get } M_2 g y = \frac{1}{2}k_0\Delta y^2$$

$$\text{We can solve for } \Delta y \text{ to find that } \Delta y = \frac{2M_2 g}{k}$$

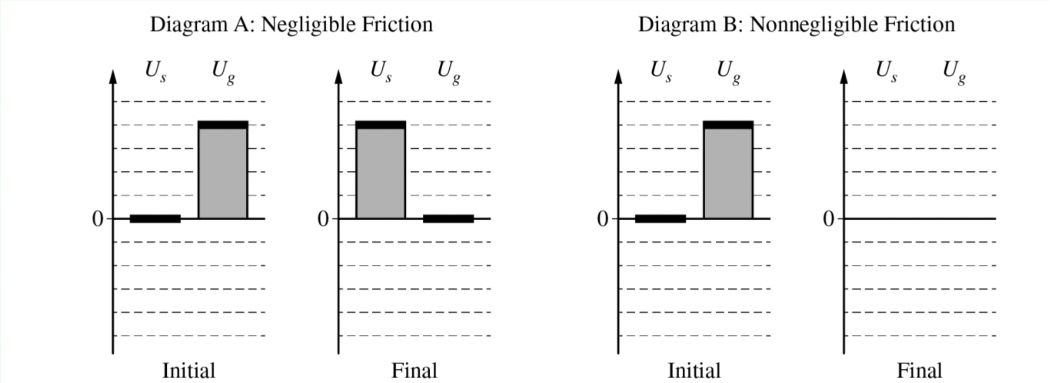
Solution to part b: Mechanical energy **does not change**. The reason is that there are no conservative forces such as friction acting on the system. Thus, mechanical energy stays the same.

Problem 4.0.28 — 2022 AP Physics 1 FRQ Continued

This is a continuation to the FRQ from above. Make sure to refer back to it to solve this part.

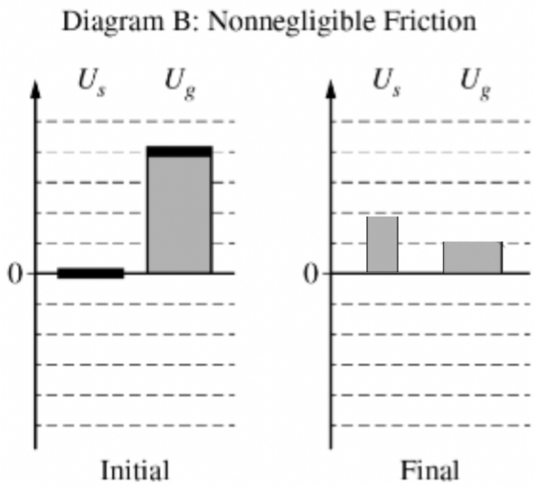
Consider the system that includes the spring, Earth, both blocks, and the string, but not the surface. Let the initial state be when the blocks are at rest just before they start moving, and let the final state be when the blocks first come momentarily to rest. Diagram A at left below is a bar chart that represents the energies in the scenario where there is negligible friction between block 1 and the surface.

The shaded-in bars in the energy bar charts represent the potential energy of the spring and the gravitational potential energy of the blocks-Earth system, U_s and U_g, respectively, in the initial and final states. Positive energy values are above the zero-point line ("0") and negative energy values are below the zero-point line.



(c) Complete diagram B (at right above) for the scenario in which friction is non-negligible. The energies for the initial state are already provided. Shade in the energies in the final state using the same scale as in diagram A.

Solution to part c: Since friction is non-negligible, energy will now be lost. This means that the final sum of U_s and U_g must be less than the initial sum. The initial sum is 4 units. As long as our diagram has a sum of less than 4 units for the final stage, then we will earn the point on the AP exam.



Note that the height of the graph matters. The width doesn't. Don't waste time trying to get equally wide graphs. I made the graph for U_g wider to explain that only the heights of the bars matter.

Unit 5 Momentum

Have you ever wondered what makes collisions so dangerous? If a car collides with a wall, then why is it so dangerous?

To answer such questions, you must dive deep into this unit and learn about momentum.

Note 5.0.1

Momentum

The momentum (p) of an object is it's mass times velocity (m · v).
The momentum of a *system* is defined as the sum of each object's momentum in that system

Clearly, since momentum is proportional to mass, an object with a larger mass will have a larger momentum. A lighter mass will have a smaller momentum.

Similarly, an object moving at a high speed will have a greater magnitude of momentum in contrast to a slow object.

To conceptually think about this scenario, try to picture a photon of light coming towards you. Light is the fastest thing. However, when it strikes you, you won't feel anything.
The reason is that a photon of light has negligible mass. It's mass is so so so small, then it's momentum is negligible. However, if a car runs straight into you, then the momentum will be great. Even though the car has a lower velocity than light, the mass of a car is much larger. That is why a car colliding with a person can have disastrous effects.

Note 5.0.2

Impulse

On an object, if there is a force F applied for a time t, then we say that the impulse on the object is

$$J = F\Delta t.$$

Another important thing is that the change in momentum of an object is the impulse applied on it:

$$J = \Delta p = m\Delta v.$$

In addition, if we know the average force on an object, then we can also say that the impulse is $F_{avg}t$

If you are given a force-time graph, then the area under the curve is the impulse.

If you know your change in momentum, then you can divide it by time to find the average force.

$$F_{\text{avg}} = \frac{\Delta p}{t}$$

Now, we know how to find the average force given the impulse. Why does this make colliding with a wall so dangerous?

In an accident, a car rapidly comes to rest. This causes the average force on it to be extremely high. This average force can shatter the car and cause the passengers many serious health problems. The large impulse occurs over such a small period of time, and this maximizes the average force.

Note 5.0.3

The Law of Conservation of Momentum states that as long as there are no external forces, the momentum of a system will be conserved.

In AP Physics conservation of momentum problems, you should make an equation which equates the initial and final momentum, such as the following:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$

Some values will be given while some will be unknown. You can use such an equation to find the unknown values.

Note 5.0.4

Elastic Vs. Inelastic Collisions

In an elastic collision, not only is momentum conserved, but kinetic energy is also conserved.

However, in an inelastic collision, although momentum is conserved, kinetic energy is NOT conserved.

In an inelastic collision, some energy is lost through heat or sound. In a **completely inelastic collision**, the objects stick together and move at the same velocity after colliding.

If a problem tells you that two objects stick together after collision, then you can automatically say that it's an inelastic collision. Momentum will be conserved, but kinetic energy will not.

If you are not told whether a collision is inelastic or elastic, you should find the initial kinetic energy of the system and the final kinetic energy. If both quantities equate, then it's an elastic collision. However, if they don't equate, then it's an inelastic collision because kinetic energy wasn't conserved.

Problem 5.0.5 — Two sumo wrestlers are in a match. At the start of the match, they both lunge at each other. They hit and miraculously come to a stand still. One wrestler was 200kg and traveling at a velocity of 2.3ms at the instance of collision. If the other wrestler was traveling at 2.9ms, what is his mass?

Solution: In this case, we can apply the law of conservation of momentum. Since they come to a "stand still" after the collision, the final momentum is zero. Hence, the initial momentum must also be zero. That is,

$$m_1 v_{1i} + m_2 v_{2i} = 0$$

$$\implies m_1 = \frac{m_2 v_{2i}}{v_1} = \frac{(200)(2.3)}{2.9} = \boxed{159\text{kg}}.$$

Problem 5.0.6 — Initially, a car of mass m_1 is moving at a speed v_1 towards another car of mass m_2 at rest. Eventually, the two cars collide, and a completely inelastic collision occurs. What is the speed of both cars after they collide.

Solution: Since this is a collision, momentum is conserved. However, energy is not because it is given that the collision is inelastic. After colliding, both cars move at the same speed which we assume is v_f.

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$

m_2 is at rest initially, so $m_1 v_1 = m_1 v_f + m_2 v_f$

$$\text{Dividing both sides by } m_1 + m_2 \text{ gives that } v_f = \frac{m_1 v_1}{m_1 + m_2}$$

Problem 5.0.7 — A 0.15kg baseball is thrown with a speed of 40m/s. If it takes 0.7s for the baseball to come to rest in the catcher's glove, what is the average force the catcher experiences due to the ball?

Recall that $J = Ft$. In this case, F varies, so the equation is

$$J = F_{\text{avg}} t$$

$$\implies F_{\text{avg}} = \frac{J}{t} = \frac{\Delta mv}{t} = \frac{(0.15)(40)}{0.7} = \boxed{8.57\text{N}}.$$

Note 5.0.8

Center Of Mass Center Of Mass (COM) is the point where the mass is balanced in a gravitational field.

$$x_{\text{cm}} = \frac{\sum_{i=1}^{N} m_i x_i}{M}$$

To find the center of mass in a two (or even three) dimensional coordinate system, we can find the center of mass in each dimension using the one dimensional formula above. This will give us a final coordinate (x_{cm}, y_{cm}, z_{cm}), which is our center of mass in the 3 dimensional space.

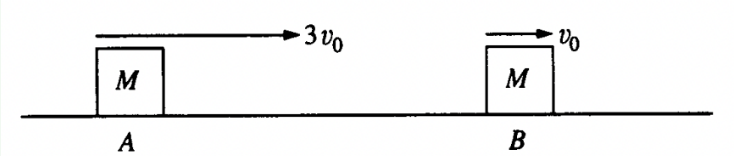
An important idea to know is that if the net force is 0, then the acceleration of the COM is 0.

It is very unlikely that you will need to use the formula for the center of mass on the AP Exam. However, you should conceptually understand what the center of mass is and the idea behind it.

Many of you might be wondering the reason regarding why momentum is conserved in a collision.

We know that when two objects collide, they will each apply a force on each other. However, that force is an **internal force** when we consider both objects to be part of the same system. Although that force changes the velocity of each individual object, the velocity of the **center of mass** remains constant. That is why momentum is conserved in a collision.

Problem 5.0.9 — 1996 AP Physics B FRQ



Two identical objects A and B of mass M move on a one-dimensional, horizontal air track. Object B initially moves to the right with speed v_0. Object A initially moves to the right with speed $3v_0$, so that it collides with object B. Friction is negligible. Express your answers to the following in terms of M and v_0:

a. Determine the total momentum of the system of the two objects.

b. A student predicts that the collision will be totally inelastic (the objects stick together on collision). Assuming this is true, determine the following for the two objects immediately after the collision.

i. The speed

ii. The direction of motion (left or right)

When the experiment is performed, the student is surprised to observe that the objects separate after the collision and that object B subsequently moves to the right with a speed $2.5v_0$.

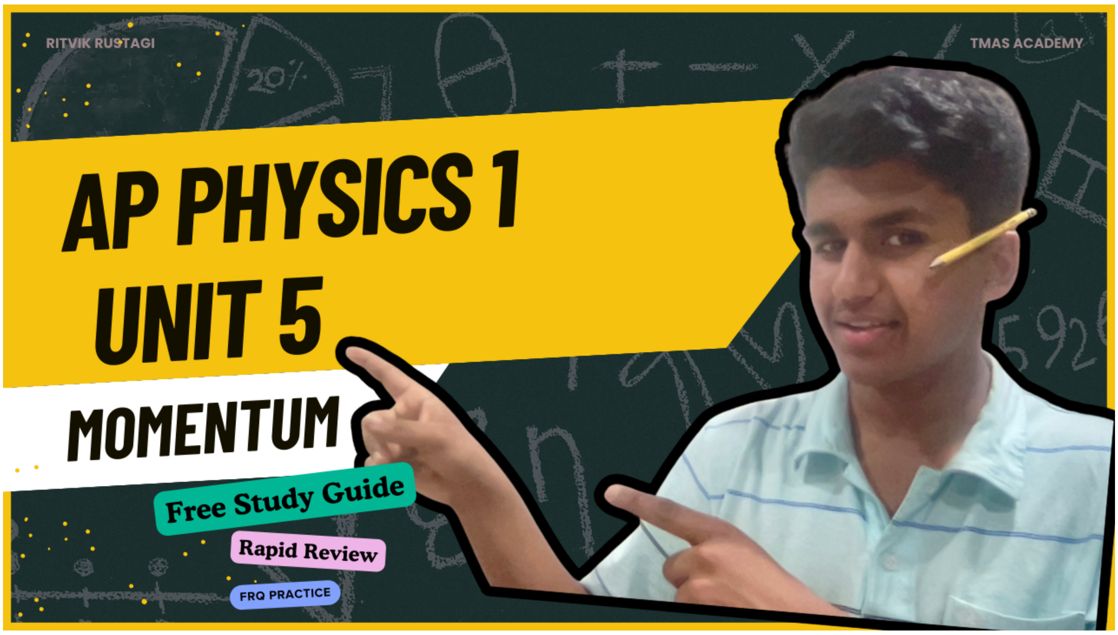
c. Determine the following for object A immediately after the collision.

i. The speed

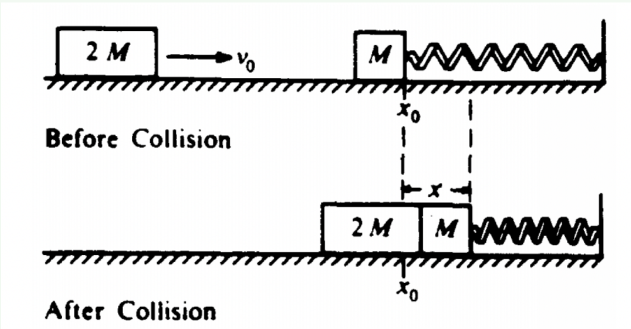
ii. The direction of motion (left or right)

d. Determine the kinetic energy dissipated in the actual experiment.

Solution: Video Solution



Problem 5.0.10 — 1983 AP Physics B FRQ

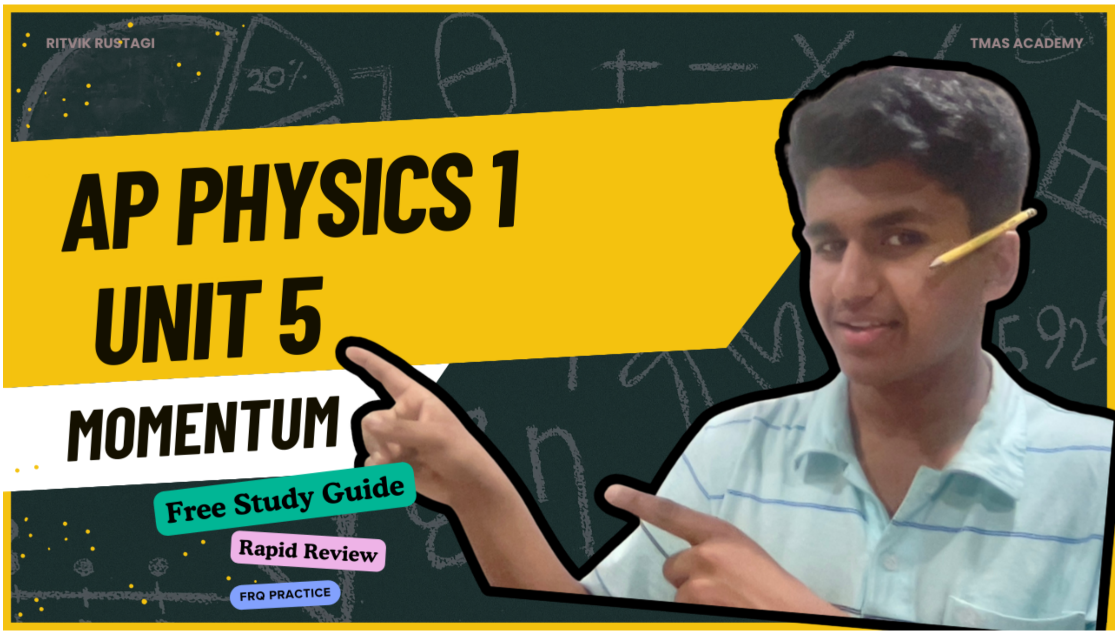


A block of mass M is resting on a horizontal, frictionless table and is attached as shown above to a relaxed spring of spring constant k. A second block of mass $2M$ and initial speed v_0 collides with and sticks to the first block. Develop expressions for the following quantities in terms of M, k, and v_0.

a. v, the speed of the blocks immediately after impact

b. x, the maximum distance the spring is compressed

Solution: Video Solution



Problem 5.0.11 — 2008 AP Physics B FRQ

A 70 kg woman and her 35 kg son are standing at rest on an ice rink, as shown above. They push against each other for a time of 0.60 s, causing them to glide apart. The speed of the woman immediately after they separate is 0.55 m/s. Assume that during the push, friction is negligible compared with the forces the people exert on each other.

(a) Calculate the initial speed of the son after the push.

(b) Calculate the magnitude of the average force exerted on the son by the mother during the push.

(c) How do the magnitude and direction of the average force exerted on the mother by the son during the push compare with those of the average force exerted on the son by the mother? Justify your answer.

(d) After the initial push, the friction that the ice exerts cannot be considered negligible, and the mother comes to rest after moving a distance of 7.0 m across the ice. If their coefficients of friction are the same, how far does the son move after the push?

Solution to part a: Before they separate from each other, the momenta of both people is simply 0. The reason is that both aren't moving which means velocity is 0.

However, after they separate from each other, the woman and her son move in opposite directions. We can conserve momentum in this problem.

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

m_1 is the mass of the woman while m_2 is the mass of the son.

Since initial velocity for both is 0, the equation becomes

$$0 = m_1 v_{1f} + m_2 v_{2f}$$

From here, we are already given that v_{1f} is 0.55 m/s. We can use this value to find the final velocity for the son.

Plugging in our values gives $0 = 70 \cdot 0.55 + 35 \cdot v_{2f}$

We can solve this to get that v_{2f} is -1.1. Since the problem asks for the speed, we can simply take the absolute value of -1.1 to get 1.1 m/s as our final answer.

Solution to part b: Whenever a problem related to a collision asks us to find the force, we should think about the impulse-momentum theorem since it relates force to the change in momentum.

$$F\Delta t = m(v_f - v_i)$$

We already know that our Δt is 0.6 s since that's the amount of time the collision lasted.

The son's final speed was 1.1 and their initial speed was 0. We can plug these values in to find the average force.

$$F \cdot 0.6 = 35(1.1 - 0)$$

We can find that $F = 64.17$ N.

Solution to part c: The force exerted by the mom on the son and the force exerted by the son on the mom satisfy the Newton's third law pair. This means that the forces are equal and in opposite directions.

Solution to part d: Since the mother comes to a stop because of friction, her final velocity is 0. We already know that her initial velocity (after separating) was 0.55m/s. On top of that, we know the displacement of the motion to be 7m.
Thus, we can apply the kinematics equation $v_f^2 = v_i^2 + 2 \cdot a \cdot \Delta x$

Plugging in our values gives $0^2 = 0.55^2 + 2 \cdot a \cdot 7$

Solving for acceleration (a) gives 0.02161 m/s^2.

Since acceleration is only caused by the frictional force, we can see that the net force in the x direction is μmg (the frictional force). We can equate this to ma to find the acceleration to be μg.

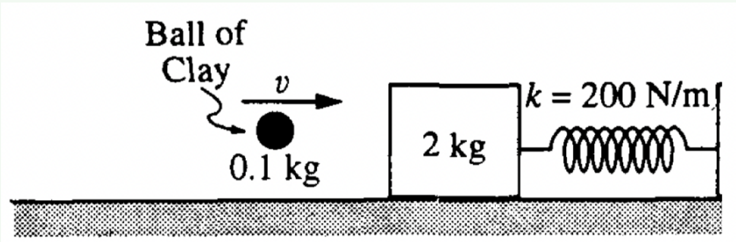
This means that acceleration is **not** dependent on mass. It will be the same for both the mom and the son. That means we can use this acceleration value to see how far the son moves.

We know that the son's initial velocity (velocity after separating) is 1.1m/s. We also know that his final velocity is simply 0. We can again use the equation $v_f^2 = v_i^2 + 2 \cdot a \cdot \Delta x$.

Plugging in our values gives $0^2 = 1.1^2 + 2 \cdot 0.02161 \cdot \Delta x$

Solving for Δx gives us an answer of 28 m.

Problem 5.0.12 — 1994 AP Physics C: Mechanics FRQ



A 2-kilogram block is attached to an ideal spring (for which $k = 200$ N/m) and initially at rest on a horizontal frictionless surface, as shown in the diagram above.

In an initial experiment, a 100-gram (0.1 kg) ball of clay is thrown at the 2-kilogram block. The clay is moving horizontally with speed v when it hits and sticks to the block. The spring is attached to a wall as shown. As a result, the spring compresses a maximum distance of 0.4 meters.

(a) Calculate the energy stored in the spring at maximum compression.

(b) Calculate the speed of the clay ball and 2-kilogram block immediately after the clay sticks to the block but before the spring compresses significantly.

(c) Calculate the initial speed v of the clay.

Solution to part a: The energy stored in a spring is $\frac{1}{2}kx^2$ where k is the spring constant and x is the distance the spring has been compressed.

Clearly, $k = 200$ as already given in the problem statement. On top of that, the maximum distance compressed x is already given as 0.4
We can plug these in to find that the energy stored in the spring is $\frac{1}{2} \cdot 200 \cdot 0.4^2 = 16$ J.

Solution to part b: As soon as the ball collides with the box, both move at the same velocity. Their kinetic energy is converted to spring potential energy.
The kinetic energy of the ball is $\frac{1}{2}mv^2$ while the kinetic energy of the box is $\frac{1}{2}Mv^2$
This means that the combined kinetic energy is $\frac{1}{2}(m + M)v^2$
This combined kinetic energy converts into spring potential energy which was already found to be 16J in part a.

Since $KE = U_s$, $\frac{1}{2}(m+M)v^2 = 16$.
We can multiply both sides by 2 to get $(m+M)v^2 = 32$
Now we can divide both sides by $m+m$ to get $v^2 = \frac{32}{m+M}$

Since we know that $m = 0.1$ kg and $M = 2$ kg, we can plug this in to get that $v^2 = \frac{32}{0.1+2}$

Simplifying gives that $v = 3.9$ m/s

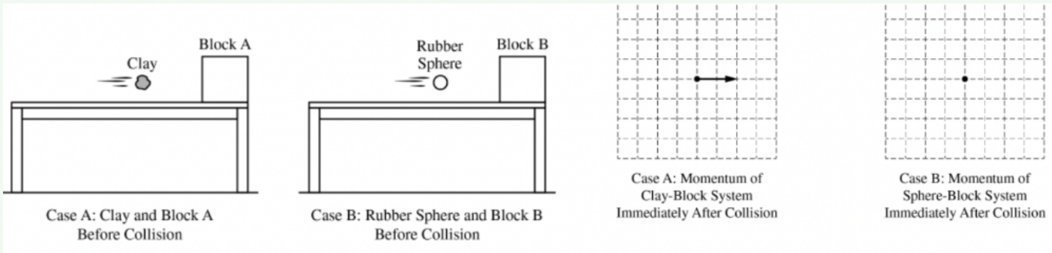
Solution to part c: Momentum is conserved in this motion. If the initial speed of the clay is v, then the initial momentum of the system is mv (the box is at rest at this moment).

The final momentum is $(m+M)v_f$
We know that $m = 0.1$kg and $M = 2$kg and $v_f = 3.9\frac{m}{s}$ from part b. Note, that 3.9 is the velocity for both the clay and box right at the moment when the clay collides with the box. Plugging these variables in gives that the final momentum is $2.1 \cdot 3.9$ which is 8.19

Since $p_i = p_f$ (initial momentum = final momentum), we can write the equation $mv = 8.19$
Since $m = 0.1$, we can divide both sides by 0.1 to get that v is $81.9\frac{m}{s}$.
Thus, The initial velocity of the clay is 81.9 m/s.

Problem 5.0.13 — 2022 AP Physics 1 FRQ
A student has a piece of clay and a rubber sphere, both of the same mass. Both objects are thrown horizontally at the same speed at identical blocks that are at rest at the edge of the identical tables, as shown, where friction between the blocks and the table is negligible. After the collisions, both blocks fall to the floor.

In case A, the clay sticks to Block A after the collision. In Case B, the rubber sphere bounces off of Block B after the collision.



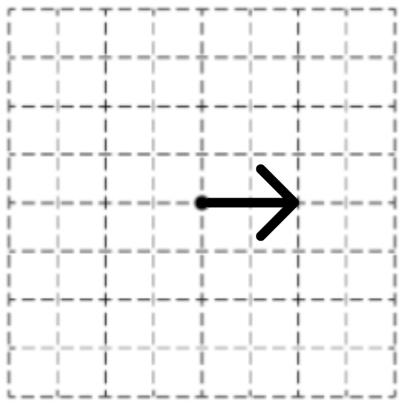
(a) In the figure at left above, the arrow represents the momentum immediately after the collision for the clay-block system in Case A. In the figure at right above, draw an arrow starting on the dot to represent the momentum of the sphere-block system immediately after the collision in Case B. If the momentum if zero, write "zero" next to the dot. The momentum, if it is not zero, must be represented by an arrow starting on, and pointing away from, the dot. The length of the vector, if not zero, should reflect the magnitude of the momentum relative to Case A.

(b) After the clay and Block A collide, Block A lands a horizontal distance d_A from the edge of the table. Does Block B land on the floor at a horizontal distance from the edge of the table that is greater than, less than, or equal to d_A? In a clear, coherent, paragraph-length response that may also contain equations and/or drawings, explain your reasoning. Neglect any frictional effects due to the table or air resistance.

Solution to part a: Instead of finding the momentum after collision, we can think about the momentum before collision. The reason is that the momentum before collision is equivalent to the value of momentum after collision due to the conservation of momentum!

Before collision, there was a clay thrown in case A. In case B, there was a rubber sphere that was thrown. Since both have the same mass and they were thrown with the same velocity, the initial momentum for each system was the same before collision. Since the initial momentum was the same, the final momentum for each case has to be the same as the initial momentum.

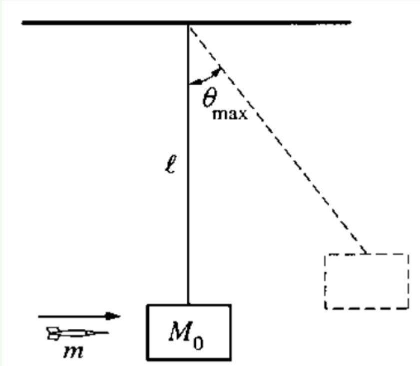
Thus, the arrow for case B will point rightwards and will be drawn with the same length as in case A.



Case B: Momentum of Sphere-Block System Immediately After Collision

Solution to part b: For case A, since the clay sticks to block A, the initial momentum from the clay must be shared between the clay and block A.
However, in case B, the rubber sphere bounces back after colliding with block B. This negative momentum must be counteracted with a greater momentum for block B after collision. This means that block B must have a greater momentum than block A. Since both blocks have the same mass, greater momentum in block B means that it has a greater velocity.
Since block B has a greater velocity, it will travel further than block A.

Problem 5.0.14 — 1999 AP Physics C: Mechanics FRQ



In a laboratory experiment, you wish to determine the initial speed of a dart just after it leaves a dart gun. The dart, of mass m, is fired with the gun very close to a wooden block of mass M0 which hangs from a cord of length l and negligible mass, as shown above. Assume the size of the block is negligible compared to l, and the dart is moving horizontally when it hits the left side of the block at its center and becomes embedded in it. The block swings up to a maximum angle from the vertical. Express your answers to the following in terms of m, M_0, l, $\theta_m ax$, and g.

(a) Determine the speed v_0 of the dart immediately before it strikes the block.

(b) The dart and block subsequently swing as a pendulum. Determine the tension in the cord when it returns to the lowest point of the swing.

Solution to part a: Since the dart sticks with the block after collision, we know that it's an inelastic collision.
In an inelastic collision, both objects move with the same velocity after they collide.

Let's assume the speed that the dart and block combined move with is v_f right after they collide.

The initial speed of the dart is v_o. Now, let's conserve momentum.
$p_i = p_f$
Since the initial speed of the block is 0 (only the dart is moving before the collision), the initial momentum of the system is just mv_o

The final momentum is the sum of both masses multiplied by their common velocity which is v_f.
Equating momentum gives us the equation $mv_o = (m + M)v_f$

Now, we can conserve energy using θ_{max}. When the combined dart and block reach the angle θ_{max}, there will be no kinetic energy at that moment. All the kinetic energy will turn into potential energy.

Let's say that the place where the dart and wooden block collide are at height 0 (our reference level). This means that the height of θ_{max} will be $l - l\cos(\theta_{max})$

We know that $KE_i + U_i = KE_f + U_f$ since energy is conserved.

The initial gravitational potential energy is 0. The final kinetic energy is also 0.

We can simplify the equation to $KE_i = U_f$

The initial kinetic energy is $KE_i = \frac{1}{2}(m+M)v_f^2$ (we use v_f since this is the variable for our combined velocity after collision).

The final gravitational potential energy is $(m+M)gl(1-\cos(\theta_{max}))$

Setting both expressions equal gives

$$\frac{1}{2}(m+M)v_f^2 = (m+M)gl(1-\cos(\theta_{max}))$$

We can cancel out $m+M$ and then multiply both sides by 2 to get $v_f^2 = 2gl(1-\cos(\theta_{max})$
This simplifies to $v_f = \sqrt{2gl(1-\cos(\theta_{max})}$

Since we now find the speed of the dart and block as soon as they collide, we can now plug this into our equation for conservation of momentum.
From conserving momentum, we know $mv_o = (m+M)v_f$
We can divide both side by m to get $v_o = \frac{m+M}{m}v_f$

Now, we simply plug in $v_f = \sqrt{2gl(1-\cos(\theta_{max})}$ to get $v_o = \dfrac{m+M}{m}\sqrt{2gl(1-\cos(\theta_{max}))}$

In summary, we were able to solve this problem by conserving momentum and energy.

Solution to part b: At the bottom, we know that there will be a centripetal force created from the tension force and gravitational force.
We can write a Newton's Law equation: $T-(m+M)g = (m+M)a = \frac{(m+M)v^2}{l}$

We can add $(m+M)g$ to both sides to get $T = (m+M)(g+\frac{v^2}{l})$
Note, that in this problem the radius has a length of l, the length of the cord. We need to use l instead of r.

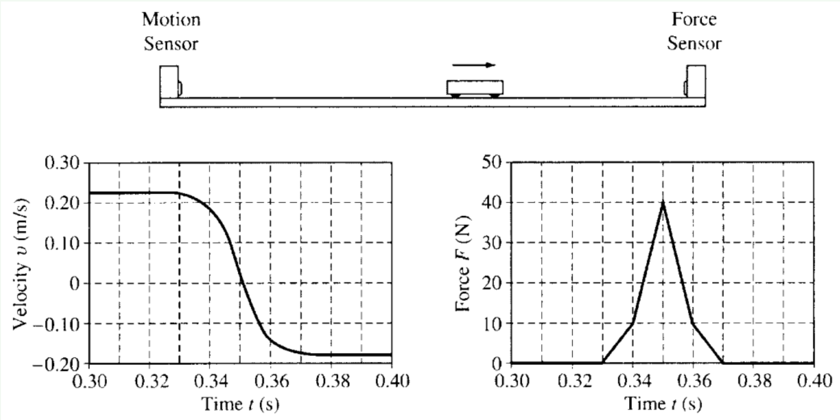
Now, the question is what value of velocity should we use. Should we use the velocity before collision or after collision?

The answer is that we use the velocity right after collision which is $v_f = \sqrt{2gl(1-\cos(\theta_{max}))}$.
The reason is that we want the velocity for the COMBINED mass.

We can plug this value of velocity into $T = (m+M)(g+\frac{v^2}{l})$
Doing so gives that $T = (m+M)(g+2g-2g\cos(\theta_{max}))$ which simplifies to

$$T = (m+M)(3g-2g\cos(\theta_{max}))$$

Problem 5.0.15 — 2001 AP Physics C: Mechanics FRQ



A motion sensor and a force sensor record the motion of a cart along a track, as shown above. The cart is given a push so that it moves toward the force sensor and then collides with it. The two sensors record the values shown in the above graphs.

(a) Determine the cart's average acceleration between $t = 0.33$ s and $t = 0.37$ s.

(b) Determine the magnitude of the change in the cart's momentum during the collision.

(c) Determine the mass of the cart.

(d) Determine the energy lost in the collision between the force sensor and the cart.

Solution to part a: To find the average acceleration in that time interval, we need to use the graph.
The average acceleration is $\frac{v_f - v_i}{\Delta t}$

The velocity at $t = 0.37$s can be estimated to be $-0.18\frac{m}{s}$.
Similarly, the velocity at $t = 0.33$s can be estimated to be $0.22\frac{m}{s}$.
We also know that Δt is jut 0.04

Plugging these variables in gives that average acceleration is $\frac{-0.18-0.22}{0.04} = 10\frac{m}{s^2}$

Solution to part b: We know that the area under a force-time graph is impulse which is the same thing as change in momentum.

We can use the graph on the right and try to find the area under it. We can break the graph up into simple shapes such as triangles and rectangles to find that the area is 0.6
This means that the change in momentum is 0.6kg $\cdot$ m/s

Solution to part c: The change in momentum is mass times change in velocity.
$\Delta p = m\Delta v$
Our change in velocity is $-0.18 - 0.22 = -0.4\frac{m}{s}$

We also already know that Δp is 0.6 from part b.

We can rearrange the equation $\Delta p = m\Delta v$ to $m = \frac{\Delta p}{\Delta v}$

$$\text{Plugging our values in gives that } m = \frac{0.6}{0.4} = 1.5 \text{ kg}$$

Solution to part d: The energy lost would be the difference in kinetic energy before and after.
We need to find ΔK which is change in change in kinetic energy.

$$\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{m}{2}(v_f^2 - v_i^2)$$

We know that $m = 1.5$ kg. We can plug in $v_f = -0.18$ and $v_i = 0.22$ to find ΔK

$$\Delta K = \frac{1.5}{2}((-0.18)^2 - (0.22)^2) = -0.012 \text{ J}$$

This means that the energy lost in the collision between the force sensor and the cart is 0.012 J.

Problem 5.0.16 — 1992 AP Physics B FRQ
A 30-kilogram child moving at 4.0 meters per second jumps onto a 50-kilogram sled that is initially at rest on a long, frictionless, horizontal sheet of ice.

(a) Determine the speed of the child-sled system after the child jumps onto the sled.
(b) Determine the kinetic energy of the child-sled system after the child jumps onto the sled.

After coasting at constant speed for a short time, the child jumps off the sled in such a way that she is at rest with respect to the ice.
(c) Determine the speed of the sled after the child jumps off it.
(d) Determine the kinetic energy of the child-sled system when the child is at rest on the ice.
(e) Compare the kinetic energies that were determined in parts (b) and (d). If the energy is greater in (d) than it is in (b), where did the increase come from? If the energy is less in (d) than it is in (b), where did the energy go?

Solution to part a: We will conserve momentum in this problem since we have an inelastic collision.

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

m_1 represents the mass of the child while m_2 represents the sled.

Initially, the sled is at rest which means $v_{2i} = 0$.
This means our equation becomes $m_1v_{1i} = m_1v_{1f} + m_2v_{2f}$

In addition, since this is an inelastic collision, both objects will move together after. This means that their final speed will be the same. We can denote the common final speed as v_f

This further simplifies the equation to $m_1v_{1i} = (m_1 + m_2)v_f$

We know that $m_1 = 30$, $m_2 = 50$, and $v_{1i} = 4$.

We can plug this in to get $30 \cdot 4 = (30 + 50)v_f$
We can solve this to find that $v_f = 1.5$ m/s

Solution to part b: After the child jumps onto the sled, the kinetic energy of the child-sled system will be $\frac{1}{2}(30 + 50) \cdot 1.5^2 = 90$ J
In this problem, we are able to combine the masses and treat them as one because they are moving together.

Solution to part c: We can again use conservation of momentum.

$$(m_1 + m_2)v_i = m_1 v_{1f} + m_2 v_{2f}$$

$m_1 = 30$, $m_2 = 50$, and $v_i = 1.5$

Initially, both the sled and person are moving at the same speed.

After collision, $v_{1f} = 0$ since the child is now at rest.

We can plug these variables in to get $(30 + 50)1.5 = 0 + 50v_{2f}$
We can solve this to get $v_{2f} = 2.4$ m/s.

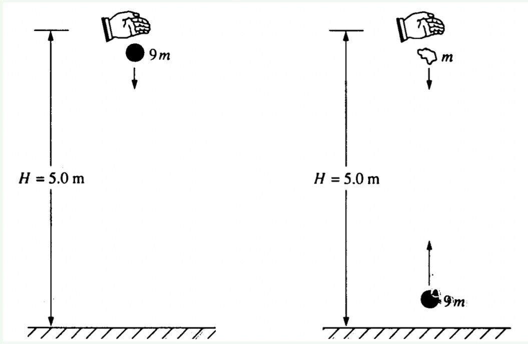
Solution to part d: The kinetic energy of the system can be found by summing up the individual kinetic energies for the child and sled.
Since the child is at rest, the kinetic energy is 0.

The sled's kinetic energy can be found through the equation $K = \frac{1}{2}mv^2$. Since it has a velocity of 2.4 and mass of 50, its kinetic energy is $\frac{1}{2} \cdot 50 \cdot 2.4^2 = 144$ J.

The sum of both kinetic energies is $0 + 144$ which is just 144 J.

Solution to part e: Clearly the kinetic energy is now greater than part *b*. 144 J is greater than 90 J. The reason for this can be due to the child doing work on the sled. The work that is done on the sled by the child causes kinetic energy to be greater.

Problem 5.0.17 — 1992 AP Physics C: Mechanics FRQ



A ball of mass $9m$ is dropped from rest from a height $H = 5.0$ meters above the ground, as shown above on the left. It undergoes a perfectly elastic collision with the ground and rebounds. At the instant that the ball rebounds, a small blob of clay of mass m is released from rest from the original height H, directly above the ball, as shown above on the right. The clay blob, which is descending, collides with the ball 0.5 seconds later, which is ascending. Assume that $g = 10\text{m/s}^2$, that air resistance is negligible, and that the collision process takes negligible time.

(a) Determine the speed of the ball immediately before it hits the ground.
(b) Determine the rebound speed of the ball immediately after it collides with the ground, justify your answer.
(c) Determine the height above the ground at which the clay-ball collision takes place.
(d) Determine the speeds of the ball and the clay blob immediately before the collision.
(e) If the ball and the clay blob stick together on impact, what is the magnitude and direction of their velocity immediately after the collision?

Solution to part a: The speed of the ball immediately before it hits the ground can be found by conserving energy. You can also apply kinematics if you want to.
The balls potential energy will all convert to kinetic energy by the time it hits the ground.

Conservation of energy says $K_i + U_i = K_f + U_f$

We can make the ground our reference level. The potential energy at the top can be represented as mgh where mass is $9m$ and h is 5 m.

Our initial kinetic energy is obviously 0 since the ball is dropped from rest. The final potential energy is also 0 since our reference level is the ground.

Our equation just becomes $U_i = K_f$

We can rewrite this as $mgh = \frac{1}{2}mv^2$

m cancels out so we are left with $gh = \frac{v^2}{2}$

We can multiply both sides by 2 and then take the square root of both sides to get $v = \sqrt{2gh}$

Since our height is 5, we can plug that in to get that $v = \sqrt{2 \cdot 9.8 \cdot 5}$ which is 9.9 m/s.

Solution to part b: Since the ball undergoes an elastic collision, the rebound speed will be the same as the speed it collides with. The speed it collides with is 9.9 m/s (found in part a)
This means that the rebound speed will also be 9.9 m/s.

The reason regarding why the rebound speed is the same is that the ground is super "heavy." If you think logically, of course a ball bouncing on the ground will not cause the ground to move. Earth is too heavy. The ball is way too light in comparison. Also, many people might try to think logically about this problem. For example, you may have experience bouncing a tennis ball or even a basketball. You have probably noticed that unless you keep applying force with your hand, both of the balls will stop. Their rebound height will not be the same. The reason is that the ground absorbs some of the energy. HOWEVER, in this problem it says that the ground makes perfectly elastic collisions with the ground. That means no energy is lost which is why the rebound speed is the same.

Solution to part c: We can find the height above the ground by observing the clay blob.
We know that they collide in 0.5 s (it says that in the problem statement), so we can use that information to find the height.

The clay blob's initial velocity $v_i = 0$ and $t = 0.5$ s. On top of that, $a = g = 9.8$

We can use the kinematics equation $\Delta y = v_i t + \frac{1}{2}at^2$
We can plug in our variables to find that $\Delta y = 0 + \frac{1}{2} \cdot g \cdot 0.5^2 = 1.225$ m.

Since this is the distance the clay ball displaces from the top, it is $5 - 1.225 = 3.775$ m above the ground.

Solution to part d: We will use kinematics equations two times to find the speeds for each object.
For the ball, $v_f = v_i + at$. We know that $t = 0.5$ and $a = g$ and $v_i = 9.9$.
Using these values, we can find that $v_f = 9.9 - 9.8 \cdot 0.5 = 5$ m/s.

Similarly, for the clay, we can use the same equation $v_f = v_i + at$. We know that $t = 0.5$ and $a = g$ and $v_i = 0$ since it is dropped from rest.
Using these values, we can find that the clay is moving at $v_f = 0 - 9.8 \cdot 0.5 = -4.9$ m/s.

Solution to part e: Since both objects stick together after impact, we know that an inelastic collision happens. Their common velocity will be the same after impact, and we can denote it as v_f

We will conserve momentum since we have a collision.

$$p_i = 9m \cdot 5 - m \cdot 4.9 = 40.1m \text{ Expression for initial momentum)}$$

At the same time, $p_f = (9m + m)v_f$

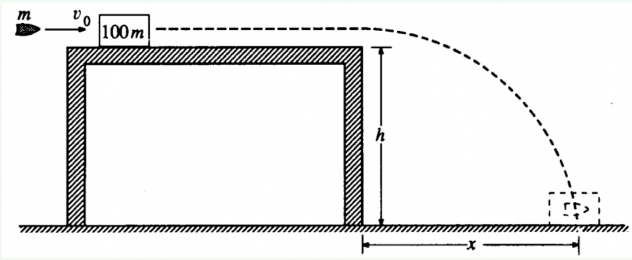
Since $p_i = p_f$, we can plug in both expressions to get

$$40.1m = (9m + m)v_f$$

We can cancel out m from both sides to get $40.1 = 10v_f$

After dividing 10 from both sides, we get that $v_f = 4.01$ m/s.

Problem 5.0.18 — 1990 AP Physics B FRQ



A bullet of mass m is moving horizontally with speed v_o when it hits a block of mass $100m$ that is at rest on a horizontal frictionless table, as shown above. The surface of the table is a height h above the floor. After the impact, the bullet and the block slide off the table and hit the floor a distance x from the edge of the table. Derive expressions for the following quantities in terms of$m, h, v)o$, and appropriate constants:

(a) the speed of the block as it leaves the table
(b) the change in kinetic energy of the bullet-block system during impact
(c) the distance x

Suppose that the bullet passes through the block instead of remaining in it.
(d) State whether the time required for the block to reach the floor from the edge of the table would now be greater, less, or the same. Justify your answer.

(e) State whether the distance x for the block would now be greater, less, or the same. Justify your answer.

Solution to part a: After collision, the bullet and block travel together. This means that we have an inelastic collision.
Momentum will be conserved. The initial momentum is mv_o (since only the bullet is moving).
The final momentum will be $(m + 100m)v_f$ since both masses will move with the same speed.

We can set both expressions equal to each other and write $mv_o = (m + 100m)v_f$
We can divide m from both sides to get $v_o = 101v_f$

$$\text{After dividing 101 from both sides, we get } v_f = \frac{v_o}{101}$$

Solution to part b: Change in kinetic energy can be represented as $\Delta K = K_f - K_i$

We can first find the initial kinetic energy. It is simply $\frac{1}{2}mv_o^2$ since only the bullet moves.
The final kinetic energy can again be found using the equation $K = \frac{1}{2}mv^2$. Our mass will be $m + 100m$ which is $101m$. We combine the masses since both objects move together. The velocity of both will be $\frac{v_o}{101}$.

$$\text{We can plug this in to find that } K_f = \frac{1}{2} \cdot 101m \cdot (\frac{v_o}{101})^2$$

$$\text{We can simplify the expression to find that } K_f = \frac{mv_o^2}{202}$$

Now, we can compute $\Delta K = K_f - K_i$. We can plug in our expressions to find that

$$\Delta K = \frac{mv_o^2}{202} - \frac{mv_o^2}{2} = -\frac{50mv_o^2}{101}$$

Solution to part c: The distance x can be found using kinematics equations.
After the bullet and block leave the table, they undergo projectile motion.
We must remember that the vertical motion is independent from the horizontal motion.
Since the initial vertical speed is 0, the vertical displacement is h, and the acceleration is g, we can find the time it takes to fall using the equation $\Delta y = v_i t + \frac{1}{2}at^2$

After we plug in our variable, we get $h = 0 + \frac{1}{2} \cdot g \cdot t^2$
We can solve for t to find that it is $\sqrt{\frac{2h}{g}}$

Since our horizontal velocity remains the same in projectile motion, we can simply multiply the horizontal velocity by the time taken to fall to find the horizontal distance travelled.
This means that $x = v_x t$. We already know that $v_x = \frac{v_o}{101}$ from part a.

$$\text{This means } x = \frac{v_o}{101}\sqrt{\frac{2h}{g}}$$

Solution to part d: The time would be the same. We can figure this out by considering motion in the y-direction. The vertical displacement, acceleration in vertical direction, and initial vertical velocity will all remain the same. Even though the horizontal velocity will differ, the motion in the vertical direction will remain the same. This will cause time to be the same.

Solution to part e: Previously, all of the bullet's momentum would be transferred to the block. However, now only part of it will be transferred. The reason is that the bullet will preserve some of its momentum to move through the block. This causes the momentum that the block receives to be lower. This will lead to a lower velocity in the x-direction.
Since the horizontal velocity is now lower for the block, the distance x travelled will also be lower.

Unit 6 Simple Harmonic Motion

Simple Harmonic Motion is a new type of motion that combines all of the concepts we've learned until now. This motion is the name for repetitive back and forth motion, such the grandfather clock. In AP Physics C: Mechanics, we will cover 2 types of SHM: the spring and the pendulum.

Note 6.0.1

Spring Review

Although we all have a general idea of what a spring is, the three main characteristics of it is that the force applied by the spring is proportional to the distance compressed/stretched *from the equilibrium position*. That is,

$$F_{\text{spring}} = -kx$$

where k is the spring constant, and x is the distance it's compressed/stretched.

The other characteristic is that the potential energy of the spring at any given point is

$$U_{\text{spring}} = \frac{1}{2}kx^2$$

where x is the distance it is stretched/compressed from the equilibrium position.

Note that the potential energy is 0 when the spring is at equilibrium, and is maximized when the spring is maximally stretched (or compressed).

The period and frequency of the spring is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

and

$$f = \frac{1}{T} = 2\pi\sqrt{\frac{k}{m}}$$

Remember that simple harmonic motion is also known as periodic motion. It occurs due to a **restoring force** that causes an object to move back and forth.

For those that don't know, remember that period is equivalent to the time it takes for one cycle while frequency is the number of cycles that occurs in 1 second. Let's derive something that is extremely important to be able to understand simple harmonic motion.

Pay close attention to the scenario that I'm about to describe.
Let's say that the spring force is the only force on an object in the horizontal direction. From Newton's Second Law, we know that $F_{net} = -kx = ma$

We can divide both sides by the mass to get $a = -\frac{kx}{m}$

In general, whenever you're deriving period for an unknown scenario, you can find an expression for the acceleration. After that, use the fact that $a = -w^2x$. This relates acceleration to the angular frequency and x (distance from equilibrium).

Please note that the angular frequency variable occurs due to simple harmonic motion being **sinusoidal** motion. This means that if you look at simple harmonic motion graphs, then they will remind you of trigonometric graphs.

Please memorize the relationship $a = -w^2x$ and $a = -\frac{kx}{m}$. $a = -w^2x$ means that the acceleration is proportional to x (distance). We can set both expressions equal to each other.

$$-w^2x = -\frac{kx}{m}$$

We can cancel out like terms to get $w^2 = \dfrac{k}{m}$

Square-rooting both sides gives $w = \sqrt{\dfrac{k}{m}}$

Thus, since $T = \dfrac{2\pi}{w}$, we can find that $T = 2\pi\sqrt{\dfrac{m}{k}}$

Also, remember that the period T and frequency f are related to each other. $T = \frac{1}{f}$.

Note 6.0.2

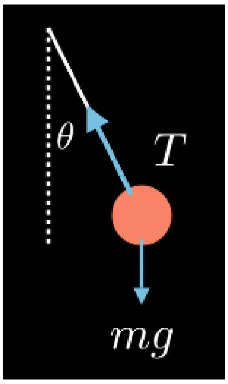
Mechanical energy is always conserved in an oscillating system with a spring and a mass.

The maximum potential energy exists when the spring is stretched to its maximum displacement. At this point, the spring will instantaneously be at rest causing the kinetic energy to be 0.

This means that the total mechanical energy is equivalent to the maximum potential energy which is $\frac{1}{2}kA^2$ (and A is the amplitude). Maximum kinetic energy will occur at the equilibrium point for a spring.

On top of a spring, oscillations can occur for pendulums. Note that the equilibrium point for pendulums occurs when it's hanging straight down.
For a pendulum, we displace it by a small angle θ. Then, the gravitational force will cause it to rotate back and forth.

For a pendulum, the period T is $T = 2\pi\sqrt{\frac{L}{g}}$ where L is the length of the pendulum.



I will partially show you how the period of a pendulum is derived. It uses topics from Unit 7, so you can skip it for now. However, to gain a deeper understanding, please come back to this part after learning torque and rotational motion.

The above image shows a pendulum. There is a ball of mass m attached at the end of the pendulum. If we find the components of the gravitational force, then we can see that $mg\sin(\theta)$ is perpendicular to the pendulum.

Thus, the component $mg\sin(\theta)$ will apply a torque, and the torque on the pendulum is

$$-mgL\sin\theta$$

We know that $\tau_{net} = I\alpha$ (Newton's Second Law for Rotation).
We also know that $I = mL^2$ because the mass of our ball is M and it is a distance L away from the pivot.

We can plug torque and rotational inertia in to find that

$$\alpha = \frac{\tau_{net}}{I} = \frac{-mgL\sin\theta}{mL^2} = -\frac{g\sin\theta}{L}$$

Since our θ will be a very very small angle to make it an oscillation, we can use the approximation $\sin\theta = \theta$.
This means that $\alpha = -\frac{g\theta}{L}$.

For a spring, we know that $a = -w^2x$. Similarly, for a pendulum, $\alpha = -w^2\theta$. This shows that the angular acceleration is proportional to the angle displaced.

We can set both expressions for α equal to each other.

$$-w^2\theta = -\frac{g\theta}{L}$$

We can cancel out like terms to get $w^2 = \dfrac{g}{L}$

We can square-root both sides to get $\omega = \sqrt{\dfrac{g}{L}}$

This means that $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{L}{g}}$

Problem 6.0.3 — A spring with constant k is initially at rest, with a block of mass M attached to it. It is then stretched a distance d and released.

What is the velocity of the block when the spring reaches equilibrium?

Solution: Initially, the energy of the spring is $\frac{1}{2}kd^2$ using the formula $U_s = \frac{1}{2}kx^2$. When the spring reaches the equilibrium point, all the spring potential energy will convert to kinetic energy.

$$\text{Thus, } \frac{1}{2}kd^2 = \frac{1}{2}Mv^2$$

Solving, we get that $v = d\sqrt{\frac{k}{M}}$, at the equilibrium position.

Problem 6.0.4 — A spring with constant k hangs from a ceiling, with a ball of mass m attached. How much does the spring stretch?

Solution: Note that the force applied on the ball by gravity is mg, and the force applied by the spring is kx. Since the ball is in equilibrium, we know that $mg = kx$ since the net force must be zero, which means that $x = \frac{mg}{k}$.

This is the main difference between a **horizontal** and **vertical** spring. In a horizontal spring, gravitational force will not cause the block to accelerate. In a vertical spring, gravitational force plays a role on the other hand. That is why for a horizontal spring, the spring force is 0 at the equilibrium point while for a vertical spring, spring force isn't 0 at the equilibrium point.

Note 6.0.5

The Pendulum Review

A pendulum oscillates due to the force of gravity. In a way, it performs repeated back-and-forth circular motion.

The period of a pendulum (time it takes to make one "tick" and one "tock", which is one back-and-forth motion) is

$$T = 2\pi\sqrt{\frac{L}{g}}$$

The frequency (number of ticks and tocks per second, which is the number of back-and-forth motions in one second) of it is

$$f = 2\pi\sqrt{\frac{g}{L}}$$

Note that both of these formulas are independent of the original displacement of the pendulum. (Think about it, this is the reason grandfather clocks are still accurate! No matter how much the air resistance reduces the displacement of the pendulum, the time between each tick and each tock is the same.)

Problem 6.0.6 — A pendulum has a period of 5 seconds. If the length of the string of the pendulum is quadrupled, what is the new period of the pendulum?

Solution: Using the formula $T = 2\pi\sqrt{\frac{L}{g}}$, if the length is multiplied by 4, the whole formula is multiplied by $\sqrt{4} = 2$. Hence the new period is $5 \cdot 2 = 10$ seconds.

Problem 6.0.7 — A ball of mass 2 kg is attached to a string of length $4m$, forming a pendulum. If the string is raised to have an angle of 30 degrees above the vertical and released, what is the velocity of the ball as it passes through its lowest point?

Solution: The change in potential energy from the initial position to the final position is

$$U_f - U_i = mg(4 - 4\cos(30^\circ)) \approx 0.54mg$$

So,

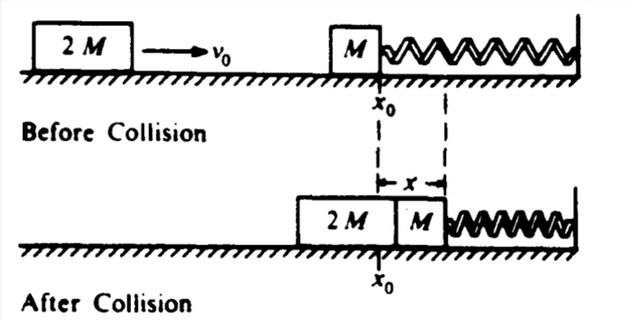
$$\frac{1}{2}mv^2 = 0.54mg$$
$$\implies v = \boxed{3.3} \text{ m/s}$$

Problem 6.0.8 — Find the total mechanical energy of a spring given a spring constant of 5 N/m and an amplitude of 2 meters.

Solution: The total mechanical energy is the sum of the potential and kinetic energies. When the spring is stretched to its amplitude, the kinetic energy will be 0. All of the energy will be in the form of spring potential energy. Thus, we can simply find the spring potential energy at the amplitude to find the total mechanical energy.

$$\frac{1}{2}kx^2 = \frac{1}{2} \cdot 5 \cdot 2^2 = 10 \text{ J}$$

Problem 6.0.9 — 1983 AP Physics B FRQ



A block of mass M is resting on a horizontal, frictionless table and is attached as shown above to a relaxed spring of spring constant k. A second block of mass $2M$ and initial speed v_0 collides with and sticks to the first block. Develop expressions for the following quantities in terms of M, k, and v_0:

(a) v, the speed of the blocks immediately after impact

(b) x, the maximum distance the spring is compressed

(c) T, the period of the subsequent simple harmonic motion

Solution to part a: The speed of the blocks after impact can be found by conserving momentum.
$p_i = p_f$
The initial momentum simply comes from the block of mass $2M$. Since it moves at a speed v_o, the initial momentum is $2Mv_o$

After collision, both of the blocks move together since it is an inelastic collision. This means they will move with the same speed, which we can assume to be v_f.
The combined mass of both blocks is $2M + M$ which is $3M$. This means that the final momentum is $3Mv_f$

After equating initial and final momentum, we can write the equation $2Mv_o = 3Mv_f$
We can solve this to find that $v_f = \frac{2v_o}{3}$

Solution to part b: We can find the maximum distance the spring is compressed by conserving energy.
The kinetic energy of both blocks after collision will convert to spring potential energy.

$$K = U_s$$

U_s (spring potential energy) can be represented as $\frac{1}{2}kx^2$
We can find kinetic energy by using the equation $\frac{1}{2}mv^2$. The combined mass is $3M$ and velocity is $\frac{2v_o}{3}$. We can plug this into the equation to find that $K = \frac{2Mv_o^2}{3}$

Now, we can equate our expressions for spring potential energy and kinetic energy.

$$\frac{2Mv_o^2}{3} = \frac{1}{2}kx^2$$

We can multiply both sides by $\frac{2}{k}$ to find that $x^2 = \frac{4Mv_o^2}{3k}$

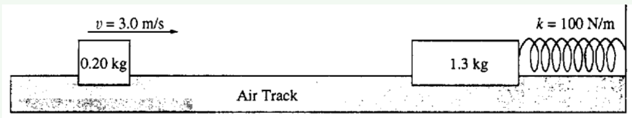
We can take the square root of both sides to find that $x = \sqrt{\frac{4Mv_o^2}{3k}}$ which can be further simplified to

$$x = 2v_o\sqrt{\frac{M}{3k}}$$

Solution to part c: Period is simply found through the equation $T = 2\pi\sqrt{\frac{m}{k}}$ Since our combined mass is $3M$, we can plug that in to find that the period T is

$$2\pi\sqrt{\frac{3M}{k}}$$

Problem 6.0.10 — 1995 AP Physics B FRQ



As shown above, a 0.20-kilogram mass is sliding on a horizontal, frictionless air track with a speed of 3.0 meters per second when it instantaneously hits and sticks to a 1.3-kilogram mass initially at rest on the track. The 1.3-kilogram mass is connected to one end of a massless spring, which has a spring constant of 100 newtons per meter. The other end of the spring is fixed.

(a) Determine the following for the 0.20-kilogram mass immediately before the impact.

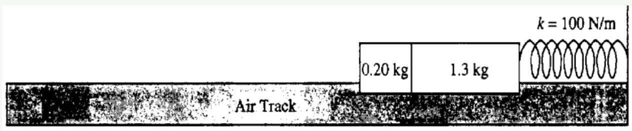
i. Its linear momentum

ii. Its kinetic energy

(b) Determine the following for the combined masses immediately after the impact.

i. The linear momentum

ii. The kinetic energy



(c) After the collision, the two masses undergo simple harmonic motion about their position at impact.

Determine the amplitude of the harmonic motion.

(d) After the collision, the two masses undergo simple harmonic motion about their position at impact.

Determine the period of the harmonic motion.

Solution to part a: The initial linear momentum can be found from $p = mv$ (the formula for momentum).

Since the mass is 0.20 kg and velocity is $3.0\frac{m}{s}$, we find that the initial momentum is $0.20 \cdot 3.0$ which is $0.6\frac{\text{m}\cdot\text{kg}}{s}$
On top of that, the initial kinetic energy can be found from $\frac{1}{2}mv^2$.

$$\text{This means that } K_i = \frac{1}{2} \cdot 0.20 \cdot 3.0^2 = 0.9 \text{ J}$$

Solution to part b: Momentum is conserved in a collision. This means that we can just use our formula for conservation of momentum which says $p_i = p_f$
We already found in part a that $p_i = 0.6$. In this collision, the linear momentum will be conserved so $p_f = 0.6$ $0.6\frac{\text{m}\cdot\text{kg}}{s}$

However, kinetic energy is not conserved in an inelastic collision. We must find the velocity of the two blocks right after collision.
We know that the two blocks move with a common velocity after collision. We can denote it with the variable v_f
This means that the final momentum is sum of the two masses times v_f (since the two blocks move together).

$$p_f = (0.2 + 1.3)v_f = 1.5v_f$$

We also already know that $p_f = 0.6$.
We can set both expressions to each other to get $1.5v_f = 0.6$
Solving it gives that $v_f = 0.4$ m/s.

Now, we can find the final kinetic energy. It can be found using the equation $K = \frac{1}{2}mv^2$
In this case, the two masses move together so we sum up the masses to get 1.5 kg. We also know that $v_f = 0.4$

$$\text{We can plug this in to find } K_f = \frac{1}{2} \cdot 1.5 \cdot 0.4^2 = \boxed{0.12 \text{ J}}$$

Solution to part c: To find the amplitude, we simply apply conservation of energy to find the maximum compression.
We equate the kinetic energy right after the two blocks collide and equate it to spring potential energy.

$$K = U_s = \frac{1}{2}kx^2$$

From part b, we already know that the kinetic energy is 0.12

We can substitute that in to get $0.12 = \frac{1}{2}kx^2$

We also know that the spring constant $k = 100$. We can plug that into the equation.

Plugging the value of k in gives $0.12 = \frac{1}{2} \cdot 100 \cdot x^2$

We can simplify the equation to find that $x^2 = 0.0024$

Now, we can take the square root of both sides to find that the amplitude $\boxed{A = 0.049 \text{ m}}$

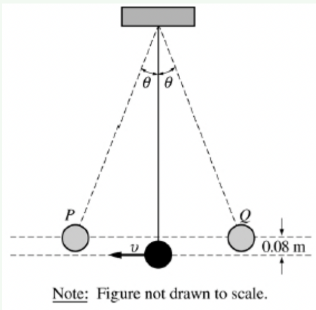
Solution to part d: To find the period, we simply need to apply our formula.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

The above is the formula to find period. For m, we will use the sum of the masses which is 1.5 kg.
We also know that the spring constant is 100.

$$\text{We can plug these values in to get } T = 2\pi\sqrt{\frac{1.5}{100}} = \boxed{0.77 \text{ s}}$$

Problem 6.0.11 — 2005 AP Physics B FRQ
A simple pendulum consists of a bob of mass 0.085 kg attached to a string of length 1.5 m. The pendulum is raised to point Q, which is 0.08 m above its lowest position, and released so that it oscillates with small amplitude θ between the points P and Q as shown below.



(a) On the figures below, draw free-body diagrams showing and labeling the forces acting on the bob in each of the situations described.

i. When it is at point P

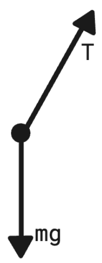
ii. When it is in motion at its lowest position

(b) Calculate the speed v of the bob at its lowest position.

(c) Calculate the tension in the string when the bob is passing through its lowest position.

(d) Describe one modification that could be made to double the period of oscillation.

Solution to part a i: The only forces on the bob are tension and gravitational force. Tension force always points along the string/rope.



Solution to part a ii: The only forces on the bob are again tension and gravitational force. However, this time tension force will point directly upwards instead of at an angle. Also, the size of the arrows don't need to represent which one's larger by magnitude since the problem doesn't explicitly say so. However, for those that are curious, tension force will be larger in magnitude. The reason is that there is an acceleration vector (centripetal acceleration) that points upwards. Thus, the net force must also be upwards which means tension must be larger.

Solution to part b: The speed can be found using conservation of energy.
We will apply conservation of energy by using point P as the initial position and the bottom point as the final position.
We know that $K_i + U_i = K_f + U_f$

To swing down from point P to the lowest point, the bob moves down a height h. This means that it gravitational potential energy decreases by mgh. This energy will convert to kinetic energy.

$$\text{This means } mgh = K_f$$

Kinetic energy can be represented as $\frac{1}{2}mv^2$

We can plug that in for K_f to get $mgh = \frac{1}{2}mv^2$

We can cancel out m and rearrange the equation to get $v = \sqrt{2gh}$

$$\text{Since } h = 0.08 \text{ m, we can find that } v = \sqrt{2 \cdot 9.8 \cdot 0.08} = \boxed{1.252 \text{ m/s}}$$

Solution to part c: We can find the tension at the bottom point by using Newton's Second Law along with some concepts from our centripetal motion chapter.

We can use Newton's Second Law to write $T - mg = ma$
We know that $a = \frac{v^2}{r}$.

$$\text{We can plug that in to get } T - mg = \frac{mv^2}{r}$$

$$\text{We can add } mg \text{ to both sides to get } T = m(g + \frac{v^2}{r})$$

We know that $m = 0.085$ kg, v at the bottom point is 1.252, and $r = 1.5$ m (since that's the length of the string).

$$\text{We can plug those values in to get } T = 0.085(9.8 + \frac{1.252^2}{1.5}) = \boxed{0.922 \text{ N}}$$

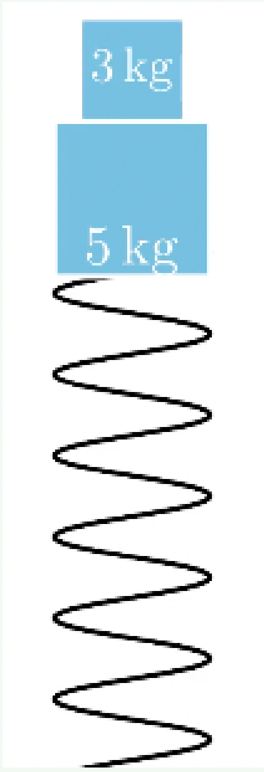
Solution to part d: We know that the period of oscillation for a pendulum can be represented using the formula below.

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Clearly, L (length of string) and g are the two variables that affect period.

We can simply quadruple the length of the string to double the period of oscillation.

Problem 6.0.12 — 1990 AP Physics FRQ
A 5-kilogram block is fastened to an ideal vertical spring that has an unknown spring constant. A 3-kilogram block rests on top of the 5-kilogram block, as shown to the right.



(a) When the blocks are at rest, the spring is compressed to its equilibrium position a distance of $\Delta x_1 = 20\,\text{cm}$, from its original length. Determine the spring constant of the spring.

The 3 kg block is then raised $50\,\text{cm}$ above the 5 kg block and dropped onto it.
(b) Determine the speed of the combined blocks after the collision.

(c) Setup, plug in known values, but do not solve an equation to determine the amplitude Δx_2 of the resulting oscillation.

(d) Determine the resulting frequency of this oscillation.

(e) Where will the block attain its maximum speed, explain.

(f) Is this motion simple harmonic?

Solution to part a: At equilibrium, the net force on the two block system will be 0. The two forces on the two block system are spring force and gravitational force. They both must be equal in magnitude for the net force to be 0.

This means that $mg = k\Delta x$

Our value of m will be the sum of the masses of the two blocks which is 8

Since we know that $\Delta x = 20 \text{ cm} = 0.2 \text{ m}$, we can plug that into the equation

Plugging our values in gives $8 \cdot 9.8 = k \cdot 0.2$

Dividing both sides by 0.2 gives $k = 392 \text{ N/m}$

Solution to part b: We know that the 3 kg block was raised 50 cm above the other block. We will write 50 cm in meters to get 0.5 m.

We will apply conservation of energy on the 3 kg block to find it's speed right before colliding with the other block.

$K_i + U_i = K_f + U_f$

We know that the 3 kg block displaces vertically be 0.5 m before colliding with the 5 kg block. This means that the gravitational potential energy will decrease as it moves down. That energy will convert to kinetic energy.

This means $\Delta U = K_f$
We know that $\Delta U = mgh$ while $K_f = \frac{1}{2}mv^2$ where $m = 3$ kg.

Equating both gives $mgh = \frac{1}{2}mv^2$.

We can cancel out m and rearrange the equation to get $v = \sqrt{2gh}$.

Since $h = 0.5$ m, we can plug that in to get

$$v = \sqrt{2 \cdot 9.8 \cdot 0.5} = 3.13\text{m/s}$$

We know that 3.13 is the light block's velocity after being dropped. However, it will now inelastically collide with the 5 kg block.

This means that momentum is conserved. The initial momentum $p_i = mv = 3 \cdot 3.13 = 9.39$

We know that $p_i = p_f$

Since this is an inelastic collision, both blocks will move with the same velocity after. We can denote the common final velocity as v_f.

This means that $p_f = (3 + 5)v_f = 8v_f$. We can equate this to p_i which is 9.39

Doing so gives $9.39 = 8v_f$

We can solve this to find $v_f = \dfrac{9.39}{8} = 1.174\text{m/s}$

Solution to part c: We know that maximum amplitude will occur at the **equilibrium point**.

We can set up the equation $K + U_g = U_s = \frac{1}{2}k\Delta x^2$

The left side represents the energy at the top. We can't forget that there will also be GRAVITATIONAL POTENTIAL ENERGY. We will make our reference level the equilibrium point. This will cause us to have gravitational potential energy at the top.

The initial kinetic energy of the two blocks can be found using $K = \frac{1}{2}mv^2$

We know that $m = 3 + 5 = 8$ kg (since both blocks move together we can treat them as one mass)

In addition, the gravitational potential energy at the top will be mgh which is $8 \cdot 9.8 \cdot (0.2 + \Delta x)$.

On top of that, $v = 1.174$m/s. (we found this in part b). We also know that the spring constant $k = 392$
We can plug in these values into the equation for energy which is

$$\frac{1}{2}mv^2 + mgh = \frac{1}{2}k\Delta x^2$$

$$\text{Plugging the values in gives } \frac{1}{2} \cdot 8 \cdot 1.174^2 + 8 \cdot 9.8 \cdot (0.2 + \Delta x) = \frac{1}{2} \cdot 392 \cdot \Delta x^2$$

Solution to part d: We know that frequency is $\frac{1}{T}$.
We also know that the period for a spring is $T = 2\pi\sqrt{\frac{m}{k}}$.
We can plug that expression into $f = \frac{1}{T}$
Doing so gives that $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

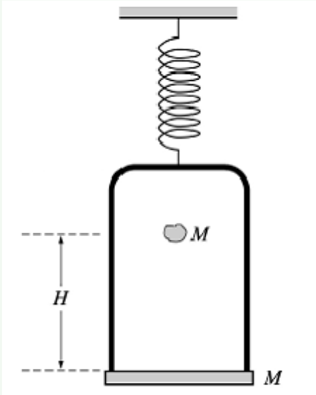
Since our combined mass is 8 kg and the spring constant $k = 392$, we can plug those values in.

$$\text{Plugging those values in gives that } f = \frac{1}{2\pi}\sqrt{\frac{392}{8}} = \boxed{1.114 \text{ Hz}}$$

Solution to part e: Yes. The block will have maximum speed at equilibrium point. The reason is that the net force is 0 at that point. Past this point, acceleration will oppose velocity causing the speed to decrease.

Solution to part f: Yes. For a motion to count as simple harmonic, it has to satisfy the equation $F = k\Delta x$
Clearly, the force in this problem is proportional to the stretched distance causing this motion to be simple harmonic.

Problem 6.0.13 — 2003 AP Physics C: Mechanics FRQ



An ideal massless spring is hung from the ceiling and a pan suspension of total mass M is suspended from the end of the spring. A piece of clay, also of mass M, is then dropped from a height H onto the pan and sticks to it. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the speed of the clay at the instant it hits the pan.

(b) Determine the speed of the clay and pan just after the clay strikes it.

(c) After the collision, the apparatus comes to rest at a distance H/2 below the current position. Determine the spring constant of the attached spring.

(d) Determine the resulting period of oscillation.

Solution to part a: The speed of the clay when it hits the pan can be found using conservation of energy.

Since the clay descends a height H, it's gravitational potential energy will decrease. It will convert to kinetic energy.

On top of that, we know that the initial kinetic energy K_i must be 0 since the clay is dropped from rest so it's initial speed is 0.

This means we can simplify the conservation of energy equation to $U_i = K_f$ (this means that the gravitational potential energy converts to kinetic energy)

We can plug in our formulas for U and K to get $mgh = \frac{1}{2}mv^2$

Cancelling m and simplifying gives that $v = \sqrt{2gh}$

We know that the height where the clay is dropped from is denoted as H in this problem.

We can plug H in replacement of h to get $\boxed{v = \sqrt{2gH}}$

Solution to part b: The clay will collide with the pan. We will have an inelas-

tic collision.
We know that momentum is conserved. This means $p_i = p_f$ from the conservation of momentum formula.

We know that $p_i = mv = M\sqrt{2gH}$
To write an expression for p_f, we can denote the common velocity of both masses as v_f.
We know that $p_f = mv = (M + M)v_f = 2Mv_f$

$$\text{Setting both expressions equal to each other gives } M\sqrt{2gH} = 2Mv_f$$

Dividing both sides by $2M$ gives $\boxed{v_f = \frac{\sqrt{2gH}}{2}}$

Solution to part c: We can use conservation of energy in this problem.

$$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}$$

The initial point that we consider is the point of collision. The final point will be the point when the apparatus is at rest.

We will make our reference level be the point where the apparatus comes to rest. This causes U_{gf} (final gravitational potential energy) to be 0. Also, $K_f = 0$ since the apparatus comes to rest.

We will set U_{si} to 0. We will set the position immediately after collision to 0 spring potential energy.

We can find U_{sf} by finding spring potential energy for the additional stretch distance that comes after the collision.

$$\text{We know that } U_{sf} = \frac{1}{2}k(\frac{H}{2})^2 = \frac{kH^2}{8}$$

We can plug these values into our conservation of energy formula to get $K_i + U_{gi} = U_{sf}$

We also know that $K = \frac{1}{2}mv^2$. Since our combined mass is $2M$ and velocity is $v_f = \frac{\sqrt{2gH}}{2}$, we can plug that in to get

$$K_i = \frac{1}{2} \cdot 2M \cdot (\frac{\sqrt{2gH}}{2})^2 = \frac{MgH}{2}$$

Similarly, U can be represented as mgh. In our case, the mass is $2M$ and height is $\frac{H}{2}$. This means that $U_i = MgH$

$$\text{We can plug these values in to get } \frac{MgH}{2} + MgH = \frac{kH^2}{8}$$

$$\text{We can divide both sides by } H \text{ to get } \frac{Mg}{2} + Mg = \frac{kH}{8}$$

The left side is $\frac{3Mg}{2}$ which means the equation is $\frac{3Mg}{2} = \frac{kH}{8}$
We can multiply both sides by $\frac{8}{H}$ to get $\boxed{k = \frac{12Mg}{H}}$

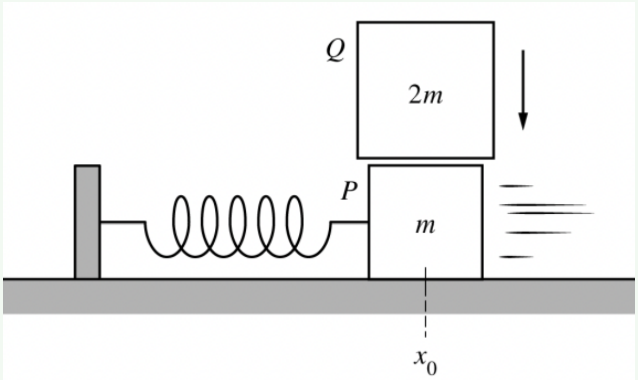
Solution to part d: We know that period can be represented as $2\pi\sqrt{\frac{m}{k}}$

We can plug in $2M$ for mass and $\frac{12Mg}{H}$ for spring constant.

Doing so gives that $T = 2\pi\sqrt{2M \div \frac{12Mg}{H}} = \boxed{2\pi\sqrt{\frac{H}{6g}}}$

Problem 6.0.14 — 2018 AP Physics 1 FRQ
Block P of mass m is on a horizontal, frictionless surface and is attached to a spring with spring constant k. The block is oscillating with period T_P and amplitude A_P about the spring's equilibrium position x_0. A second block Q of mass $2m$ is then dropped from rest and lands on block P at the instant it passes through the equilibrium position, as shown above. Block Q immediately sticks to the top of block P, and the two-block system oscillates with period T_{PQ} and amplitude A_{PQ}.

(a) Determine the numerical value of the ratio $\frac{T_{PQ}}{T_P}$.



(b) The figure is reproduced above. How does the amplitude of oscillation A_{PQ} of the two-block system compare with the original amplitude A_P of block P alone?
______ $A_{PQ} < A_P$ ______ $A_{PQ} = A_P$ ______ $A_{PQ} > A_P$
In a clear, coherent paragraph-length response that may also contain diagrams and/or equations, explain your reasoning.

Solution to part a: We know that period can be written as $T = 2\pi\sqrt{\frac{m}{k}}$
T_P only involves block P which has a mass of m. That means our period is simply $T_p = 2\pi\sqrt{\frac{m}{k}}$

However, for T_{PQ}, things are a little different. The reason is that now the combined mass of block P and Q is $3m$. This means we must plug in $3m$ instead of m into our formula for period. This means $T_{PQ} = 2\pi\sqrt{\frac{3m}{k}}$

Clearly in the ratio of $\frac{T_{PQ}}{T_P}$, everything cancels out except for the factor of 3. We can find that

$$\frac{T_{PQ}}{T_P} = \sqrt{3}$$

Solution to part b: We know that the amplitude can be found from conservation of energy.
We will first find the speed after collision. This will be done using conservation of momentum.

We know that $p_i = p_f$

If block P moves with velocity v initially, after block Q is dropped the velocity of both blocks will drop.
Since $p_i = mv$, we know that $p_f = (m + 2m)v_f$
We know that both blocks will move with a common final speed v_f due to this collision being inelastic.
We can solve $mv = (m + 2m)v_f$ to find that $v_f = \frac{v}{3}$

Before collision, we know that $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$ since the kinetic energy will convert to spring potential energy.
We can solve it to find that the amplitude A_P is $v\sqrt{\frac{m}{k}}$

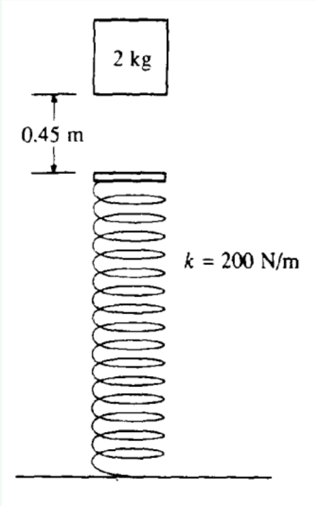
When both block P and Q are attached to the spring, we can apply the same method again.
$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$
However, now there is one difference, Instead of m, we must use $3m$ since that is the combined mass. In addition, our velocity is $\frac{v}{3}$ since that's the new velocity after collision.

We can plug those values in to get that $\frac{1}{2} \cdot 3m \cdot (\frac{v}{3})^2 = \frac{1}{2}kx^2$
We can solve this to find that the amplitude A_{PQ} is $v\sqrt{\frac{m}{3k}}$

Clearly, the amplitude is lower for blocks P and Q combined. The reason for this is the lower velocity of the two-block system which leads to lower kinetic energy. This causes the amplitude to be lower after we equate kinetic energy to maximum spring potential energy which occurs at the amplitude.

The answer is $\boxed{A_{PQ} < A_P}$

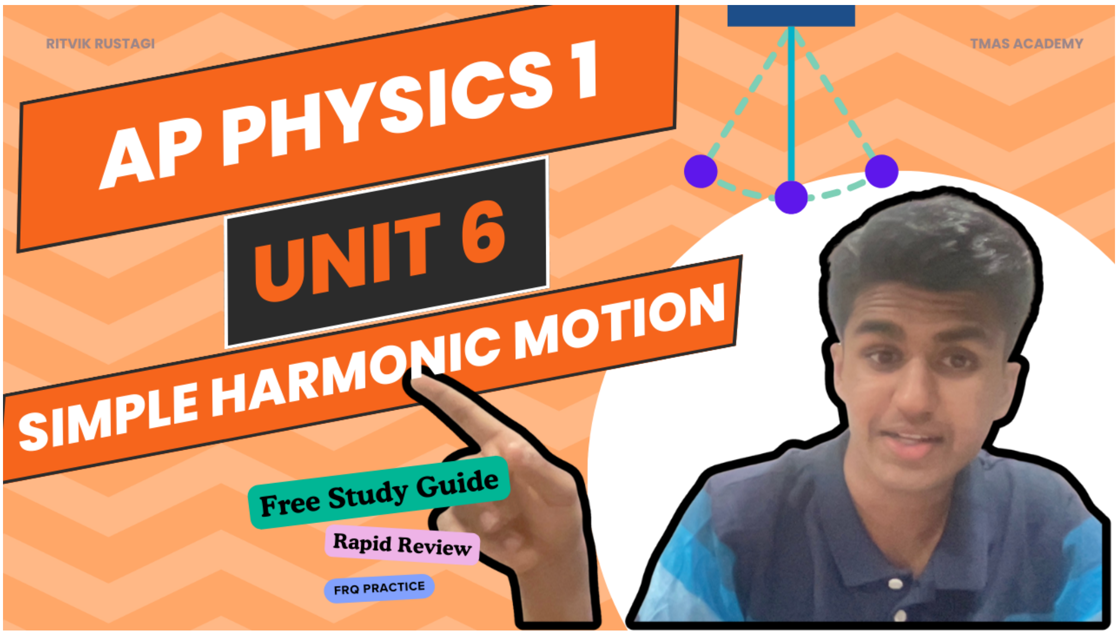
Problem 6.0.15 — 1989 AP Physics B FRQ



A 2-kilogram block is dropped from a height of 0.45 meter above an uncompressed spring, as shown above. The spring has an elastic constant of 200 newtons per meter and negligible mass. The block strikes the end of the spring and sticks to it.

a. Determine the speed of the block at the instant it hits the end of the spring.
b. Determine the force in the spring when the block reaches the equilibrium position
c. Determine the distance that the spring is compressed at the equilibrium position
d. Determine the speed of the block at the equilibrium position
e. Determine the resulting amplitude of the oscillation that ensues
f. Is the speed of the block a maximum at the equilibrium position, explain.
g. Determine the period of the simple harmonic motion that ensues

Solution: Video Solution



Unit 7 Torque and Rotational Motion

Have you ever been on a seesaw in the park near your house? Maybe you went with multiple friends. Did you notice that sometimes the seesaw would be balanced, so neither your friend nor you would move up and down. Then, you might tell your friend to move further away from you (further from the pivot). After that, you would start to rotate. Now to truly understand the famous seesaw, you must learn about torque and rotational motion.

Before you proceed to learn about torque and rotational motion, I want to define the term axis of rotation. Axis of rotation is basically a straight line that some body rotates around. For example, a carousel will have an axis of rotation since the entire ride spins around an imaginary axis.

Now let's dive into torque.

Note 7.0.1

Torque
Torque is the rotational analogue of force. If a rigid body rotates around an origin O, torque is defined as

$$\tau = FR \sin\theta,$$

where F is the magnitude of the force applied at a point at distance R from O, and the smaller angle between the line of force and the line between R and O is θ.

In simple words, you can simply use the formula $\tau = FR$ as long as the distance R you find is the perpendicular distance between the pivot and the force. This distance R is also known as the lever arm.

In addition, be very careful about the **direction** that you denote in a problem. Traditionally, the **counterclockwise** direction is defined as positive. Thus, if a torque is causing an object to spin in the counterclockwise direction, write it with a positive sign. However, if it's causing it to move in the clockwise direction, write it with a negative sign. You will see examples of this later, so don't worry if you're confused.

Thus, if you have a very long lever arm (perpendicular distance between the force and the axis of rotation), then the torque will be greater. The idea behind torque has influenced many tools such as wrenches.

You may have noticed that a very small wrench can struggle to tighten or loosen a bolt. The reason is that the lever arm is extremely small. The axis you rotate around will be close to the force you apply. This will cause the torque to be less. However, a large wrench will make it much easier to tighten or loosen that same bolt. The reason is that the lever arm is now longer, and this leads to a greater torque.

Another common example of this is a door. Try to open a door by pushing the hinge. It will be EXTREMELY hard. The reason is that the force you apply is super close to the hinge, causing the lever arm to be small. That is why a door knob is placed far from the hinge. It maximizes the lever arm allowing torque to be greater. Thus, you don't need to apply as great of a force due to the long lever arm.

We already covered kinematics in Unit 1. However, our unit of rotation also has kinematics in it. It's known as rotational kinematics. It's extremely similar to the kinematics we have learned. The formulas will be almost the same, but the variables will be different. Make analogies with unit 1 if you want to have an easy time with rotational kinematics.

Note 7.0.2

Rotational Kinematics

The angular displacement θ is defined as the radians an object has rotated.

The angular velocity ω is defined as the rate of change of θ, or $\frac{\Delta\theta}{\Delta t}$.

The angular acceleration α is defined as the rate of change of ω, or $\frac{\Delta\omega}{\Delta t}$

Similar to regular kinematics, the formulas still hold, where the linear variables are just replaced with the rotational variables. For example, θ replaces d or Δx, ω replaces v, and α replaces a.

Similar to regular kinematics, subscripts will still be used in rotational kinematics. For example, you still want to use ω_o or ω_i to denote the initial angular velocity and ω_f to denote the final angular velocity.

$$\omega_f = \omega_i + \alpha t \qquad (1.1)$$

$$\Delta\theta = \frac{\omega_i + \omega_f}{2} \cdot t \qquad (1.2)$$

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2 \qquad (1.3)$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta \qquad (1.4)$$

All the rotational kinematic equation above are true if the angular acceleration is constant. Out of the 5 variables $t, \omega_i, \omega_f, \alpha, \Delta\theta$, you must know 3 values to find the rest using the equations above.

Note that $\Delta\omega$ represents the angular displacement.

In Unit 2, we learned about Newton's Laws and the idea behind inertia. For example, an object with a larger mass would take a lot more force to move. Something similar occurs for this unit.

Note 7.0.3

Rotational Inertia

In rotational motion, rotational inertia is analogous to mass in linear motion. It is defined as

$$I = MR^2$$

for a particle, where R is the distance of the particle from the pivot. Note that I denotes rotational inertia.

The rotational inertia for multiple objects is $I = \sum m_i r_i^2$. You basically add up the rotational inertias of each object.

The rotational inertias for common rigid bodies are $\frac{2}{5}MR^2$ for a solid sphere, $\frac{2}{3}MR^2$ for a hollow sphere, $\frac{1}{12}ML^2$ for a rod rotated around the center, $\frac{1}{3}ML^2$ for a rod rotated around its end, MR^2 for a ring, MR^2 for a hollow cylinder, $\frac{1}{2}MR^2$ for a disk, and $\frac{1}{2}MR^2$ for a solid cylinder.

You don't need to memorize the rotational inertias for these solid bodies for AP Physics 1. They will be given to you in the problem. However, you should conceptually understand when the rotational inertia of a rigid body is larger or smaller than another one.

For example, let's compare a hoop and a disk. For a hoop and disk that have the same mass and same radius, the rotational inertia will be larger for the hoop. The reason is that the mass is concentrated towards the ends. However, a disk has mass throughout the entire area unlike a hoop (which only has mass near the circumference).

If you can compare the rotational inertias of certain objects by analyzing how the mass is distributed in them, then you will have a very strong conceptual understanding to approach multiple choice questions.

Now, there is something known as equilibrium. When an object is at complete equilibrium, then $F_{net} = 0$ and $\tau_{net} = 0$. The net force and net torque will both be 0. This is specifically known as **static equilibrium**.

Often for such problems, you will write two equations. One will be for the net torque and one will be for the net force.

Note 7.0.4

Newton's 2nd Law of Rotation is an analog to Newton's second law.

$$\tau_{net} = I\alpha$$

This means that the net torque will cause angular acceleration.
When an object is at rotational equilibrium, we can say that the net torque on it must be 0.

Now, we must learn all about another specific type of energy. For any object that has an angular velocity, it will have rotational kinetic energy.

The rotational kinetic energy is $\frac{1}{2}I\omega^2$.

If a problem asks you to find the total kinetic energy, then you must add up the rotational kinetic and translational kinetic energies.

$$\text{The total kinetic energy is } \frac{1}{2}mv^2 + \frac{1}{2}Iw^2$$

Note 7.0.5

Rotational Work is similar to work done by a force. When a torque τ is applied for an angular distance of θ, the work done is

$$W = \tau\theta$$

In terms of calculus, the work done can be found by the formula below.

Now, we will learn the tricks to solve a popular problem that always shows up on the AP exam.

When a problem describes the motion as rolling without slipping, then our object is both rotating and translating without sliding. In addition, in this scenario, static friction will act on the object, but it will do no work on that object.

In such situations, $v = \omega r$. This means that the angular velocity times the radius will be the velocity.

Note, that if the object is **slipping**, then the relationship $v = wr$ won't be true.

Now, instead of linear momentum, we will look at angular momentum. You will be surprised by how similar both topics are.

Note 7.0.6

Angular Momentum

Similar to linear momentum, angular momentum is defined as

$$L = mvr\sin(\theta)$$

This is the formula that you should use for a point mass, where θ is the angle between the radius vector and velocity vector.

For a rotating body around a specific origin with angular velocity ω and inertia I, the angular momentum should be represented as

$$L = I\omega$$

Traditionally, you should use $L = mvr\sin(\theta)$ for a point mass and $L = I\omega$ for rigid bodies such as a hoop.

Let's discuss impulse now. Remember how the impulse was equivalent to the change in linear momentum.
Something similar occurs in this unit. **Angular impulse** is the change in **angular momentum**.

Note 7.0.7

ΔL represents the change in angular momentum. It's also equivalent to the angular impulse which is $\tau \Delta t$.

$$\Delta L = I\Delta\omega = \tau\Delta t$$

Now, we can use this information to figure out what conservation of angular momentum is.
We should know by now that linear momentum is conserved when there are no external forces.
For **angular momentum to be conserved**, there should be no **external torques** on our system.

Conservation of angular momentum gives that $I_1 w_{1i} + I_2 w_{2i} = I_1 w_{1f} + I_2 w_{2f}$

Of course knowing this formula isn't enough to do well on the AP exam. You must practice many problems to be able to do well. There are many different scenarios that you might encounter on the AP exam. You should check out the AP Physics 1 Unit 7 Rapid Review video on the TMAS Academy youtube channel to learn about common examples, such as a ballerina spinning in a circle and pulling her arms inwards.

Now get ready for a big set of problems that will help you process this entire unit.

Problem 7.0.8 — A 15-kg box sits on a lever arm at a distance of 5 meters from the axis of rotation. What distance must a second 10-kg box sit to create a clockwise moment that will result in a net torque of zero?

Solution: Since we want a net torque of zero, both boxes should apply the same torque. They are both on the opposite sides of the axis of rotation, so if they both have an equal magnitude of torque, then the net torque will end up being 0.

The 15-kg block applies a torque of $\tau = Fr$.
Now, we must figure out what force is applied on the lever arm. Since the 15-kg box is sitting on the lever arm, the force will simply be equal to the magnitude of the weight which is $m_1 g$ (let's give the weight of the heavier box the variable m_1).

$$\tau = Fr = m_1 g \cdot r = 5m_1 g$$

This means that the 10-kg block must also apply that same torque. Let's denote the mass of this block as m_2. Let's say that x is the distance of the lighter box from the axis of rotation.
The torque: $\tau = Fr = m_2 g x$

Since the net torque is 0, we know that $5m_1g = m_2gx$
We can cancel out the g and rearrange to get $x = \frac{5m_1}{m_2}$

$$\text{We can plug in the masses to get } x = \frac{5m_1}{m_2} = \frac{5 \cdot 15}{10} = 7.5 \text{ m}$$

Problem 7.0.9 — The iron door of a building is x meters wide. It can be opened by applying a force of F_1 newtons at the middle of the door. Calculate the least force which can open the door in terms of F_1.

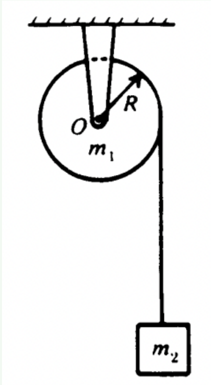
Solution: Since the door is x meters wide, the middle of the door will be $x/2$ meters from the hinge.
That means the door can be opened if a torque of $\tau = F_1 \cdot \frac{x}{2} = \frac{F_1x}{2}$ is applied.

This required torque will remain constant despite where the force is applied. However, it's the force and distance from the hinge that can vary. The force to open the door will be minimized when it's applied as far as possible from the hinge. The reason is that this will maximize the distance from the hinge, causing the required force to be less.

Thus, the force should be applied at the edge of the door so that the lever arm is x meters long.
Thus, if the force applied is F_a, then the torque will be F_ax. We can set this equal to $\frac{F_1x}{2}$ to find F_a (least force to open the door).

$$\text{We can divide both sides by } x \text{ to find that } F_a = \frac{F_1}{2}$$

Problem 7.0.10 — 1983 AP Physics FRQ



A uniform solid cylinder of mass m_1 and radius R is mounted on frictionless bearings about a fixed axis through O. The moment of inertia of the cylinder about the axis is $I = \frac{1}{2}m_1R^2$. A block of mass m_2, suspended by a cord wrapped around the cylinder as shown above, is released at time $t = 0$.

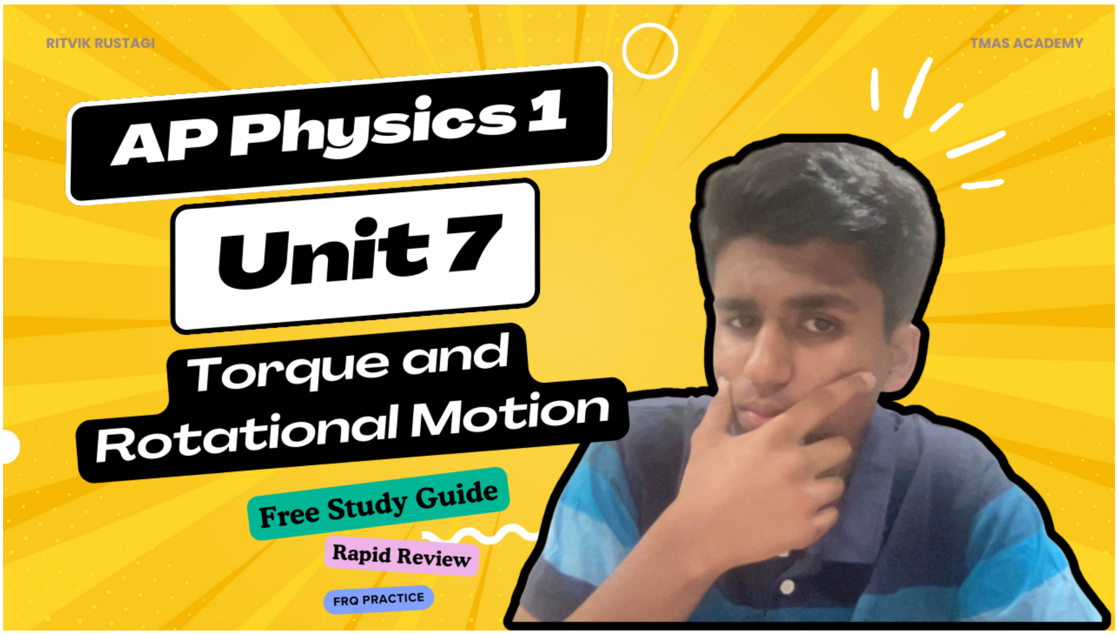
(a) Identify all of the forces acting on the cylinder and on the block, and draw free-body diagrams for both.

(b) In terms of m_1, m_2, R, and g, determine each of the following.

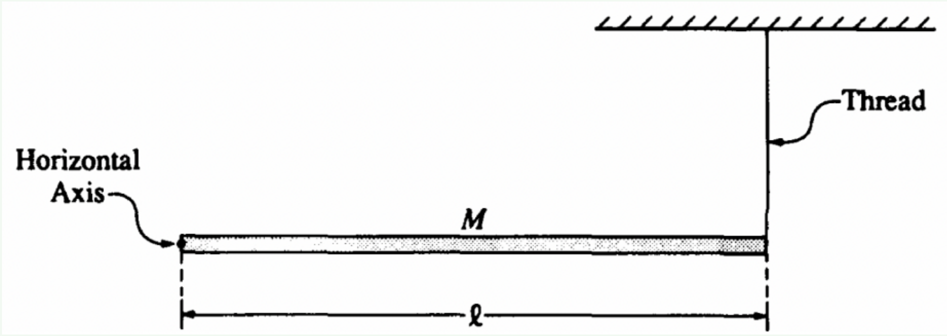
i. The acceleration of the block

ii. The tension in the cord

Solution: Video Solution



Problem 7.0.11 — 1993 AP Physics C: Mechanics FRQ



A long, uniform rod of mass M and length l is supported at the left end by a horizontal axis into the page and perpendicular to the rod, as shown above. The right end is connected to the ceiling by a thin vertical thread so that the rod is horizontal. Express the answers to all parts of this question in terms of M, L, and g.

(a) Determine the magnitude and direction of the force exerted on the rod by the axis.

(b) If the breaking strength of the thread is $2Mg$, determine the maximum distance, r, measured from the hinge axis, that a box of mass $4M$ could be placed without breaking the thread.

Solution to part a: Let's say that force from the horizontal axis will be denoted as F and it will point upwards.
Then, our summation of forces is $F + T = Mg$ (T is the tension force from the thread)
Now, we will apply the concept of torque about the horizontal axis.
We know that $\tau_{net} = \sum F_i r_i$.

About the horizontal axis, the tension force is a distance L away while the gravitational force is $\frac{L}{2}$ away from the pivot(since gravity acts at the center)

Clearly, $\tau_{net} = T \cdot L - Mg \cdot \frac{L}{2}$
We know that our net torque must be 0 since this system is static (there is no motion going on). Since there is no rotational acceleration, net torque must be 0.
This means that $T \cdot L - Mg \cdot \frac{L}{2} = 0$
We can rearrange this and solve to find that $T = \frac{Mg}{2}$

Now, we can plug this into the equation $F + T = Mg$.
Doing so gives that $\boxed{F = \frac{Mg}{2}}$. Since our force is positive, the direction we assumed initially for this force is the correct direction.

Solution to part b: We are given the maximum possible tension force T to be $2Mg$.

We can use our horizontal axis as the pivot point. This means that

$$\tau_{net} = T \cdot L - Mg \cdot \frac{L}{2} - 4Mg \cdot r$$

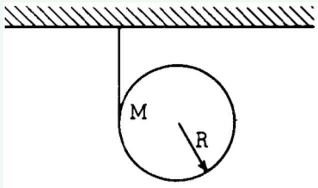
We know that $\tau_{net} = 0$ since the system is in static equilibrium. We can plug in our maximum possible value of tension which is $2Mg$
Doing so simplifies the equation to

$$2MgL - \frac{MgL}{2} - 4Mgr = 0$$

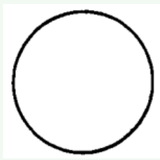
We can further simplify this to $\frac{3MgL}{2} = 4Mgr$

Now, we can divide both sides by $4Mg$ to get that $r = \frac{3L}{8}$

Problem 7.0.12 — 1976 AP Physics FRQ



A cloth tape is wound around the outside of a uniform solid cylinder (mass M, radius R) and fastened to the ceiling as shown in the diagram above. The cylinder is held with the tape vertical and then released from rest. As the cylinder descends, it unwinds from the tape without slipping. The moment of inertia of a uniform solid cylinder about its center is $1/2MR^2$.



(a) On the circle above draw vectors showing all the forces acting on the cylinder after it is released. Label each force clearly.

(b) In terms of g, find the downward acceleration of the center of the cylinder as it unrolls from the tape.

(c) While descending, does the center of the cylinder move toward the left, toward the right, or straight down? Explain.

Solution to part a: The only forces on the cylinder are tension force and gravitational force.
Sometimes the problem will say to draw the forces at the center. In that case, no matter from what point the force stems from, you should still draw all forces pointing from the center. In this case, the problem doesn't say list such a condition which is why I'll draw tension force from the side (since that's where the rope is connected to the cylinder at).

Solution to part b: We will write an equation for net force and net torque.

$$F_{net} = Mg - T = Ma$$

We will write an equation for net torque around the center of the cylinder.

$$\tau_{net} = T \cdot R = I\alpha$$

Since the cylinder rolls without slipping, we know that $\alpha = \frac{a}{R}$. This relates acceleration to angular acceleration.
We can plug that into our equation for net torque to get $T \cdot R = I \cdot \frac{a}{R}$

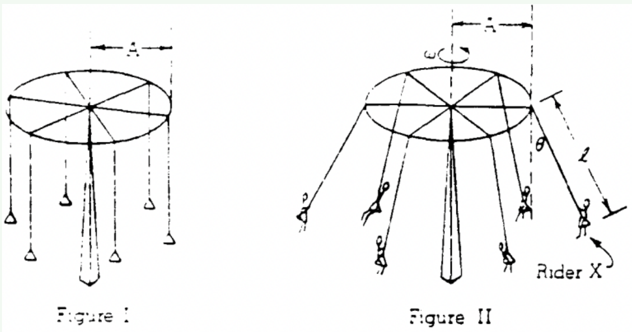
Since we know that $I = \frac{1}{2}MR^2$, we can plug that in to get $T \cdot R = \frac{1}{2}MR^2 \cdot \frac{a}{R} = \frac{MRa}{2}$

We can divide both sides by R to find that $T = \frac{Ma}{2}$

Now, we can plug this into the equation for net force which is $Mg - T = Ma$
Doing so gives $Mg - \frac{Ma}{2} = Ma$
We can rearrange this equation and solve it to find that $\boxed{a = \dfrac{2g}{3}}$

Solution to part c: Clearly, there are no forces in the horizontal direction. It means that the cylinder can't move to the left or to the right. It must move $\boxed{\text{straight down}}$.

Problem 7.0.13 — 1978 AP Physics FRQ



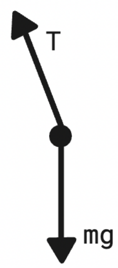
An amusement park ride consists of a ring of radius A from which hang ropes of length l with seats for the riders as shown in Figure I. When the ring is rotating at a constant angular velocity ω, each rope forms a constant angle θ with the vertical as shown in Figure II. Let the mass of each rider be m and neglect friction, air resistance, and the mass of the ring, ropes, and seats.

(a) Draw and label all the forces acting on rider X under the constant rotating condition of Figure II. Clearly define any symbols you introduce.

(b) Derive an expression for ω in terms of A, l, θ, and the acceleration of gravity g.

(c) Determine the minimum work that the motor that powers the ride would have to perform to bring the system from rest to the constant rotating condition of Figure II. Express your answer in terms of m, g, l, θ, and the speed v of each rider.

Solution to part a: The two forces on the rider will be tension force and gravitational force. T is used to denote tension force while mg is the gravitational force.



Solution to part b: We know that a centripetal force points towards the center. The forces in the vertical direction will balance out. However, there will be a net horizontal force and that is what leads to centripetal acceleration.

In the vertical direction, we know that $T\cos(\theta) - mg = 0$

In the horizontal direction, $T\sin(\theta) = ma = mw^2r$

It is the horizontal component of tension that leads to centripetal acceleration. Also, mw^2r is the same thing as $\frac{mv^2}{r}$. The only difference is that one uses angular velocity while the other uses velocity itself in the expression.

Now, we must find an expression for our radius which currently is written as r. Clearly, the rider is a distance A (radius of the ring) summed up with the horizontal length of the rope. This means that $r = A + l\sin(\theta)$

We can plug this into the equation for net force in x-direction to get

$$T\sin(\theta) = mw^2(A + l\sin(\theta))$$

Using the equation in the vertical direction, we can find that $T = \frac{mg}{\cos(\theta)}$
We can plug our expression for tension into $T\sin(\theta) = mw^2(A + l\sin(\theta))$

$$\frac{mg}{\cos(\theta)} \cdot \sin(\theta) = mw^2(A + l\sin(\theta))$$

We can simplify the equation to $w^2 = \frac{g\tan(\theta)}{A+l\sin(\theta)}$
Now, we can take the square root of both sides to get $w = \sqrt{\frac{g\tan(\theta)}{A+l\sin(\theta)}}$

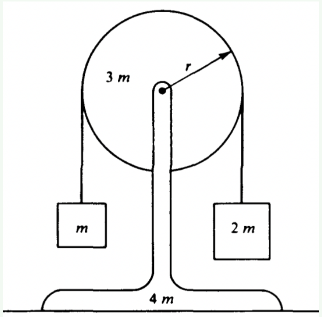
Solution to part c: We know that the work done is change in energy.
$W = \Delta K + \Delta U$

Clearly, $\Delta K = \frac{1}{2}mv^2$
Also, $\Delta U = U_f - U_i = mgh - 0 = mgl(1 - \cos(\theta))$

This means that the work done for one rider is $W = \frac{1}{2}mv^2 + mgl(1 - \cos(\theta))$
However, since we have 6 riders, we must multiply that expression by 6.

Doing so gives that the total work done is $3mv^2 + 6mgl(1 - \cos(\theta))$

Problem 7.0.14 — 1985 AP Physics FRQ



A pulley of mass $3m$ and radius r is mounted on frictionless bearings and supported by a stand of mass $4m$ at rest on a table as shown above. The moment of inertia of this pulley about its axis is $1.5mr^2$. Passing over the pulley is a massless cord supporting a block of mass m on the left and a block of mass $2m$ on the right. The cord does not slip on the pulley, so after the block-pulley system is released from rest, the pulley begins to rotate.

(a) On the diagrams below, draw and label all the forces acting on each block.

(b) Use the symbols identified in part a. to write each of the following.

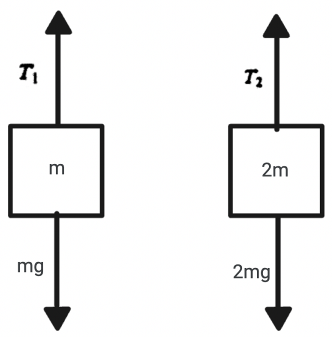
i. The equations of translational motion (Newton's second law) for each of the two blocks

ii. The analogous equation for the rotational motion of the pulley

(c) Solve the equations in part b. for the acceleration of the two blocks.

(d) Determine the tension in the segment of the cord attached to the block of mass m.

Solution to part a: The only forces on each block are tension force and gravitational force. The length of the force vectors don't matter as long as all the forces are labelled. The reason is that the problem doesn't explicitly say to make the lengths of the force vectors comparable by magnitude.



Solution to part b i: We will use Newton's Second Law for both blocks.
For block of mass $2m$, we get $2mg - T_2 = 2ma$
For block of mass m, we get $T_1 - mg = ma$

Solution to part b ii: We can use Newton's Second Law for rotation which says $\tau_{net} = I\alpha$
Clearly, $\tau_{net} = T_2 r - T_1 r = r(T_2 - T_1)$
This means that $r(T_2 - T_1) = I\alpha$

Solution to part c: Since the cord does not slip on the pulley, we know that $\alpha = \frac{a}{r}$
This relates our angular acceleration to acceleration itself.

We can plug that into $r(T_2 - T_1) = I\alpha$ to get $r(T_2 - T_1) = \frac{Ia}{r}$

Now, we can substitute our expression for I which is $1.5mr^2$

Plugging it in turns the equation into $r(T_2 - T_1) = 1.5mra$

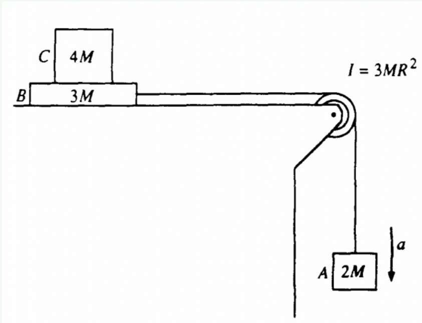
We can divide r from both sides to get $T_2 - T_1 = 1.5ma$

Now, we can solve the two equations we got using Newton's Second Law in part b i. The two equations that we got were $2mg - T_2 = 2ma$ and $T_1 - mg = ma$
We can add both of these equations to get $mg - (T_2 - T_1) = 3ma$
Now, we can substitute the expression we found previously which was $T_2 - T_1 = 1.5ma$

Doing so turns the equation into $mg - 1.5ma = 3ma$
After adding $1.5ma$ to both sides, we can find that $\boxed{a = \frac{2g}{9}}$

Solution to part d: We can find the tension in the cord for the block of mass m by using the equation we found for that block using Newton's Second Law.
We found that $T_1 - mg = ma$
We can rearrange the equation to get $T_1 = m(a + g)$
We can plug in $a = \frac{2g}{9}$ to get $T_1 = m(\frac{2g}{9} + g) = \boxed{\frac{11mg}{9}}$

Problem 7.0.15 — 1989 AP Physics FRQ



Block A of mass $2M$ hangs from a cord that passes over a pulley and is connected to block B of mass $3M$ that is free to move on a frictionless horizontal surface, as shown above. The pulley is a disk with frictionless bearings, having a radius R and moment of inertia $3MR^2$. Block C of mass $4M$ is on top of block B. The surface between blocks B and C is NOT frictionless. Shortly after the system is released from rest, block A moves with a downward acceleration a, and the two blocks on the table move relative to each other.

In terms of M, g, and a, determine the
(a) tension T_v in the vertical section of the cord

(b) tension T_h in the horizontal section of the cord

If $a = 2$ meters per second squared, determine the
(c) coefficient of kinetic friction between blocks B and C

(d) acceleration of block C

Solution to part a: We will find the net force on the block of mass $2M$. The forces on it are the gravitational force and tension in the vertical section (T_v)

The equation for Newton's Second Law on that block is $2Mg - T_v = 2Ma$
We can rearrange the equation to find that $\boxed{T_v = 2M(g - a)}$

Solution to part b: Some people might use Newton's Second Law on blocks B and C to find T_h.
However, this won't work since friction force also exists and we don't know it's magnitude. However, we can still find T_h by using Newton's Second Law for Rotation.

We know that $\tau_{net} = I\alpha$. We can find the net torque on the pulley.

$$\tau_{net} = T_v \cdot R - T_h \cdot R = I\alpha$$

We can plug in $I = 3MR^2$ into the equation for net torque to get

$$T_v \cdot R - T_h \cdot R = 3MR^2 \cdot \alpha$$

We can divide R from both sides to get $T_v - T_h = 3MR\alpha$
We will also use the fact that $\alpha = \frac{a}{R}$. We can plug this in to get $T_v - T_h = 3Ma$

Since we know that $T_v = 2M(g-a)$ from the first part, we can plug that in to find T_h. After we make our substitution, we find that $\boxed{T_h = 2Mg - 5Ma}$

Solution to part c: Now, we will use Newton's Second Law on the block B.
Clearly, the only forces in the horizontal direction on block B are tension force and friction force.
Using Newton's Second Law, we know that $F_{net} = T_h - F_f = m_b a = 3Ma$

We also know that $F_f = \mu N$. From forces in the y-direction on block C, it's obvious that $N = 4Mg$ (this same friction force is also applied on block C but in the opposite direction as of block B)
This means that $F_f = 4\mu Mg$

We can plug this into our equation for Newton's Second Law to get $T_h - 4\mu Mg = 3Ma$
We can plug in $T_h = 2Mg - 5Ma$ to get $2Mg - 5Ma - 4\mu Mg = 3Ma$

We can rearrange the equation to get $2Mg - 8Ma = 4\mu Mg$
We can divide both sides by M to get $2g - 8a = 4\mu g$

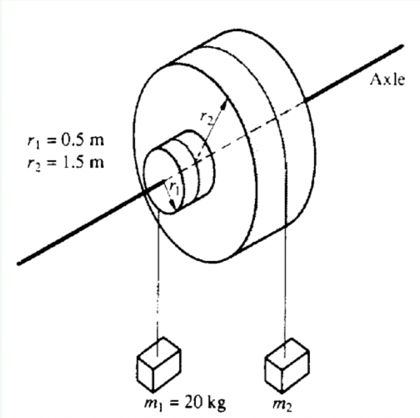
Now, we can plug in $g = 9.8$ and $a = 2$ to get $2 \cdot 9.8 - 8 \cdot 2 = 4 \cdot 9.8 \cdot \mu$
We can solve this equation to find that $\boxed{\mu = 0.092}$

Solution to part d: The acceleration of block C can be found using Newton's Second Law.
We know that $F_f = m_c a$
Clearly, $F_f = \mu N = \mu m_c g$

We can plug that in to get $\mu m_c g = m_c a$
After dividing both sides by m_c, we find that $a = \mu g$

Since $\mu = 0.092$, we know that $a = 0.092 \cdot 9.8 = \boxed{0.9016 \frac{m}{s^2}}$

Problem 7.0.16 — 1991 AP Physics FRQ



Two masses, m_1 and m_2, are connected by light cables to the perimeters of two cylinders of radii r_1 and r_2, respectively, as shown in the diagram above. The cylinders are rigidly connected to each other but are free to rotate without friction on a common axle. The moment of inertia of the pair of cylinders is $I = 45\,\text{kg}\cdot\text{m}^2$. Also, $r_1 = 0.5\,\text{m}$, $r_2 = 1.5\,\text{m}$, and $m_1 = 20\,\text{kg}$.

(a) Determine m_2 such that the system will remain in equilibrium.

The mass m_2 is removed, and the system is released from rest.
(b) Determine the angular acceleration of the cylinders.

(c) Determine the tension in the cable supporting m_1.

(d) Determine the linear speed of m_1 at the time it has descended 1.0 m.

Solution to part a: For the system to remain in equilibrium, the net torque should be 0. On top of that, the net force should also be 0.

We can write the summation of forces for both block of masses m_1 and m_2

Assuming that the force of tension of block of mass m_1 is T_1, we know that by Newton's Second Law, $T_1 - m_1 g = 0$. This means that $T_1 = m_1 g$

We can do something similar for the block of mass m_2. If tension force is T_2, then Newton's Second Law tells us that $T_2 - m_2 g = 0$ which means that $T_2 = m_2 g$

We also know that net torque (τ_{net}) must be 0 on the cylinder.
The net torque about the axle can be written as $\tau_{net} = T_1 \cdot r_1 - T_2 \cdot r_2$

Since $\tau_{net} = 0$, we know that $T_1 \cdot r_1 = T_2 \cdot r_2$
We can plug in $T_1 = m_1 g$ and $T_2 = m_2 g$ to get $m_1 g \cdot r_1 = m_2 g \cdot r_2$

$$\text{We can isolate } m_2 \text{ to get } m_2 = \frac{m_1 r_1}{r_2} = \frac{20 \cdot 0.5}{1.5}$$

This means that $\boxed{m_2 = 6.67 \text{ kg}}$

Solution to part b: To find the angular acceleration, we use the equation $\tau_{net} = I\alpha$
After m_2 is removed, the only force that applies a torque on the cylinder is the tension force from mass m_1.

We can find that $\tau_{net} = T_1 \cdot r_1$ and we know that this equals to $I\alpha$

Now, we can write another equation using Newton's Second Law on block of mass m_1.
Since the only forces on it are tension and gravity, we know that $m_1 g - T_1 = m_1 a$

Now, we can use the relation $\alpha = \frac{a}{r_1}$.
We can plug that into our first equation to get $T_1 \cdot r_1 = \frac{Ia}{r_1}$

We can rewrite this equation as

$$T_1 = \frac{Ia}{r_1^2}$$

Now, we can plug that into $m_1 g - T_1 = m_1 a$ to get

$$m_1 g - \frac{Ia}{r_1^2} = m_1 a$$

Since $I = 45$, $m_1 = 20$, and $r_1 = 0.5$, we can plug all of that into the equation above and solve it to get $a = 0.98 \frac{m}{s^2}$

Now, we can plug this back into $\alpha = \frac{a}{r_1}$ to find angular acceleration.
We can plug in $a = 0.98$ and $r_1 = 0.5$ to get $\boxed{\alpha = 1.96 \text{ rad}/s^2}$

Solution to part c: To find the tension force, we can plug in our value of angular acceleration.
We can plug it into $T_1 \cdot r_1 = I\alpha$
Since $r_1 = 0.5$, $I = 45$, and $\alpha = 1.96$, we can find that $\boxed{T_1 = 176.4 \text{ N}}$

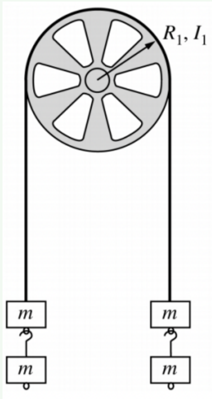
Solution to part d: To find the linear speed, we will use our acceleration (not angular acceleration).

We already found that $a = 0.98 \frac{m}{s^2}$. We also know that $v_i = 0$. On top of that, we know that $\Delta y = 1$ m.
We can use the kinematics equation $v^2 = v_i^2 + 2a\Delta y$
Plugging in our values gives

$$v^2 = 0^2 + 2 \cdot 0.98 \cdot 1 = 1.96$$

After we take the square root of both sides, we find that $\boxed{v = 1.4 \text{ m/s}}$

Problem 7.0.17 — 2000 AP Physics FRQ

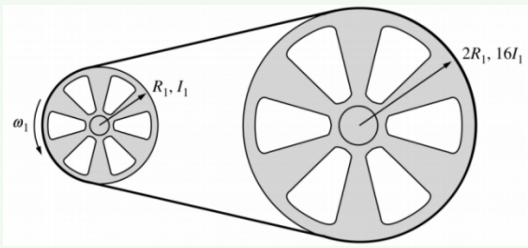


A pulley of radius R_1 and rotational inertia I_1 is mounted on an axle with negligible friction. A light cord passing over the pulley has two blocks of mass m attached to either end, as shown above. Assume that the cord does not slip on the pulley.
(a) Determine the tension T in the cord.
(b) One block is now removed from the right and hung on the left. When the system is released from rest, the three blocks on the left accelerate downward with an acceleration $g/3$. Determine the following.

i. The tension T_3 in the section of cord supporting the three blocks on the left

ii. The tension T_1 in the section of cord supporting the single block on the right

iii. The rotational inertia I_1 of the pulley



(c) The blocks are now removed, and the cord is tied into a loop, which is passed around the original pulley and a second pulley of radius $2R_1$ and rotational inertia $16I_1$. The axis of the original pulley is attached to a motor that rotates it at angular speed ω_1, which in turn causes the larger pulley to rotate. The loop does not slip on the pulleys. Determine the following in terms of I_1, R_1, and ω_1.

i. The angular speed ω_2 of the larger pulley

ii. The angular momentum L_2 of the larger pulley

iii. The total kinetic energy of the system

Solution to part a: Let's consider the two blocks on the left to be part of one system. The only two forces on the two block system are tension force and gravity.

Since it doesn't accelerate, we can use Newton's Second Law to write $T - 2mg = 0$
This means that $\boxed{T = 2mg}$

Solution to part b i: On the side with three blocks, we can consider all three of those blocks as a system. The only forces on that system are tension force and gravity. By using Newton's Second Law we know that $3mg - T_3 = 3ma$
Since $a = \frac{g}{3}$, we can plug that in above to get $\boxed{T_3 = 2mg}$

Solution to part b ii: We can again use Newton's Second Law. On the right side, we will only consider the 1 block alone as its own system. The only forces on it are tension force and gravity. It accelerates upwards, so we will consider that direction to be positive.

By using Newton's Second Law we know that $T_1 - mg = ma$
This means that $T_1 = m(a + g)$
We can plug in $a = \frac{g}{3}$ to get $\boxed{T_1 = \dfrac{4mg}{3}}$

Solution to part b iii: We can find rotational inertia by using Newton's Second Law for rotation.
We know that $\tau_{net} = I\alpha$. Now, let's write an equation using that for the torque on the pulley.

$$\tau_{net} = T_3 \cdot R_1 - T_1 \cdot R_1 = R_1(T_3 - T_1)$$

On top of that, we can use the relation $\alpha = \frac{a}{r}$

We can plug in $a = \frac{g}{3}$ and $r = R_1$ into $\alpha = \frac{a}{r}$ to find that $\alpha = \frac{g}{3R_1}$

Now we can plug that expression in to find that

$$R_1(T_3 - T_1) = \frac{Ig}{3R_1}$$

We can solve the equation for I to find that

$$I = \frac{3R_1^{22}(T_3 - T_1)}{g}$$

We can plug in $T_3 = 2mg$ and $T_1 = \frac{4mg}{3}$ to find that $\boxed{I = 2mR_1^2}$

Solution to part c i: Even though the angular speeds of the pulley don't have to be equal, the tangential speed of the cord around each pulley must be equal. The tangential speeds are equal because the cord around this system is the same. Thus, it must be moving with the same tangential speed regardless of the radius of any pulley.

The tangential speed around the original pulley is $v_1 = w_1 R_1$
Similarly, the tangential speed around the new pulley is $v_2 = w_2 \cdot 2R_1 = 2w_2 R_1$

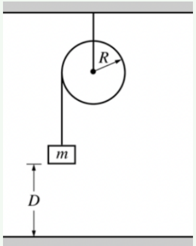
We can set v_1 and v_2 equal to each other to find that $\boxed{w_2 = \dfrac{w_1}{2}}$

Solution to part c ii: We know that angular momentum is $L = Iw$
For the larger pulley, $I = 16I_1$ and $w = w_2 = \frac{w_1}{2}$. We can plug that in to find that $L_2 = 16I_1 \cdot \frac{w_1}{2} = \boxed{8I_1 w_1}$

Solution to part c iii: The sum of the kinetic energy of the entire system is the sum of the kinetic energies for each pulley.
Since we know rotational inertia and angular velocity for each pulley, we will use $K = \frac{1}{2}Iw^2$

$$\text{The total kinetic energy is } \frac{1}{2}I_1w_1^2 + \frac{1}{2} \cdot 16I_1 \cdot (\frac{w_1}{2})^2 = \frac{5}{2}I_1w_1^2$$

Problem 7.0.18 — 2004 AP Physics FRQ



A solid disk of unknown mass and known radius R is used as a pulley in a lab experiment, as shown above. A small block of mass m is attached to a string, the other end of which is attached to the pulley and wrapped around it several times. The block of mass m is released from rest and takes a time t to fall the distance D to the floor.

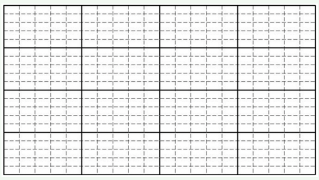
(a) Calculate the linear acceleration a of the falling block in terms of the given quantities.

(b) The time t is measured for various heights D and the data are recorded in the following table.

D (m)	t (s)
0.5	0.68
1	1.02
1.5	1.19
2	1.38

i. What quantities should be graphed in order to best determine the acceleration of the block? Explain your reasoning.

ii. On the grid below, plot the quantities determined in **b. i.**, label the axes, and draw the best-fit line to the data.



iii. Use your graph to calculate the magnitude of the acceleration.

(c) Calculate the rotational inertia of the pulley in terms of m, R, a, and fundamental constants.

(d) The value of acceleration found in b. iii, along with numerical values for the given quantities and your answer to c., can be used to determine the rotational inertia of the pulley. The pulley is removed from its support and its rotational inertia is found to be greater than this value. Give one explanation for this discrepancy.

Solution to part a: We can use kinematics to find an expression for acceleration. We know that vertical displacement Δy is D, $v_i = 0$, and we are given the various times.

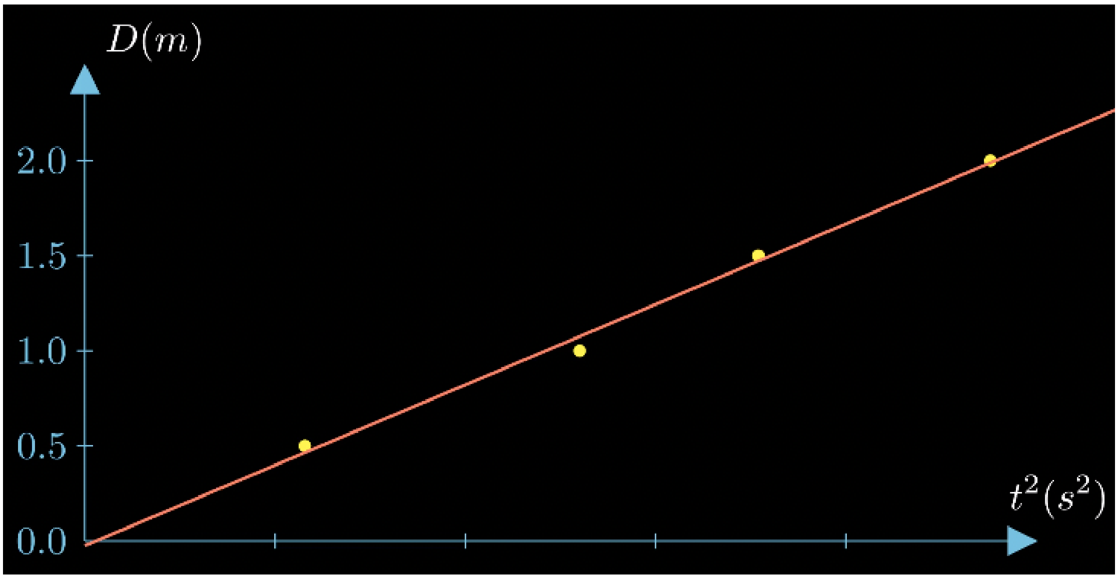
We can plug this into $\Delta y = v_i t + \frac{1}{2}at^2$

We can plug in our values to get $D = 0 + \frac{1}{2}at^2$

We can rearrange the equation to find that $\boxed{a = \dfrac{2D}{t^2}}$

Solution to part b i: We should graph D on the y-axis and t^2 on the x-axis.
The reason is that it causes our graph to be linear. Then, to find acceleration, we will simply only need to find the slope.

Solution to part b ii: The graph is shown below



Solution to part b iii: By observing the graph, it is evident that the slope is 1.
some people will immediately say that the acceleration is 1. However, they are forgetting one step.
The slope of 1 represents $\frac{D}{t^2}$. However, we want to find $\frac{2D}{t^2}$
That means we must multiply our slope by 2 to find acceleration.
$\boxed{a = 2 \text{ m}/s^2}$

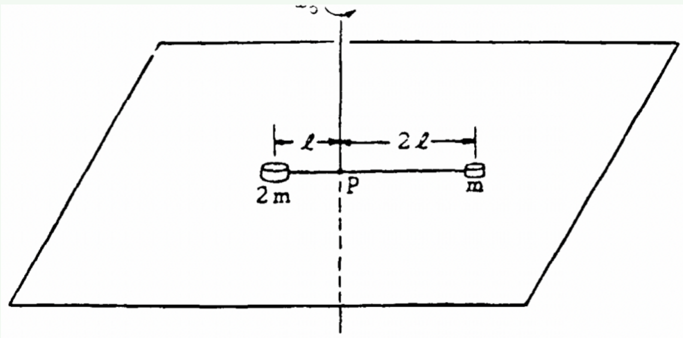
Solution to part c: We will use net torque and net force to find the rotational inertia.
Assuming that downwards is positive, using Newton's Second Law we can write $mg - T = ma$
Using net torque τ_{net}, we know that $\tau_{net} = TR = I\alpha$
Now, for the equation that used net torque, we can divide both sides by R to get $T = \frac{I\alpha}{R}$
We can plug this into $mg - T = ma$ to get $mg - \frac{I\alpha}{R} = ma$

Since $\alpha = \frac{a}{R}$, we can plug that into the equation above to get $mg - \frac{Ia}{R^2} = ma$
We can rearrange the equation to get $mg - ma = \frac{Ia}{R^2}$

After multiplying both sides by $\frac{R^2}{a}$, we get that $\boxed{I = \dfrac{mR^2(g-a)}{a}}$

Solution to part d: One possible discrepancy occurs if the string is wrapped around the pulley multiple times. The reason is that this will cause the radius of the pulley to increase. However, we used the radius of the pulley alone instead of accounting for the extra radius that came from the string being wrapped around many times.

Problem 7.0.19 — 1982 AP Physics FRQ



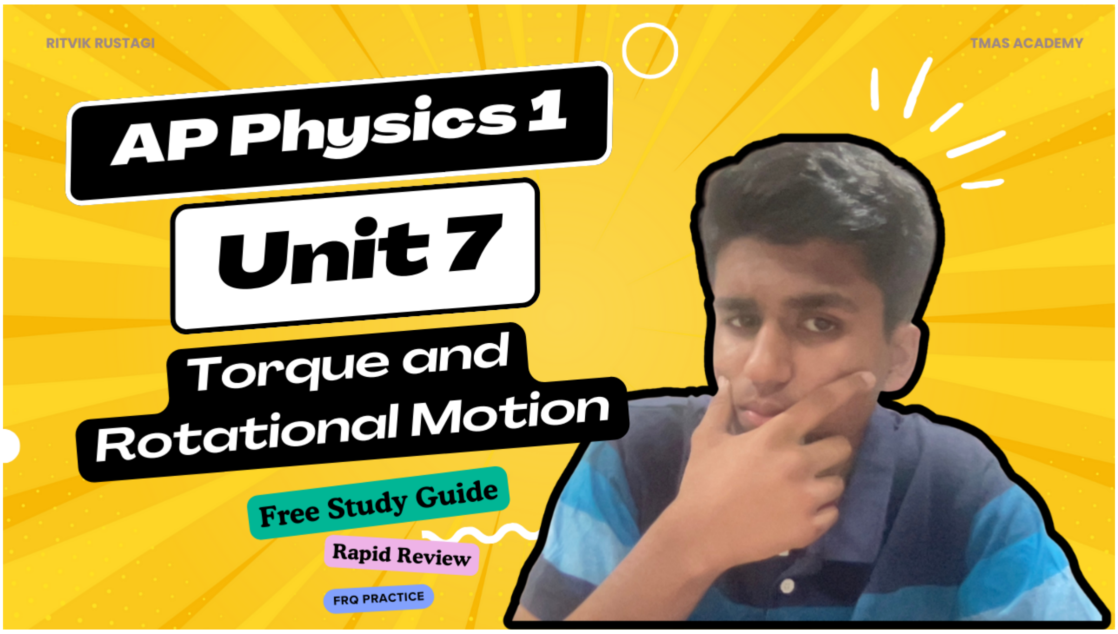
A system consists of two small disks, of masses m and $2m$, attached to a rod of negligible mass of length $3l$ as shown above. The rod is free to turn about a vertical axis through point P. The two disks rest on a rough horizontal surface; the coefficient of friction between the disks and the surface is μ. At time $t = 0$, the rod has an initial counterclockwise angular velocity ω_o about P. The system is gradually brought to rest by friction.
Develop expressions for the following quantities in terms of μ, m, l, g, and ω_o

(a) The initial angular momentum of the system about the axis through P

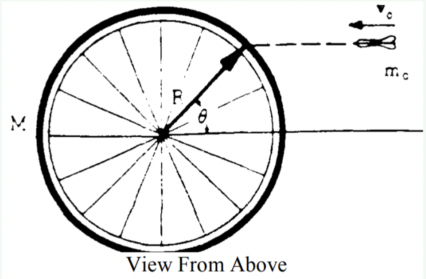
(b) The frictional torque acting on the system about the axis through P

(c) The time T at which the system will come to rest.

Solution: Video Solution



Problem 7.0.20 — 1975 AP Physics FRQ



A bicycle wheel of mass M (assumed to be concentrated at its rim) and radius R is mounted horizontally so it may turn without friction on a vertical axle. A dart of mass m_0 is thrown with velocity v_o as shown above and sticks in the tire.

(a) If the wheel is initially at rest, find its angular velocity ω after the dart strikes.

(b) In terms of the given quantities, determine the ratio:

$$\frac{\text{final kinetic energy of the system}}{\text{initial kinetic energy of the system}}$$

Solution to part a: We can use angular momentum conservation in this problem. Conservation says that $L_i = L_f$

The initial momentum can be found using the formula $L = mvr$. We know that r represents the perpendicular distance to the axis. Clearly, in this problem, the bullet is a distance $R\sin(\theta)$ away. This means that $L_i = mv_oR\sin(\theta)$

We know that after both collide, they will rotate the vertical axle. Since the bicycle wheel has all of its mass concentrated at the rim, the rotational inertia for the wheel alone is MR^2.

After collision, the rotational inertia of the dart will be mR^2 since it spins around with the bicycle wheel with a radius R.

The total rotational inertia can be found by summing both which gives $(m + M)R^2$

We know that $L = Iw$, so we can find that $L_f = (m + M)R^2 w_f$

We can equate this to L_i which is $mv_oR\sin(\theta)$

$$mv_oR\sin(\theta) = (m + M)R^2 w_f$$

Now, we can divide both sides by $(m + M)R^2$ to get $\boxed{w_f = \frac{mv_o\sin(\theta)}{(m + M)R}}$

Solution to part b: We know that the initial kinetic energy can simply be found by using $K = \frac{1}{2}mv^2$ due to the translational motion. This means that $K_i = \frac{1}{2}mv_o^2$

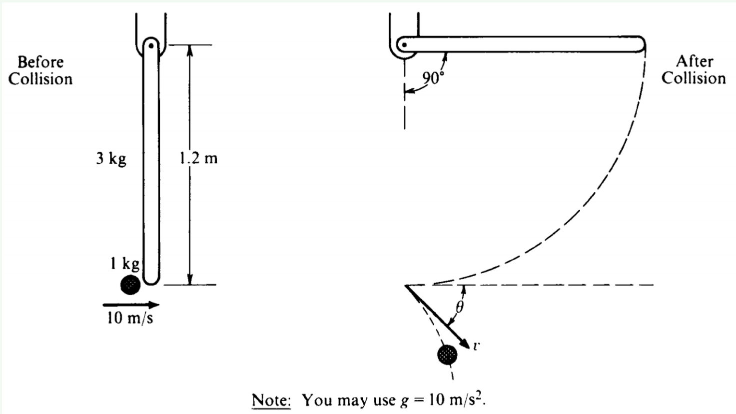
However, after collision, the wheel and bullet both spin around a common axle. The rotational kinetic energy can be found using $\frac{1}{2}Iw_f^2$

We already know that their combined rotational inertia is $(m+M)R^2$. On top of that, we know that $w_f = \frac{mv_o \sin(\theta)}{(m+M)R}$

We can plug these expressions into $K_f = \dfrac{1}{2}Iw^2$ to find that $K_f = \dfrac{m^2v_o^2\sin^2(\theta)}{m+M}$

When we take the ratio $\frac{K_f}{K_i}$, most variables cancel out.
We are left with $\boxed{\dfrac{m\sin^2(\theta)}{m+M}}$ which is the ratio.

Problem 7.0.21 — 1987 AP Physics FRQ



A 1.0-kilogram object is moving horizontally with a velocity of 10 meters per second, as shown above, when it makes a glancing collision with the lower end of a bar that was hanging vertically at rest before the collision. For the system consisting of the object and bar, linear momentum is not conserved in this collision, but kinetic energy is conserved. The bar, which has a length l of 1.2 meters and a mass m of 3.0 kilograms, is pivoted about the upper end. Immediately after the collision, the object moves with speed v at an angle θ relative to its original direction. The bar swings freely, and after the collision reaches a maximum angle of 90 with respect to the vertical. The moment of inertia of the bar about the pivot is $I_{\text{bar}} = \frac{ml^2}{3}$. Ignore all friction.

(a) Determine the angular velocity of the bar immediately after the collision.

(b) Determine the speed v of the 1-kilogram object immediately after the collision.

(c) Determine the magnitude of the angular momentum of the object about the pivot just before the collision.

Solution to part a: For the bar, its rotational kinetic energy will convert to potential energy.
Its rotational kinetic energy is $\frac{1}{2}Iw^2$

To find the change in potential energy, we will set the reference level as the center of the bar. The reason is that we only need to consider the center of mass to find the change of potential in this case. The center of mass simply moves a distance $\frac{l}{2}$ where l is the length of the bar.

This means that the change in potential energy is $mg \cdot \frac{l}{2}$ which is $\frac{mgl}{2}$
Since $-\Delta K = \Delta U$, we know that $\frac{1}{2}Iw^2 = \frac{mgl}{2}$
We can solve this for w to get $w = \sqrt{\frac{mgl}{I}}$
Now, we can plug in $I = \frac{ml^2}{3}$

Doing so gives that $w = \sqrt{\frac{3g}{l}}$
Since $l = 1.2$ m, we can plug that in to get

$$w = \sqrt{\frac{3 \cdot 9.8}{1.2}} = \boxed{4.95 \text{ rad/s}}$$

Solution to part b: We can find the initial speed of the object by conserving kinetic energy. The problem explicitly says that kinetic energy is conserved.
The initial kinetic energy can be found using the formula $K = \frac{1}{2}mv^2$. Using it gives that it is $\frac{1}{2} \cdot 1 \cdot 10^2 = 50$ J

Now, we will find the final kinetic energy. The 1 kg object moves at a speed of v after collision. This means that it's kinetic energy is simply $\frac{1}{2} \cdot 1 \cdot v^2 = \frac{v^2}{2}$

The kinetic energy of the bar can be found using $K = \frac{1}{2}Iw^2$
Since $I = \frac{ml^2}{3}$, we can plug in $m = 3$ and $l = 1.2$ to find that $I = \frac{3 \cdot 1.2^2}{3} = 1.44$
Now, we can plug $I = 1.44$ and $w = 4.95$ (from part a) into the formula $K = \frac{1}{2}Iw^2$. This gives us that the rotational kinetic energy of the bar is $\frac{1}{2} \cdot 1.44 \cdot 4.95$ which evaluates to 17.642 J.

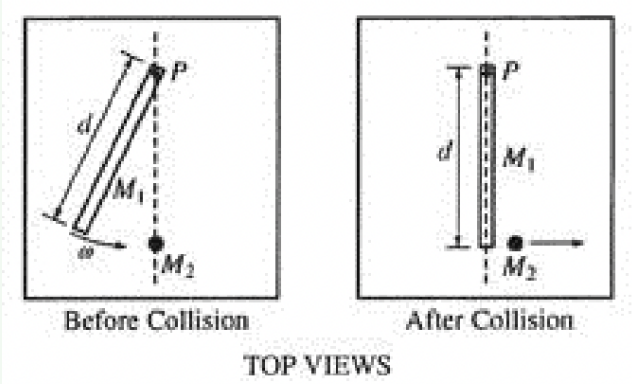
Now, since kinetic energy is conserved, we know that $K_i = K_f$
Our initial kinetic energy was found to be 50. The final kinetic energy is $\frac{v^2}{2} + 17.642$

We can set both expressions equal to each other to get $50 = \frac{v^2}{2} + 17.642$
We can solve the equation to find that $\boxed{v = 8.045 \text{ m/s}}$

Solution to part c: The initial momentum can be found using the equation $L = mvr$. Note that this equation is used often when we are finding the angular momentum for some point object. In this problem, it works because the initial angular momentum only involves the small 1-kg object.

r is the length of the bar which is 1.2 m. We also know that $m = 1$ kg while $v = 10$ m/s. We can plug these values in to find that $L_i = 1 \cdot 10 \cdot 1.2 = \boxed{12 \text{ kg} \cdot m^2/\text{s}}$

Problem 7.0.22 — 2005 AP Physics FRQ



A system consists of a ball of mass M_2 and a uniform rod of mass M_1 and length d. The rod is attached to a horizontal frictionless table by a pivot at point P and initially rotates at an angular speed ω, as shown above left. The rotational inertia of the rod about point P is $\frac{1}{3}M_1d^2$. The rod strikes the ball, which is initially at rest. As a result of this collision, the rod is stopped and the ball moves in the direction shown above right. Express all answers in terms of M_1, M_2, ω, d, and fundamental constants.

(a) Derive an expression for the angular momentum of the rod about point P before the collision.

(b) Derive an expression for the speed v of the ball after the collision.

(c) Assuming that this collision is elastic, calculate the numerical value of the ratio $\frac{M_1}{M_2}$.

Solution to part a: We know that angular momentum for the rod can be written as $L_i = Iw$
The rotational inertia about point P is $\frac{1}{3}M_1d^2$ (the formula for rotational inertia of a rod about its end)
Since the initial angular velocity is w, we multiply both expressions together to get $L_i = \frac{M_1d^2w}{3}$

Solution to part b: We know that angular momentum will be conserved. Since the rod stops rotating after colliding, it means that its final angular momentum is 0. However, the ball moves at a speed of v after.
This means that the final angular momentum is $L_f = M_2vd$ (from the formula $L = mvr$)

We must equate both our expressions: $L_i = L_f$
We plug in $L_i = \frac{M_1d^2w}{3}$ and $L_f = M_2vd$ to get

$$\frac{M_1d^2w}{3} = M_2vd$$

We can solve this for v to find that $\boxed{v = \frac{M_1dw}{3M_2}}$

Solution to part c: If the collision is elastic, then kinetic energy is conserved.

The initial kinetic energy $K_i = \frac{1}{2}Iw^2$ (only the rod moves)
We can plug in $I = \frac{1}{3}M_1d^2$ and w to find that $K_i = \frac{1}{6}M_1d^2w^2$

The final kinetic energy only comes from the ball. We will use the formula for kinetic energy $K = \frac{1}{2}mv^2$ to find that $K_f = \frac{1}{2}M_2v^2$
We already know that $v = \frac{M_1dw}{3M_2}$ from part b. We can plug that in to find that

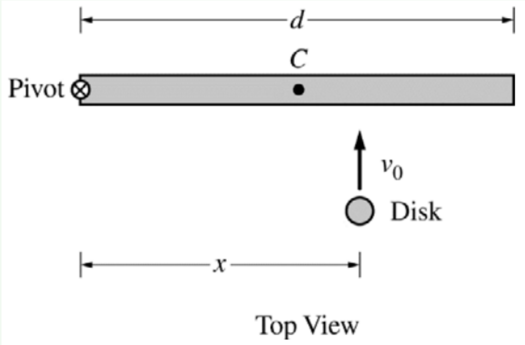
$$K_f = \frac{1}{2} \cdot M_2 \cdot (\frac{M_1dw}{3M_2})^2 = \frac{M_1^2d^2w^2}{18M_2}$$

Now, we can equate K_i and K_f.

$$\frac{1}{6}M_1d^2w^2 = \frac{M_1^2d^2w^2}{18M_2}$$

We can cancel out many variables to find that $\frac{M_1}{M_2} = 3$

Problem 7.0.23 — 2017 AP Physics 1 FRQ



The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0 perpendicular to the rod, and the disk will stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(a) Suppose the rod is much more massive than the disk. To give the rod as much angular speed as possible, should the student make the disk hit the rod to the left of point C, at point C, or to the right of point C?
_____ To the left of C _____ At C _____ To the right of C
Briefly explain your reasoning without manipulating equations.

(b) On the Internet, a student finds the following equation for the post-collision angular speed ω of the rod in this situation: $\omega = \frac{m_{\text{disk}} x v_0}{I}$. Regardless of whether this equation for angular speed is correct, does it agree with your qualitative reasoning in part (a)? In other words, does this equation for ω have the expected dependence as reasoned in part (a)?
_____ Yes _____ No
Briefly explain your reasoning without deriving an equation for ω.

(c) Another student deriving an equation for the post-collision angular speed ω of the rod makes a mistake and comes up with $\omega = \frac{I x v_0}{m_{\text{disk}} d^4}$. Without deriving the correct equation, how can you tell that this equation is not plausible—in other words, that it does not make physical sense? Briefly explain your reasoning.

Solution to part a: We know that angular momentum will be conserved. We want to try to maximize the initial angular momentum of the disk. Since we know the angular momentum of the disk is mvx, we want it to be as far as possible from the pivot.
The distance x from the pivot point will be maximized when when the disk hits the rod to $\boxed{\text{the right of C}}$.
You could also say that to the right of point C, torque applied on the rod will be maximized due to the large distance x from the pivot.

Solution to part b: Yes. This equation matches with our reasoning in part a. The reason is that as x goes up (meaning distance from pivot increases), then the angular velocity goes up. Clearly this matches with our reasoning in part a.

Solution to part c: We know that the larger the mass of the disk is, then the higher the angular momentum will be. The reason is that the formula for angular momentum of a point object is $L = mvr$.

Since the angular momentum goes up as m_{disk} goes up, the angular velocity must also go up.
However, in the given equation for angular velocity, that isn't the case. In fact, m_{disk} is in the denominator which means as the mass increases, the angular velocity goes down. This contradicts the statement we previously made, proving that the equation is not plausible.

Problem 7.0.24 — Continuation of previous 2017 AP Physics FRQ
For parts (d) and (e), do NOT assume that the rod is much more massive than the disk.
(d) Immediately before colliding with the rod, the disk's rotational inertia about the pivot is $m_{\text{disk}}x^2$ and its angular momentum with respect to the pivot is $m_{\text{disk}}v_0x$. Derive an equation for the post-collision angular speed ω of the rod. Express your answer in terms of $d, m_{\text{disk}}, I, x, v_0$, and physical constants, as appropriate.

(e) Consider the collision for which your equation in part (d) was derived, except now suppose the disk bounces backward off the rod instead of sticking to the rod. Is the post-collision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the post-collision angular speed of the rod when the disk sticks to it?
______ Greater than ______ Less than ______ Equal to
Briefly explain your reasoning.

Solution to part d: In this problem, we will apply conservation of angular momentum which states that $L_i = L_f$

We are already given that $L_i = m_{\text{disk}}v_0x$
Now, we simply find L_f and equate it to our expression above.
After collision, both the disk and rod will rotate with a common angular velocity.

This means that $L_f = (I_{\text{rod}} + I_{\text{disk}})\omega$
In this problem, it is already given that $I_{\text{rod}} = I$ and $I_{\text{disk}} = m_{\text{disk}}x^2$
We can plug that into our expression for L_f to find that $L_f = (I + m_{\text{disk}}x^2)\omega$

Now, we can equate L_i and L_f to set up the expression $m_{\text{disk}}v_0x = (I + m_{\text{disk}}x^2)\omega$

We can divide both sides by $I + m_{\text{disk}}x^2$ to isolate ω.
Doing so gives that $\boxed{\omega = \dfrac{m_{\text{disk}}v_0x}{I + m_{\text{disk}}x^2}}$

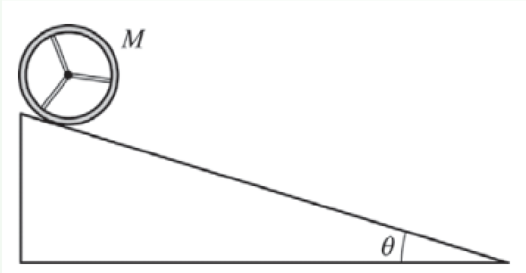
Solution to part e: We will apply the concept of angular momentum being conserved in this problem.
When the disk sticks to the rod, the disk will have an angular momentum in the same direction as the rod. This means that the rod won't receive a high amount of that

momentum since it has to share it with the disk.

However, when the disk bounces back, it will have an angular momentum in the opposite direction after collision. To counteract this angular momentum, the rod will need to have a significantly greater angular momentum.

Thus, when the disk bounces back, the post-collision angular speed will be $\boxed{\text{greater}}$.

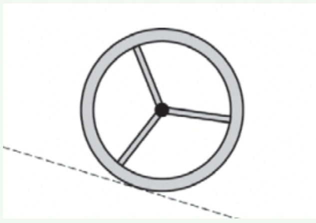
Problem 7.0.25 — 2016 AP Physics 1



A wooden wheel of mass M, consisting of a rim with spokes, rolls down a ramp that makes an angle θ with the horizontal, as shown above. The ramp exerts a force of static friction on the wheel so that the wheel rolls without slipping.

(a)

i. On the diagram below, draw and label the forces (not components) that act on the wheel as it rolls down the ramp, which is indicated by the dashed line. To clearly indicate at which point on the wheel each force is exerted, draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted. The lengths of the arrows need not indicate the relative magnitudes of the forces.



ii. As the wheel rolls down the ramp, which force causes a change in the angular velocity of the wheel with respect to its center of mass.
Briefly explain your reasoning.

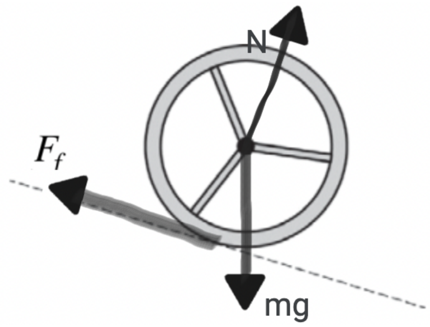
(b) For this ramp angle, the force of friction exerted on the wheel is less than the maximum possible static friction force. Instead, the magnitude of the force of static friction exerted on the wheel is 40 percent of the magnitude of the force or force component directed opposite to the force of friction. Derive an expression for the linear acceleration of the wheel's center of mass of M, theta, and physical constants, as appropriate.

(c) In a second experiment on the same ramp, a block of ice, also with mass M, is released from rest at the same instant the wheel is released from rest, and from the same height. The block slides down the ramp with negligible friction.

i. Which object, if either, reaches the bottom of the ramp with the greatest speed?
_____ Wheel _____ Block _____ Neither; both reach the bottom with the same speed.
Briefly explain your reasoning in terms of forces.

ii. Briefly explain your answer again, now reasoning in terms of energy.

Solution to part a i: The only forces on the wheel are normal force, gravitational force, and frictional force. Note, that the vector lengths of the force arrows don't matter.



Solution to part a ii: Only the friction force can change angular velocity. The reason is that you need torque to change angular velocity. However, the normal force and gravitational force apply no torque. The reason is that the length of the "lever arm" for the two forces is 0 (since the forces pass through the center).

Solution to part b: Along the ramp, there are only two forces. One of them is friction while the other is one component of gravity.
The force of gravity along the ramp is $Mg\sin(\theta)$

We know that the equation for Newton's Second Law along the ramp is

$$F_{net} = Mg\sin(\theta) - F_f = Ma$$

Since the problem says that the force of friction is 40 percent of the force directed opposite to it, we can figure out that $F_f = 0.4Mg\sin(\theta)$
We can plug this into our equation for Newton's Second Law to get

$$Mg\sin(\theta) - 0.4Mg\sin(\theta) = Ma$$

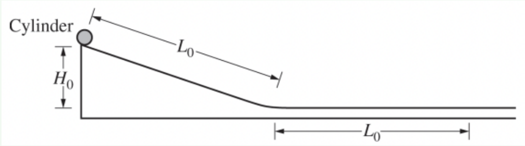
After dividing both sides by M, we find that $\boxed{a = 0.6g\sin(\theta)}$

Solution to c i: The $\boxed{\text{block}}$ reaches the bottom with the greatest speed. The reason is that the wheel has a friction force on it. However, the block doesn't. This causes the net force along the ramp to be larger for the block leading to greater acceleration.

Solution to c ii: We know that both objects have the same potential energy at the top of the ramp (assuming the reference level is the bottom of the ramp)
Once they reach the bottom of the ramp, all the potential energy will turn into kinetic energy.

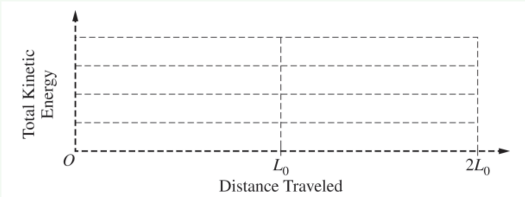
For the wheel, the kinetic energy will be in the forms of translational kinetic energy and rotational kinetic energy. However, for the ice block, there will only be translational kinetic energy.
This means that the ice block will have a greater speed due to all of its kinetic energy being in the form of translational kinetic energy.

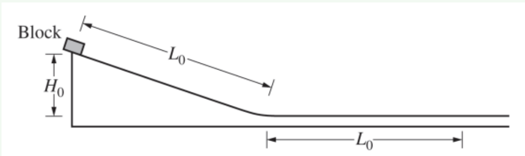
Problem 7.0.26 — 2021 AP Physics 1



A cylinder of mass m_0 is placed at the top of an incline of length L_0 and height H_0, as shown above, and released from rest. The cylinder rolls without slipping down the incline and then continues rolling along a horizontal surface.

(a) On the grid below, sketch a graph that represents the total kinetic energy of the cylinder as a function of the distance traveled by the cylinder as it rolls down the incline and continues to roll across the horizontal surface.



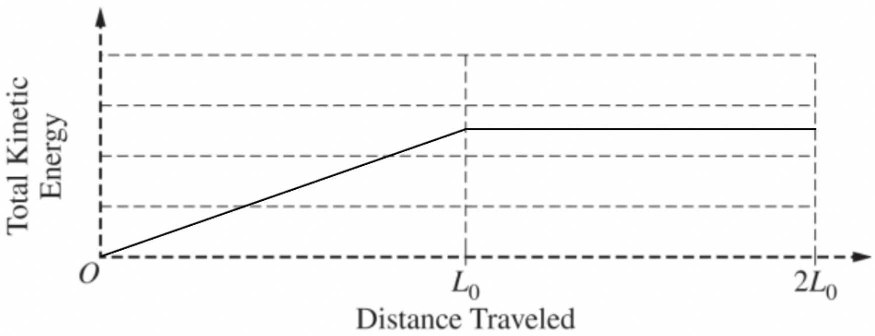


The cylinder is again placed at the top of the incline. A block, also of mass m_0, is placed at the top of a separate rough incline of length L_0 and height H_0, as shown above. When the cylinder and block are released at the same instant, the cylinder begins to roll without slipping while the block begins to accelerate uniformly. The cylinder and the block reach the bottoms of their respective inclines with the same translational speed.

(b) In terms of energy, explain why the two objects reach the bottom of their respective inclines with the same translational speed. Provide your answer in a clear, coherent paragraph-length response that may also contain figures and/or equations.

Solution to part a: For each unit of distance travelled while going down the ramp, the change in potential energy will be the same. This is due to the ramp having a constant slope.
This means that the potential energy decreases linearly. Since the sum of potential energy and kinetic energy are constant, kinetic energy must increase linearly.

When the distance travelled is between 0 and L_0 (meaning the cylinder is on the ramp), the kinetic energy will increase linearly. However, once it reaches the bottom of the ramp, the kinetic energy will remain constant since the cylinder continues to roll in the horizontal section.



Solution to part b: At the top, both the cylinder and block will have the same gravitational potential energy.
Whenever something rolls without slipping, friction doesn't dissipate its energy. The reason is that friction does no work on that object.
Thus, the cylinder's potential energy entirely converts to kinetic energy when it reaches the bottom of the ramp. Some of that kinetic energy will be in the form of translational kinetic energy while the rest will be in the form of rotational kinetic energy.
We can assume that the potential energy at the top of the ramp is U while the translational kinetic energy and rotational kinetic energy at the bottom of the ramp are K_{trans} and K_{rot} respectively.
From conservation of energy, we know that $U = K_{\text{trans}} + K_{\text{rot}}$

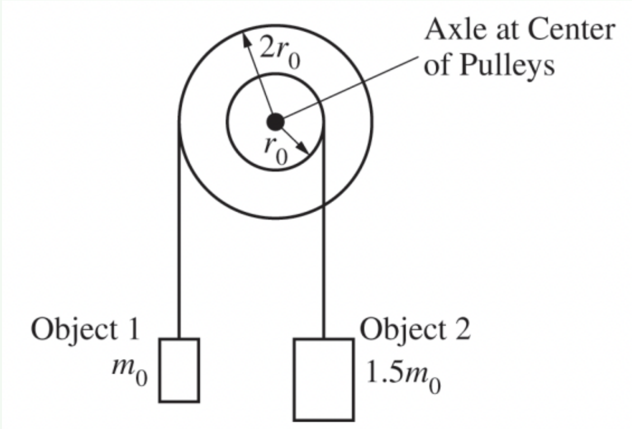
On the other hand, for the block, some energy will indeed be lost as friction. The remaining energy that is left at the bottom of the ramp will be in the form of translational kinetic energy.

Assuming that the energy lost by friction is represented as E_f, then by conservation of mechanical energy we know that $U = E_f + K_{\text{trans}}$

We can write both expressions for K_{trans}. Doing this for the cylinder gives $U - K_{\text{rot}}$ while for the block we get $U - E_f$

We know that both of these expressions equate, and that is only possible when $K_{\text{rot}} = E_f$.
This means that the rotational kinetic energy must equate to the energy lost by friction for the block.
On the real AP exam, you don't need to show all the steps above to show that the rotational kinetic energy of the cylinder must equate to the energy lost by friction for the block. I only showed all that work to highlight the intuition behind that idea.

Problem 7.0.27 — 2021 AP Physics 1 FRQ



Two pulleys with different radii are attached to each other so that they rotate together about a horizontal axle through their common center. There is negligible friction in the axle. Object 1 hangs from a light string wrapped around the larger pulley, while object 2 hangs from another light string wrapped around the smaller pulley, as shown in the figure above.

m_0 is the mass of object 1.
$1.5m_0$ is the mass of object 2.
r_0 is the radius of the smaller pulley.
$2r_0$ is the radius of the larger pulley.
(a) At time $t = 0$, the pulleys are released from rest and the objects begin to accelerate.

i. Derive an expression for the magnitude of the net torque exerted on the objects-pulleys system about the axle after the pulleys are released. Express your answer in terms of m_0, r_0, and physical constants, as appropriate.

ii. Object 1 accelerates downward after the pulleys are released. Briefly explain why.

(b) At a later time $t = t_C$, the string of object 1 is cut while the objects are still moving and the pulley is still rotating. Immediately after the string is cut, how do the directions of the angular velocity and angular acceleration of the pulley compare to each other?
☐ Same direction ☐ Opposite directions
Briefly explain your reasoning.

Solution to part a i: We know that $\tau_{net} = \sum F_i \cdot r_i$
This means that we must sum up the product of the forces times distance from pivot.

The forces on the pulley are the two tension forces. We can say that T_1 is the tension force on the left side while T_2 is the tension force on the right side.

$$\tau_{net} = T_1 \cdot 2r_0 - T_2 \cdot r_0$$

Now, we can write equations using Newton's Second Law for each block.

On object 1, the equation is $T_1 - m_0 g = 0$. This means that $T_1 = m_0 g$

Similarly, on object 2, the equation is $T_2 - 1.5m_0 g = 0$. This means that $T_2 = 1.5m_0 g$

We can plug in these values of tension into our equation for τ_{net}.

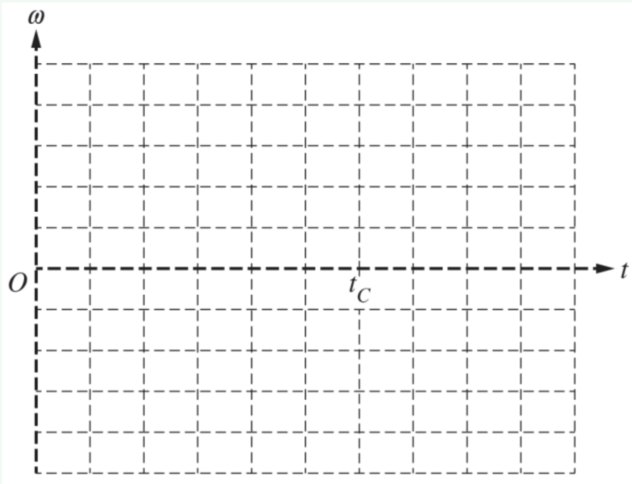
$$\tau_{net} = m_0 g \cdot 2r_0 - 1.5m_0 g \cdot r_0 = \boxed{0.5m_0 r_0 g}$$

Solution to part a ii: Object 1 accelerates downward because it exerts a larger torque on the pulley. It is also double the distance away from the axle compared to object 2 while object 2 only has a mass that is 1.5 times as great as object 1.

Solution to part b: They will move in $\boxed{\text{opposite directions}}$. The reason is that now the torque will switch directions and become clockwise causing angular acceleration to be clockwise. However, the angular velocity will still be counterclockwise temporarily. It won't change directions immediately since it was already spinning in a counterclockwise direction.

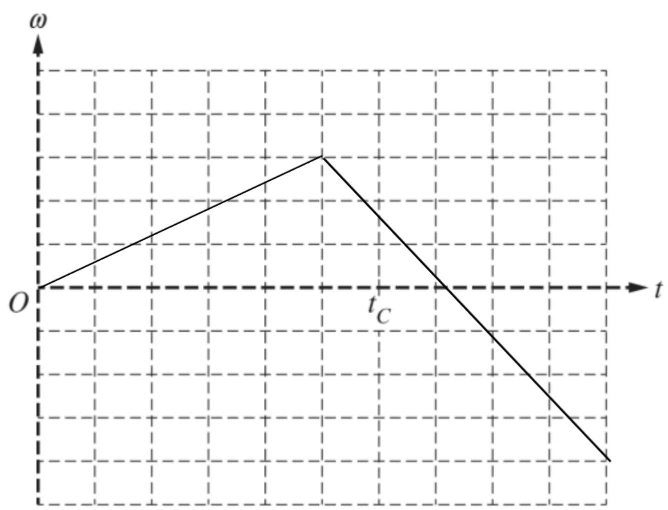
Problem 7.0.28 — Continuation of the FRQ above

(c) On the axes below, sketch a graph of the angular velocity ω of the system consisting of the two pulleys as a function of time t. Include the entire time interval shown. The pulleys are released at $t = 0$, and the string is cut at $t = t_C$.

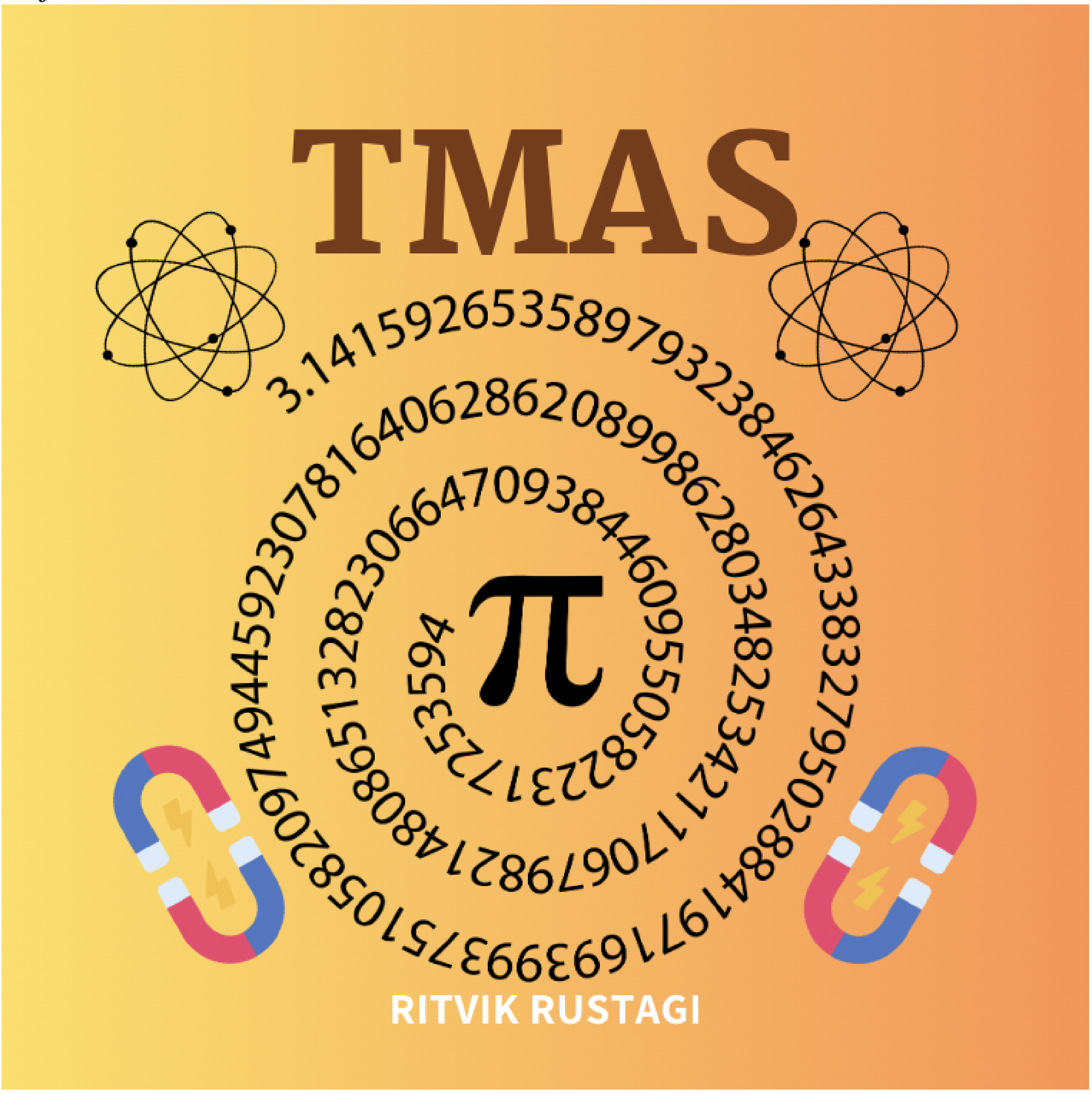


Before the string is cut, we know that the system will accelerate in the counterclockwise direction. This will cause the angular velocity to increase linearly until time t_c.

However, after t_c, we no longer have block of mass m_1 in this system. The angular acceleration is now in the clockwise direction which will cause the angular velocity to fall linearly.


ω
O
t
t_C

Thank you for going through this book!
It is an honor for me to have contributed to your academical journey in some way!



Thanks,

Ritvik Rustagi