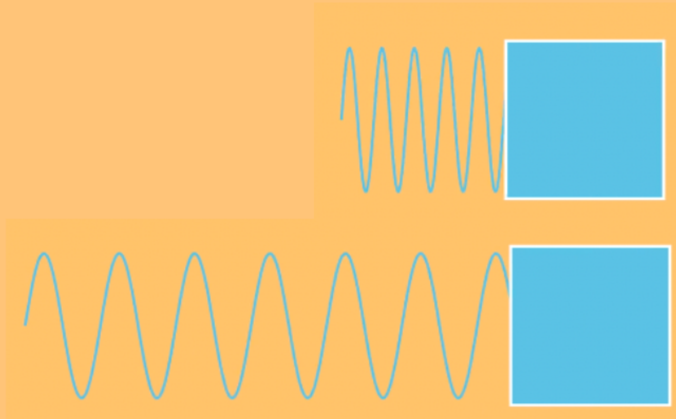
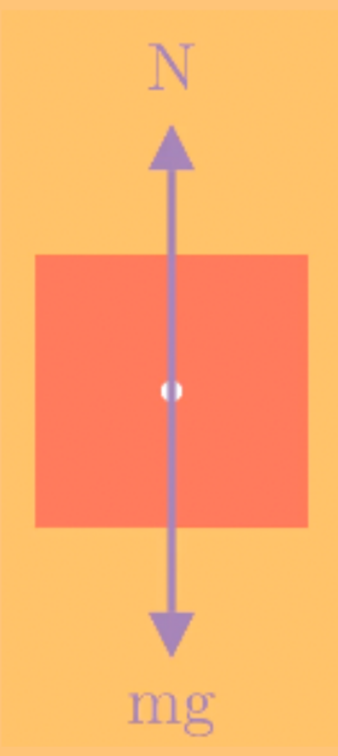
TMAS Academy



ACE

AP Physics C: Mechanics

2024



- 100+ Problems
- All Topics
- Detailed Solutions

Ritvik Rustagi

Contents

Important Information

If you found this book from any platform other than the website, then there's a high chance that you don't have the latest version with all the updates. Make sure to use the version from the official TMAS Academy website. If you are using the book from another platform, then you're at a risk of missing out on important content that is available on the new version which can always be found on the TMAS Academy website. In addition, important links in this book might not work on many platforms. However, they will work if you use the book from the official website.

About TMAS Academy

This book is brought to you by me (Ritvik Rustagi). TMAS Academy, previously known as Explore Math, was started by me in 2020. TMAS stands for The Math and Science. Currently, I have written five free books for students around the world. Those books include the *ACE The AMC 10/12*, *ACE AP Physics 1*, *ACE AP Calculus AB*, *ACE AP Physics C: Mechanics*, and *ACE AP Calculus BC*. All of the books have been designed to make preparing for these exams efficient and accessible for everyone.
You can find more info about this program on my website linked below.
Website: https://www.tmasacademy.com/

Opportunities For You To Contribute To TMAS Academy

Contributing to TMAS Academy is simple.
You can **join the team** by checking out the form below which can also be found on the website:
https://forms.gle/VXGvj27UvcZPGhiJ8

Donations: If you want to assist me in my monthly payments to run this program which includes website costs, Overleaf costs (the platform used to write such books), and filming/editing costs, then please consider donating! For those that are willing to contribute, I have listed a few ways below. **Don't forget to write a message so I know who you are which will allow me to send you a thank you note**.

- You can donate through PayPal to the email: ritvikrustagi7@gmail.com
- If you want to donate and the above method doesn't work for you, then you can send an email to ritvikrustagi7@gmail.com

You can also contribute by **subscribing** to the Youtube channel: https://www.youtube.com/@tmasacademy
Also, don't forget to join the Discord server to connect with other students and the owner: https://discord.gg/tmas-academy-1019082642794229870!

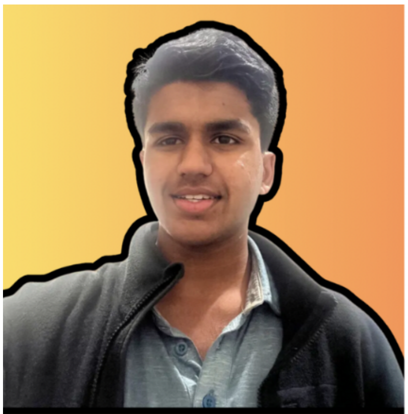
You can also follow all of our socials such as the Linkedin page and the Instagram account that is run by the media team. Also, please join the mailing list to learn about all updates and our upcoming books and videos. All of that can be found at the bottom of the site: https://www.tmasacademy.com/

About The Author: Ritvik Rustagi

My name is Ritvik Rustagi, and I am a student at Prospect High School. Some information about me is that I enjoy doing math, physics, and programming.

Some of my qualifications include qualifying for USAJMO and USAMO (United States of America Mathematical Olympiad), qualifying for USAPHO (United States of America Physics Olympiad), achieving a gold medal in the finals round for MathCon in Chicago, and qualifying for the AIME several times.
During Covid, I discovered my passion for teaching math competition topics through my Youtube channel. It also allowed me to absorb these complicated topics more efficiently since teaching can help one improve their own skills. After that, I began my journey of writing various books for math competitions and AP courses. By October of 2023, I released my first major book for the AMC 10/12. In March 2024, I released several AP books which can all be found on the website.

This book has been written to help any student aiming to do well on the AP exam and the class itself. Due to the difficulty of this exam, a good guide is necessary with a rich problem set for students to practice with. This is what the book aims to do. Many students these days struggle to prepare for AP exams due to the vast amount of content. However, productive preparation can solve that problem. That is how I got 6 5s on the following AP exams in my sophomore year of high school: AP Physics 1, AP Calculus BC, AP Physics C: EM, AP Physics C: Mech, AP World History, and AP Statistics. Anyone can do it if they believe in themselves and choose the right resources to prepare with. Tons of problems are contained within this book with well written solutions. This will allow even the most inexperienced students to have a productive session of preparation while comprehending the problems and theory.



Benefits of Taking AP Exams

Preparing for AP exams such as the AP Calculus AB/BC, AP Physics 1, and AP Physics C is a great way to expand your knowledge. These exams go a step further to deepen your knowledge of subjects that you might have previously encountered. On top of that, you will learn many concepts that will be used throughout your life. It's a great learning experience and can give you the opportunity to enrich your journey. It also improves your problem solving skills which can serve as a life skill in many situations.

What if there is an error in the book?

There are possibilities for minor errors such as typos or a mistake in latex for some of the solutions to the problems. If that's the case, then please click on this link (https://forms.gle/3mxZb4izUuBZLkmz5) to report the mistake.

If you have any other questions or concerns, then please feel free to reach out to **ritvikrustagi7@gmail.com**

Credits

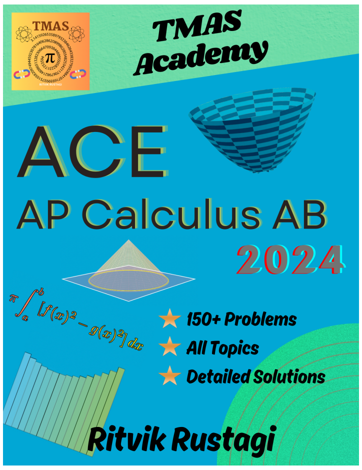
I would like to thank **The College Board** for their high quality problems that were used to teach concepts for this course.

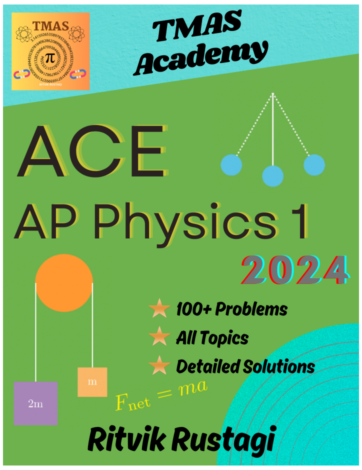
I would also like to thank **Evan Chen**, a PhD student at MIT (Massachusetts Institute of Technology), for his latex template which made it easy to format this book. I would also like to thank him for answering all of my questions regarding the format of this book.

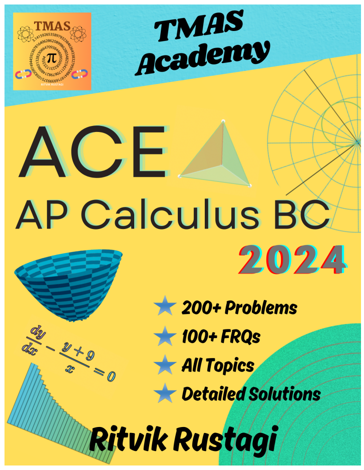
I would also like to thank my parents for supporting me with my goals and for everything else that they have done.

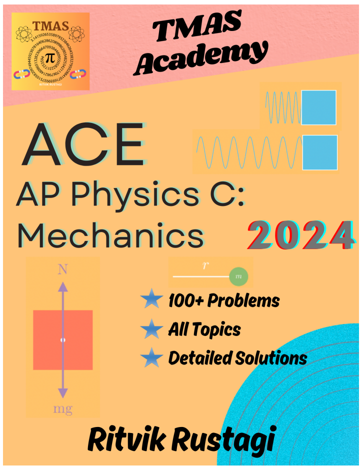
I would like to thank everyone else that has supported the content that I have made and encouraged me to continue to do so.

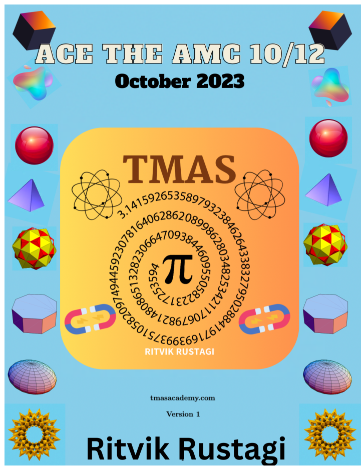
Other Important Resources











All five of these books were written by me. They can all be found on the TMAS Academy website. All five of the books are comprehensive and contain all the topics that you need to know. They have been designed to make your preparation productive through the vast number of official free response questions.

Make sure to check out the following playlists on the TMAS Academy youtube channel! These are important to learn all the topics that show up on the following AP exams: AP Physics 1, AP Calculus AB/BC, and AP Physics C: Mechanics.

AP Calculus AB/BC Playlist

AP Physics 1 Playlist

AP Physics C: Mechanics Playlist

Connect with the Author

Feel free to connect with me on Linkedin, Instagram, Discord, or through email!

I highly recommend joining the Discord server to access study and review sessions hosted in the server. You should also consider following the Instagram to access animations that will be posted there to allow you to learn.

Linkedin: https://www.linkedin.com/in/ritvik-638590210/

Instagram: https://www.instagram.com/ritvik_rustagi_tmas/

Discord: https://discord.gg/tmas-academy-1019082642794229870

Email: ritvikrustagi7@gmail.com

Unit 1 Kinematics

Kinematics is the study of motion of objects. We will deal with many variables in this unit such as velocity, time, displacement, distance, etc. After working with 1D kinematics, we will deal with 2D kinematics.

Note 1.0.1 — Scalars vs Vectors
A scalar quantity is a quantity with no direction – it only has a magnitude. On the other hand, a vector quantity has a magnitude *and* a direction.

Scalar quantities include **distance** and **speed**.

Vector quantities include **displacement** and **velocity**.

Note 1.0.2 — Scalar Quantities in Depth
Distance is defined as the amount of space traveled when moving from one point to another point. This is typically denoted by d.

Speed is defined as the rate at which the distance is traveled, and is typically denoted by s. That is, $s = \frac{d}{t}$.

Note 1.0.3 — Vector Quantities in Depth
Displacement is defined as the change in position of an object in a given time. This is denoted by $\vec{d}$ or Δx.

Velocity is defined as the rate at which the object displaces, and is typically denoted by $\vec{v}$. That is, $\vec{v} = \frac{\vec{d}}{t}$ or $\frac{\Delta x}{t}$.

Acceleration is defined as the rate at which the velocity of the object changes, and is denoted by a. That is, $a = \frac{\Delta \vec{v}}{t}$.

In general, if you want to find the speed at a given time period and you're given velocity, then you can simply take the absolute value of velocity to find the speed. The reason is that speed has no direction, only a magnitude!

Problem 1.0.4 — Matt bikes from position $x = 50$ to position $x = 100$, and then to position $x = 55$. This occurs in 25 seconds. Assume that Mike travels at constant velocity.

a) What is the distance Matt travels?

b) What is Matt's speed?

c) What is Matt's velocity?

d) What is Matt's acceleration?

Solution to part a: He travels a distance of $(100 - 50) + (100 - 55) = \boxed{95}$ meters.

Solution to part b: From part (a), Matt travels 95 meters in 25 seconds, so his speed is $\frac{95}{25} = \boxed{\frac{19}{5}}\frac{\text{m}}{\text{s}}$

Solution to part c: Matt's displacement is 5, which occurs in 25 seconds. The reason is that he starts at a position of $x = 50$ and ends up at a position of $x = 55$. Thus, $\Delta x = 5$.

We can find that his velocity is $\frac{5}{25} = \boxed{\frac{1}{5}}\frac{\text{m}}{\text{s}}$

Solution to part d: Matt moves with constant velocity, so his acceleration is $\boxed{0}\frac{\text{m}}{\text{s}^2}$

Note 1.0.5 — **Average vs. Instantaneous values**
Average velocity is simply the displacement divided by the total time. However, instantaneous velocity is the velocity of an object at a **specific** instant of time.

Similarly, this difference between average acceleration and instantaneous acceleration holds.

Note 1.0.6 — Kinematics Equations
There are 4 kinematics equations that are the key to solve all 1D kinematics problems on the exam. Note that these equations only hold true in situations with constant acceleration. Let v_i, v_f be the initial and final velocities, t be the time taken in the journey, Δx be the displacement during the journey, and a be the acceleration. The kinematics equations are all shown below.

$$v_f = v_i + at \tag{1.1}$$

$$\Delta x = \frac{v_i + v_f}{2} \cdot t \tag{1.2}$$

$$\Delta x = v_i t + \frac{1}{2} a t^2 \tag{1.3}$$

$$v_f^2 = v_i^2 + 2a\Delta x \tag{1.4}$$

Out of the 5 variables $t, v_i, v_f, a, \Delta x$, you must know 3 values to find the rest using the equations above.
Also, note that other forms of the variables can be used. For example, sometimes people label displacement as d. You can often see v_0 or v_o for the initial velocity.

Note 1.0.7 — Note that the acceleration due to gravity is g. The exact value of g is around $9.81\frac{m}{s^2}$. In AP Physics C: Mechanics, the answer choices might be in terms of g. Sometimes, you will have to use 9.81 instead. On top of that, sometimes the problem will allow you to round g and use $10\frac{m}{s^2}$

You also need to know how to read kinematics graphs.
For kinematics graphs, this is all you need to know:

When you have a **velocity-time graph**, the slope at any point is the acceleration at that specific time, and the area under the graph up to a specific time t is the displacement occurred up to that time. In simple words, the area under a velocity-time graph is the displacement.

When you have a position-time graph, the slope at any point is the velocity at that specific time.

In general, remember that d and Δx are both used to represent displacement.

Problem 1.0.8 — How long would it take a car, starting from rest and accelerating uniformly in a straight line at $5\frac{\text{m}}{\text{s}^2}$, to cover a distance of 200m?

Solution: From equation (1.3), we have

$$d = v_i t + \frac{1}{2} a t^2$$

$$200 = 0 + \frac{1}{2}(5)t^2$$

$$\implies t = \sqrt{80} \approx \boxed{9} \text{ seconds.}$$

Problem 1.0.9 — A ball is dropped off a cliff and strikes the ground with an impact velocity of $30\frac{\text{m}}{\text{s}}$. How high was the cliff?
(Use $g = 10\frac{\text{m}}{\text{s}^2}$)

Solution: From equation (1.4), we have

$$v_f^2 = v_i^2 + 2ad$$

$$30^2 = 0^2 + 2gd$$

$$\implies d = \frac{30^2}{2g} = \boxed{45} \text{ meters.}$$

Problem 1.0.10 — A stone is thrown vertically upward with an initial speed of $5\frac{\text{m}}{\text{s}}$. What is the velocity of the stone 3 seconds later?

Solution: From equation (1.1), we have

$$v_f = v_i + at$$

$$v_f = 5 + (-g)(3)$$

$$\implies v_f = 5 - 3g = \boxed{-25\frac{\text{m}}{\text{s}}}.$$

Problem 1.0.11 — A car traveling at a speed of v_i applies its brakes, skidding to a stop over a distance of x meters. Assuming that the deceleration due to the brakes is constant, what would be the skidding distance of the same car if it were traveling with twice the initial speed?

Solution: From equation (1.4),

$$v_f^2 = v_i^2 + 2ad.$$

Since v_f in this scenario is 0, we have

$$d = \frac{v_i^2}{2a}.$$

Note that doubling v_i leads to increasing d by a factor of 4. So the answer is

$$\boxed{4x} \text{ meters.}$$

Problem 1.0.12 — A rocket initially moves at constant velocity v at $t = 0$. At time t_1, it starts to accelerate upwards with acceleration a. If the rocket moves until time t_2, how much distance does it travel?

Solution: Until time t_1, the rocket moves with constant velocity, so the distance traveled during that time interval is vt_1. The rocket also accelerates for time $t_2 - t_1$, so by equation (1.3), during that time interval, it travels a distance

$$d = v_i t + \frac{1}{2}at^2 = v(t_2 - t_1) + \frac{1}{2}a(t_2 - t_1)^2.$$

The total distance traveled is

$$vt_1 + v(t_2 - t_1) + \frac{1}{2}a(t_2 - t_1)^2$$

$$= vt_1 + vt_2 - vt_1 + \frac{1}{2}a(t_2 - t_1)^2$$

$$= \boxed{vt_2^2 + \frac{1}{2}a(t_2 - t_1)^2} \text{ meters.}$$

We now know kinematics algebraically. However, we must know how to apply some basic integration and differentiation to this. The process is simple if you have some knowledge of calculus.

Note 1.0.13 — The velocity v is denoted as $\frac{dx}{dt}$. $\frac{dx}{dt}$ represents the rate of change of position with respect to time (same thing as velocity).

The acceleration a is denoted as $\frac{dv}{dt}$. $\frac{dv}{dt}$ represents the rate of change of velocity with respect to time (same thing as acceleration).

Now just remember to consider the direction in a problem. For example, if the motion happens in the x direction, you should use $v = \frac{dx}{dt}$. However, if it occurs in the y-direction, you should use $v = \frac{dy}{dt}$

Many people don't understand the importance of this. When you learn projectile motion, you will be working with motion in both x and y-direction at once. Thus, it is important to keep the axes consistent throughout and make sure the variables you denote are for the right direction.

Now, there are a few formulas that will require some integration,

Note 1.0.14 — The velocity v can be denoted as $\int adt$. $\int adt$ is the integral of acceleration with respect to time (same thing as velocity).

The displacement x can be denoted as $\int vdt$. $\int vdt$ represents the integral of velocity with respect to time (same thing as displacement).

Some multiple choice problems will be a simple application of the formulas above. For example, you might be given an expression for acceleration a written in terms of t. An example of this is $a = t^2 + 3$. Then, the problem might ask you to find the velocity in terms of t.

To do the mini example above, we need to integrate acceleration with respect to time.

We use the formula $v = \int a dt$.
We find that $v = \int [t^2 + 3] dt = \frac{t^3}{3} + 3t + C$

Also, please use the right subscripts in problems! For example, v_f typically denotes final velocity while v_i or v_0 typically denotes initial velocity. This is important since problems can involve multiple velocities. You need to know what numerical value represents what specifically. This will reduce the chances of making an error!

Now it's time for some **2D Kinematics**

Before I introduce some techniques, just remember that projectile motion/2D Kinematics problems is all about solving 2 1D kinematics problems at once. The reason is that you have motion occurring in two directions. However, even though there is motion occurring in both directions, you can work separately with quantities in the x and y direction!

Note 1.0.15 — Projectile Motion Basics
Projectile motion happens in a parabolic manner. The path of the object looks like a parabola.

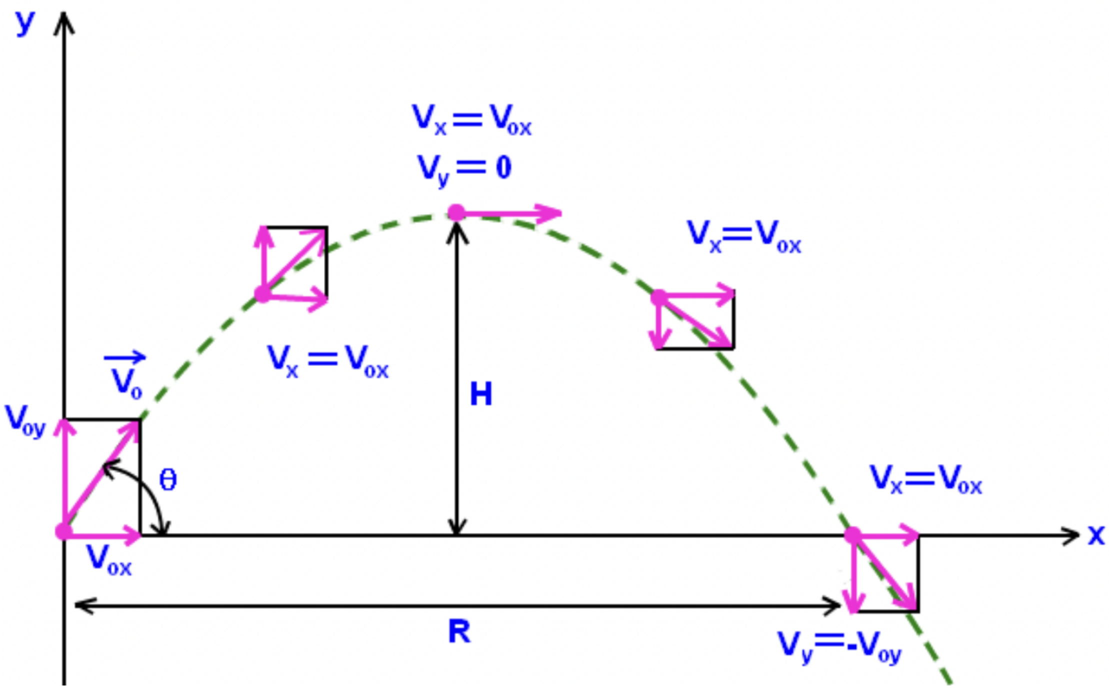
The best way to deal with such problems is to work separately with quantities in the horizontal direction and quantities in the vertical direction. Also, remember that gravity always points downward in the y direction. The acceleration in the y direction is equal to $g(9.8)$.

In projectile motion problems, one variable that is always the same in the x and y direction in AP Physics is t (time). Thus, solving for time is extremely important since it gives you information for a specific variable in both the x and y-direction.

Note 1.0.16 —
At the top of the projectile motion (at the very tip), the vertical velocity is 0. The horizontal velocity, however, remains the same always.

The horizontal velocity can change during the motion if we're dealing with rotated reference frames, but note that this is way beyond the scope of AP Physics, so you will never have to work with such a case.

Full credit to the physics and universe website for the image below.



Note 1.0.17 —
The image with the arc represents how a ball thrown in projectile motion would move. The arrow at the very top that is pointing to the right shows the velocity of the ball. There will only be one component in the horizontal direction, but no vertical velocity at all.

Also, note that in projectile motion, the object will be thrown at some angle θ with respect to the horizontal direction.

Using this information, we can write out our variables and see which ones we know (seen in the table below). Now we can calculate the time to reach the top point of the projectile motion and even find the displacement of it (seen in the equation after the table).

$$\begin{array}{cc} v_{ox} = v_o \cos\theta & v_{oy} = v_o \sin\theta \\ v_x = v_o \cos\theta & v_y = 0 \\ a_x = 0 & a_y = g \\ t & t \\ \Delta x & \Delta y \end{array}$$

Note that v_{ox} and v_{oy} represent the initial velocity in the x and y-direction while v_x and v_y represent the velocity at the very top of the arc.

Now, we can try to find time t when we're only given the initial velocity v_o and the angle θ.
We will apply kinematics equations for the motion in the **vertical** direction to find time to reach the top of the arc.
Since $v_{oy} = v_o \sin\theta$, $v_y = 0$, and $a_y = g$, we can apply the kinematics equation shown below.

$$v_f = v_i + at$$

However, we will rewrite this equation in terms of the variables we have. For example, our final velocity at the top in the y-direction is v_y, so we should use that instead of v_f. This may not seem important, but trust me on the importance of labelling with the right variables that are defined for that problem. It will also reduce the chances of getting an error.

$$v_y = v_{oy} + a_y t$$

Now, we can plug in our values to get

$$0 = v_o \sin(\theta) + gt$$

Just remember that in this problem, upwards direction is considered to be positive. This means that the acceleration in the y-direction is actually $-g$ (since it opposes the velocity that is upwards)

Now, we can solve for t to find that $\boxed{t = \frac{v_o \sin(\theta)}{g}}$

We can use another kinematics equation to find the vertical displacement.

$$v_y^2 = v_{oy}^2 + 2a\Delta y$$

$$0 = (v \sin\theta)^2 - 2g\Delta y$$

$$\boxed{\Delta y = \frac{v^2 \cdot (\sin\theta)^2}{2g}}$$

Note 1.0.18 — Tips for Solving Projectile Motion Problems

1. Separate the variables in the x direction from the y direction. You are basically solving two separate kinematics problems, but it's part of one.

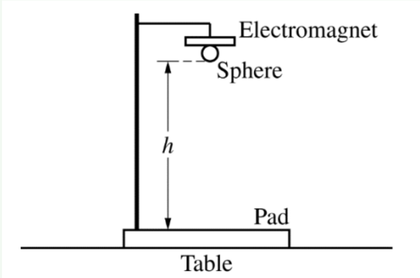
2. Write out all your variables as shown below. It's important to note that gravity is always in the y direction.

$$\begin{array}{cc} v_{ox} = v\cos\theta & v_{oy} = v\sin\theta \\ v_x = v\cos\theta & v_y = 0 \\ a_x = 0 & a_y = g \\ t & t \\ \Delta x & \Delta y \end{array}$$

3. In most projectile motion problems, remember that at the very top of the motion, the velocity in the y direction is 0. Also remember that the horizontal component of velocity stays the same (initial = final) in the majority of AP Physics C: Mechanics problems.

4. Remember that t (time) is the same for both the x and y axis. Thus, solving for time t can be extremely useful since it's a quantity that can be applied for both directions.

Problem 1.0.19 — 2018 AP Physics C: Mechanics FRQ (Modified)



A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

(a) While taking the first data point, the student notices that the electromagnet actually releases the sphere after the timer begins. Would the value of g calculated from this one measurement be greater than, less than, or equal to the actual value of g at the student's location? Justify your answer.

The electromagnet is replaced so that the timer begins when the sphere is released. The student varies the distance h. The student measures and records the time Δt of the fall for each particular height, resulting in the following data table.

h (m)	0.10	0.20	0.60	0.80	1.00
Δt (s)	0.105	0.213	0.342	0.401	0.451

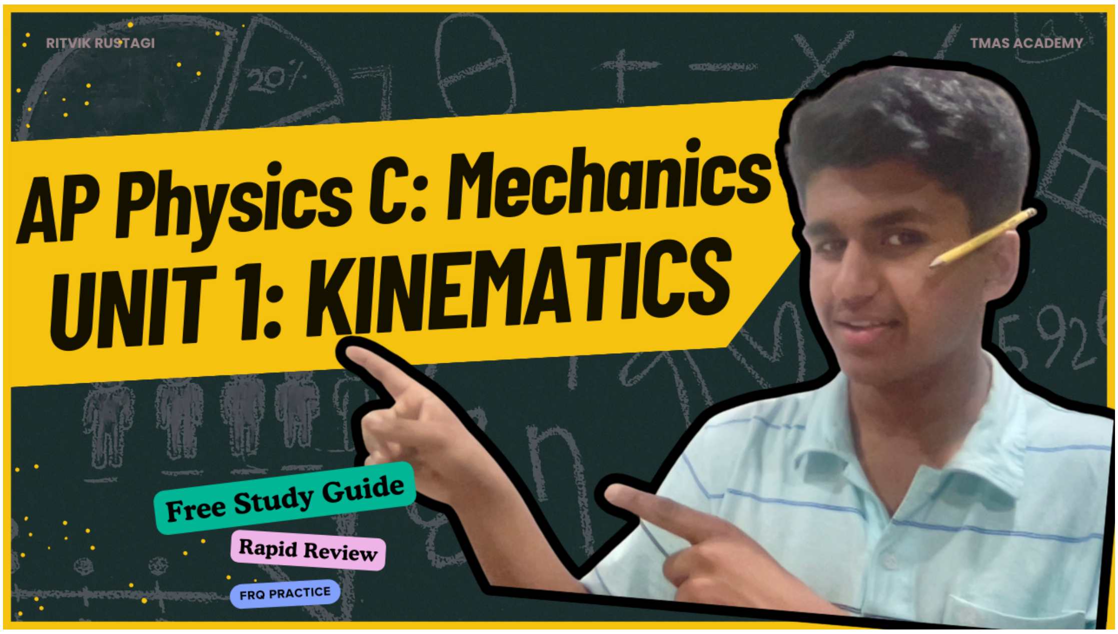
(b) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for g. Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given in the table. Label each row you use and include units.

(c) Plot the data points for the quantities indicated in part (b) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.

(d) Using the straight line, calculate an experimental value for g.
Another student fits the data in the table to a quadratic equation. The student's equation for the distance fallen y as a function of time t is $y = At^2 + Bt + C$, where $A = 5.75\text{m/s}^2$, $B = -0.524\text{m/s}$, and $C = +0.080$ m. Vertically down is the positive direction.

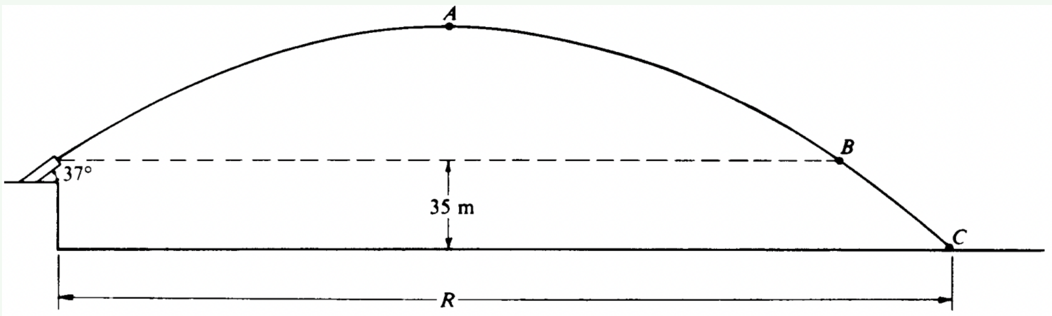
(e) Using the student's equation above, derive an expression for the velocity and acceleration of the sphere as a function of time.

Solution: Video Solution



Problem 1.0.20 — 1985 AP Physics C: Mechanics FRQ
A projectile is launched from the top of a cliff above level ground. At launch the projectile is 35 meters above the base of the cliff and has a velocity of 50 meters per second at an angle 37° with the horizontal. Air resistance is negligible. Consider the following two cases and use $g = 10\text{m/s}^2$, $\sin(37°) = 0.60$, and $\cos(37°) = 0.80$.

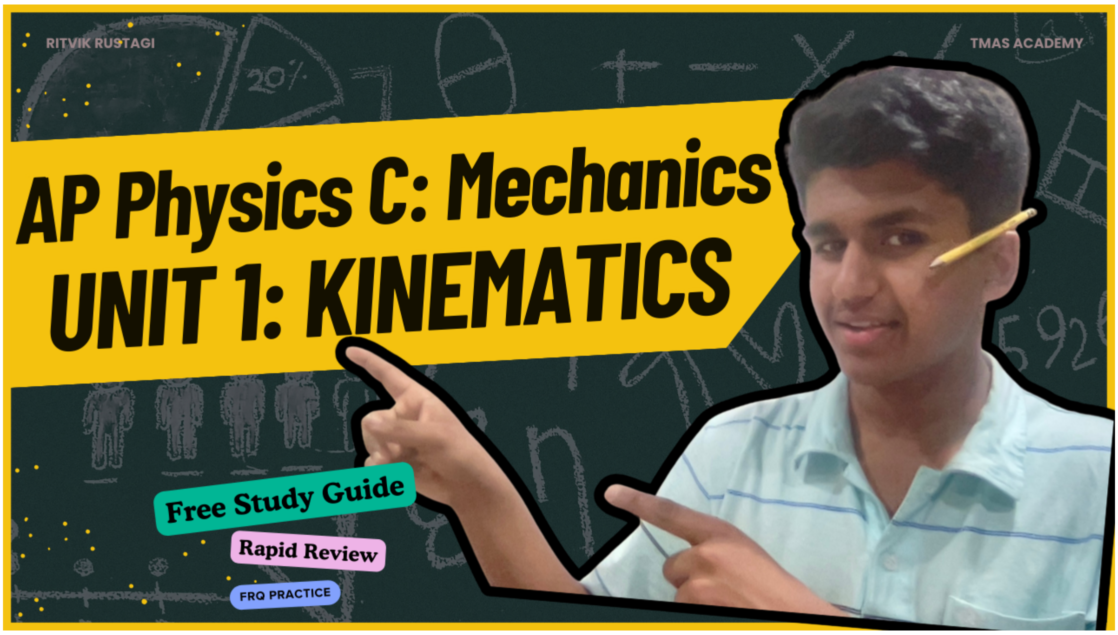
The projectile follows the path shown by the curved line in the following diagram



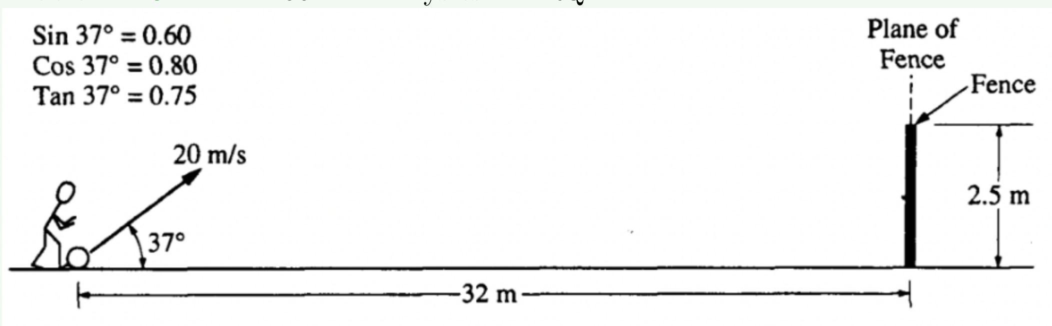
(a) Calculate the total time from launch until the projectile hits the ground at point C.

(b) Calculate the horizontal distance R that the projectile travels before it hits the ground.

Solution: Video Solution



Problem 1.0.21 — 1994 AP Physics B FRQ



A ball of mass 0.5 kilogram, initially at rest, is kicked directly toward a fence from a point 32 meters away, as shown above. The velocity of the ball as it leaves the kicker's foot is 20 meters per second at an angle of 37° above the horizontal. The top of the fence is 2.5 meters high. The ball hits nothing while in fight and air resistance is negligible.

(a) Determine the time it takes for the ball to reach the plane of the fence.

(b) Will the ball hit the fence? If so, how far below the top of the fence will it hit? If not, how far above the top of the fence will it pass?

Solution to part a: In this problem, we will first write out all the variables that we have.

$$\begin{array}{cc} v_{ox} = v_0 \cos\theta & v_{oy} = v_0 \sin\theta \\ v_x = v_{ox} & v_y = ? \\ a_x = 0 & a_y = g \\ \text{t} & \text{t} \\ \Delta x = 32 & \Delta y = \ ? \end{array}$$

In this problem, we don't know the final velocity in the y direction (v_y). The final velocity now occurs when the times hits the plane of the fence. It's no longer the situation when we considered the final velocity to be at the top of the arc (which was 0, or at the end of a perfectly symmetrical projectile problem).

However, we seem to have much more information about the variables in the x direction. Thus, let's investigate the motion in the x-direction.

We know that $v_x = v_{ox}$ because the velocity in the horizontal direction remains the same in projectile motion.

Also, v_0 is simply the initial velocity which is $20\frac{\text{m}}{\text{s}}$.

We can apply our kinematics equations to find t (time). We'll do this in the x direction.

Since velocity in the x direction is constant, acceleration in the x direction is 0. We can simply apply the equation $\Delta x = vt$ (which is true when the acceleration is 0).

Since Δx is 32 and $v = v_{ox} = 20\cos(37) = 16$, $t = \frac{32}{16} = \mathbf{2}$ **seconds**.

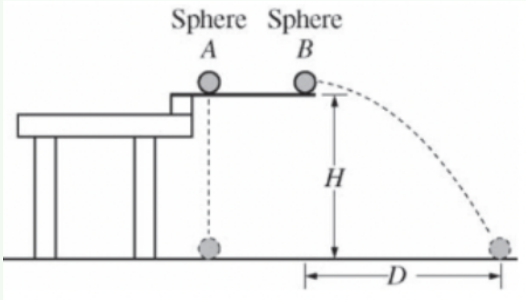
Solution to part b: Since we know the time of the motion, we will use kinematics in the y direction to see the position of the ball in the vertical direction after this time period.

$$\begin{array}{cc} v_{ox} = v_0 \cos\theta & v_{oy} = v_0 \sin\theta \\ v_x = v_{ox} & v_y = ? \\ a_x = 0 & a_y = g \\ t = 2 & t = 2 \\ \Delta x = 32 & \Delta y = \, ? \end{array}$$

Since $v_{oy} = v_0 \sin\theta = 20\sin(37) = 12.036$, $a_y = g = -9.8$, and $t = 2$, we can use the equation $y = v_{oy}t + \frac{1}{2}a_y t^2$.

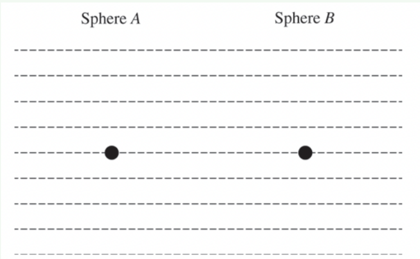
Plugging in our values gives $y = 12.036 \cdot 2 - \frac{9.8 \cdot 2^2}{2}$ which is 4.472 m. Since this is more than the height of the fence (2.5 m), the ball lands above the top. The distance above the top is $4.472 - 2.5 = \mathbf{1.972}$ **m**.

Problem 1.0.22 — 2016 AP Physics 1 FRQ

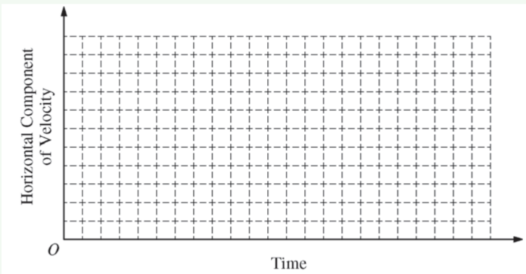


Two identical spheres are released from a device at time $t = 0$ from the same height H, as shown above. Sphere A has no initial velocity and falls straight down. Sphere B is given an initial horizontal velocity of magnitude v_0 and travels a horizontal distance D before it reaches the ground. The spheres reach the ground at the same time t_f, even though sphere B has more distance to cover before landing. Air resistance is negligible.

(a) The dots below represent spheres A and B. Draw a free-body diagram showing and labeling the forces (not components) exerted on each sphere at time $\frac{t_f}{2}$.

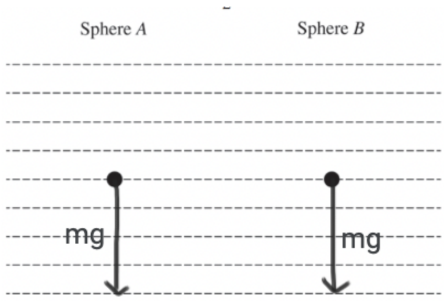


(b) On the axes below, sketch and label a graph of the horizontal component of the velocity of sphere A and of sphere B as a function of time.



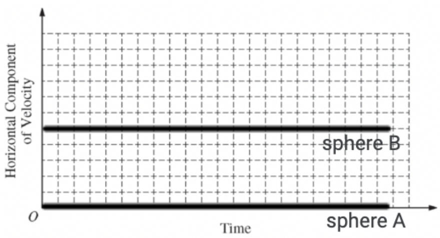
(c) In a clear, coherent, paragraph-length response, explain why the spheres reach the ground at the same time even though they travel different distances. Include references to your answers to parts (a) and (b).

Solution to part a: The only forces on each sphere are the gravitational force. The gravitational force on each sphere will be drawn with the same length since their masses are the same.



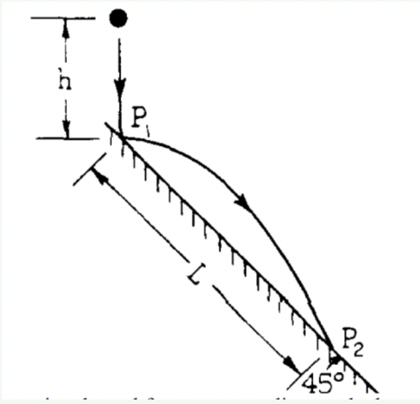
Solution to part b: Sphere A has no horizontal velocity. The reason is that it is simply in free fall. There is only vertical velocity.

On the other hand, Sphere B will have horizontal velocity. Since there is no acceleration in the x-direction, the horizontal velocity on sphere B will remain constant.



Solution to part c: We must know that the horizontal motion is independent from the vertical motion. If we consider motion in the vertical direction, then the initial velocity for both spheres is the same. The acceleration will be g for both. Since the vertical displacement for both is the same (h), the time it will take for both spheres to fall will also be the same. Both spheres reach the ground at the same time since they travel the same vertical displacement with the same initial vertical velocity.

Problem 1.0.23 — 1979 AP Physics B FRQ



A ball of mass m is released from rest at a distance h above a frictionless plane inclined at an angle of 45° to the horizontal as shown above. The ball bounces horizontally off the plane at point P_1 with the same speed with which it struck the plane and strikes the plane again at point P_2. In terms of g and h, determine each of the following quantities:

(a) The speed of the ball just after it first bounces off the plane at P_1.

(b) The time the ball is in flight between points P_1 and P_2.

(c) The distance L along the plane from P_1 to P_2.

(d) The speed of the ball just before it strikes the plane at P_2.

Solution to part a: For the motion towards point P_1, the ball starts with initial velocity v_i that is 0 (since it's simply dropped from rest).

The vertical displacement is h. The acceleration in this direction is g.
We can use the kinematics equation $v_f^2 = v_i^2 + 2a\Delta y$

We can plug in our variables to find that $v_f^2 = 0 + 2gh$
Now, we can take the square root of both sides to find $v_f = \sqrt{2gh}$

Solution to part b: The problem says that the ball bounces horizontally off the plane. This means that the initial vertical velocity is 0 for the motion from P_1 to P_2

The vertical displacement between point P_1 and P_2 is Δy which is $L\sin(45) = \frac{L\sqrt{2}}{2}$.
The acceleration in this direction is g.
We can now use the equation $\Delta y = v_i t + \frac{1}{2}at^2$

Plugging in our variables gives that $\frac{L\sqrt{2}}{2} = 0 + \frac{1}{2}gt^2$
We can isolate t^2 to find that $t^2 = \frac{L\sqrt{2}}{g}$
However, we need to replace L with something since our expression must be in terms of g and h.

Thus, we must consider motion in the horizontal direction. In the horizontal direction, the velocity will always be $\sqrt{2gh}$. The ball got this velocity due to its initial drop when it was released from a height of h. This velocity will stay the same (since in projectile motion, horizontal velocity is constant).

We can use the equation $\Delta x = v_x t$

We know that $\Delta x = L\cos(45) = \frac{L\sqrt{2}}{2}$

We can plug this in to find that $\frac{L\sqrt{2}}{2} = \sqrt{2gh}t$

Dividing both sides by $\sqrt{2gh}$ gives that

$$t = \frac{L\sqrt{2}}{2\sqrt{2gh}}$$

Since we already have an expression for t^2 from the vertical motion, we can square the time we found for the horizontal motion. Then, we can equate both expressions.

Squaring $t = \frac{L\sqrt{2}}{2\sqrt{2gh}}$ gives that $t^2 = \frac{L^2}{4gh}$
We can set this equal to the other expression which was $t^2 = \frac{L\sqrt{2}}{g}$
Equating both expressions gives that $\frac{L^2}{4gh} = \frac{L\sqrt{2}}{g}$

Now, we can cancel out some terms to find that $L = 4h\sqrt{2}$

Now, we can plug in $L = 4h\sqrt{2}$ into $t^2 = \frac{L^2}{4gh}$.

$$t^2 = \frac{(4h\sqrt{2})^2}{4gh} = \frac{8h}{g}$$

We can take the square root of both sides to find that $t = \sqrt{\frac{8h}{g}}$

Solution to part c: In part b, we already found that $L = 4h\sqrt{2}$. On the real AP exam, you probably won't have to find the answer for a later part in an earlier part. However, if you have to, it would be beneficial to show the work again to guarantee the points. For now, I will skip the solution to this part since it was already show in part b.

Solution to part d: To find the speed of the ball at point P_2, we can find the velocity in the x-direction and y-direction.

The reason is that the total speed will be $\sqrt{v_x^2 + v_y^2}$. We must account for both components of velocity!

In the x-direction, velocity will stay the same since there is no acceleration. We already found in part a that $v_x = \sqrt{2gh}$.

In the y-direction, we know that the acceleration is g. On top of that, the initial vertical velocity is 0. We also know from part b that the time is $t = \frac{8h}{g}$.
We can plug this into the equation $v_y = v_{iy} + at$
Plugging in our variables gives $v_y = 0 + g \cdot \sqrt{\frac{8h}{g}} = \sqrt{8gh}$

Since $v_x = \sqrt{2gh}$ and $v_y = \sqrt{8gh}$, we can plug this into $\sqrt{v_x^2 + v_y^2}$ to find the speed. Doing so gives that the speed is $\sqrt{(\sqrt{2gh})^2 + (\sqrt{8gh})^2} = \sqrt{10gh}$

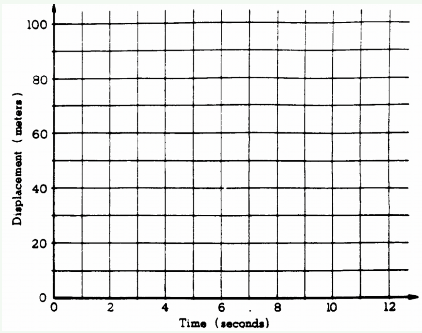
Problem 1.0.24 — 1982 AP Physics B FRQ
The first 10 meters of a 100-meter dash are covered in 2 seconds by a sprinter who starts from rest and accelerates with a constant acceleration. The remaining 90 meters are run with the same velocity the sprinter had after 2 seconds.

(a) Determine the sprinter's constant acceleration during the first 2 seconds.

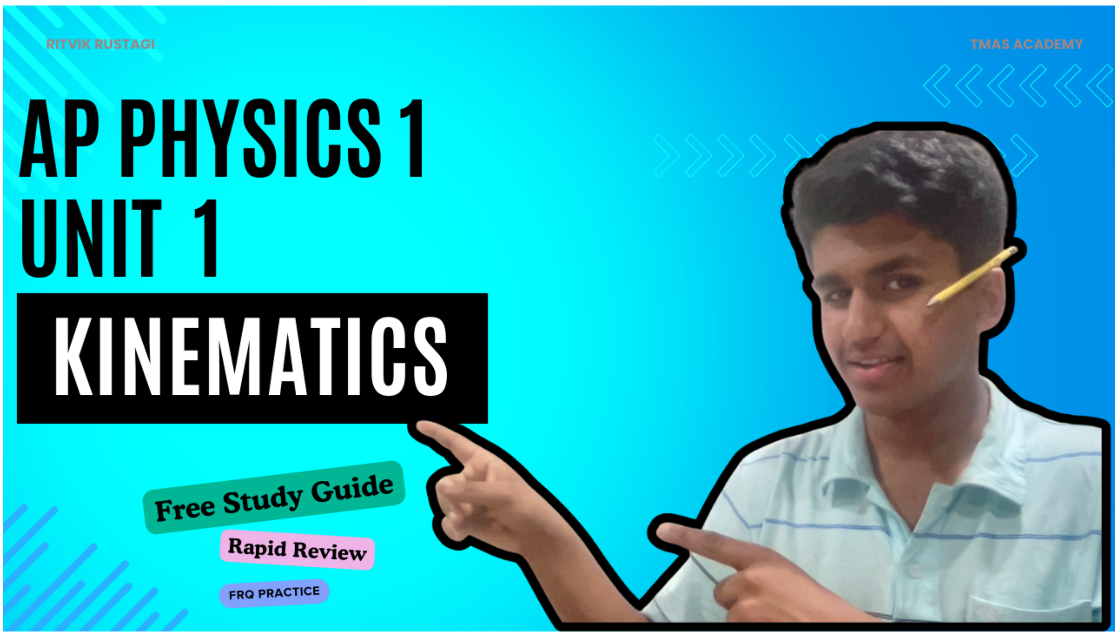
(b) Determine the sprinter's velocity after 2 seconds have elapsed.

(c) Determine the total time needed to run the full 100 meters.

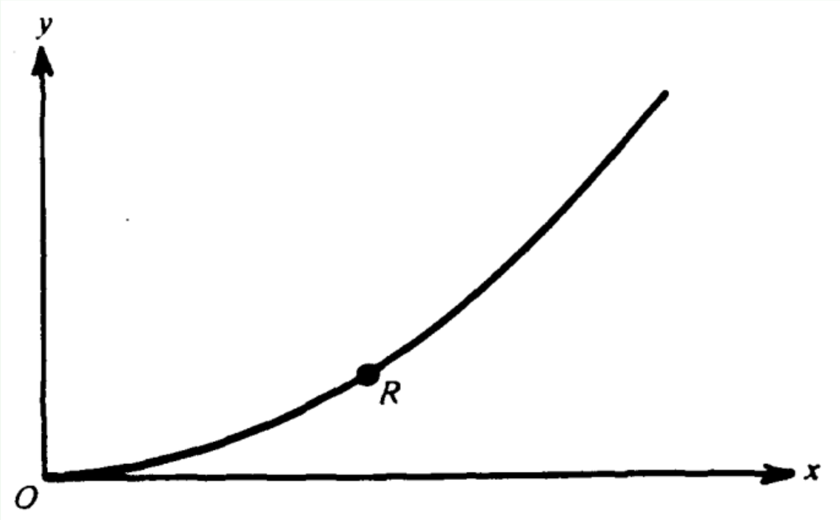
(d) On the axes provided below, draw the displacement-time curve for the sprinter.



Solution: Video Solution



Problem 1.0.25 — 1983 AP Physics C: Mechanics FRQ



A particle moves along the parabola with equation $y = \frac{1}{2}x^2$ shown above.
(a) Suppose the particle moves so that the x-component of its velocity has the constant value $v_x = C$; that is $x = Ct$.

i. On the diagram above, indicate the directions of the particle's velocity vector v and acceleration vector a at point R, and label each vector.

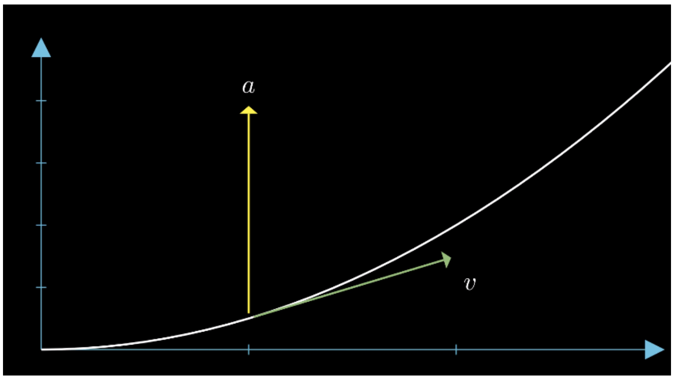
ii. Determine the y-component of the particle's velocity as a function of x.

iii. Determine the y-component of the particle's acceleration.

(b) Suppose, instead, that the particle moves along the same parabola with a velocity whose x-component is given by $v_x = \frac{C}{(1+x^2)^{1/2}}$
Show that the particle's speed is constant in this case.

Solution to part a: The velocity vector will be tangent to the position. Thus, for our velocity vector, we draw a line tangent to the curve at our specific point.

Now, we know that our horizontal velocity is constant (given in the problem). Thus, there is no acceleration in the x-direction. However, there will be acceleration in the y-direction. We can simply draw an arrow pointing upwards to represent the acceleration that occurs in the y-direction.



Solution to part a ii: To find the velocity in the y-direction with respect to x, we must use some calculus.

We already know an equation for the particle's motion: $y = \frac{1}{2}x^2$

We should take the derivative of the equation with respect to time so that on the left side we get $\frac{dy}{dt}$ which is velocity in the y-direction. To differentiate, we must use the chain rule on the right side. If you are unfamiliar with this, check out TMAS Academy's AP Calculus AB or BC books that are also both written by Ritvik Rustagi.

$$\text{From the Chain Rule, we know that } \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

Since $y = \frac{1}{2}x^2$, we can differentiate this with respect to x to get $\frac{dy}{dx} = x$

We can use that to find $\frac{dy}{dt} = v_y = x\frac{dx}{dt}$

We are already given that the velocity in the x-direction (same thing as $\frac{dx}{dt}$) is C. That means $\frac{dy}{dt} = v_y = Cx$

Solution to part a iii: The acceleration in the y-direction is the derivative of velocity in the y-direction.

$$\text{We know that } a_y = \frac{dv_y}{dt} = \frac{dv_y}{dx} \cdot \frac{dx}{dt}$$

Since we know that $v_y = Cx$, it's obvious that $\frac{dv_y}{dx} = C$

$$\text{We can plug that in to find that } a_y = \frac{dv_y}{dt} = C \cdot \frac{dx}{dt}$$

We again already know that $\frac{dx}{dt} = C$ (since velocity in the x-direction was given to be C). This means that $a_y = C^2$

Solution to part b We know that speed is $\sqrt{v_x^2 + v_y^2}$
We are already given v_x. We must find v_y to see if the speed is constant.
Since we know that $y = \frac{1}{2}x^2$, we can take the derivative of this equation with respect to time.

$$\text{Doing so gives } \frac{dy}{dt} = x\frac{dx}{dt}$$

We also know that $\frac{dx}{dt}$ represents the velocity in the x-direction which is given to be

$$\frac{C}{(1+x^2)^{\frac{1}{2}}}$$

$$\text{We can plug this in to find that } v_y = \frac{dy}{dt} = \frac{Cx}{\sqrt{1+x^2}}$$

$$\text{Now we can plug in } v_x = \frac{C}{\sqrt{1+x^2}} \text{ and } v_y = \frac{Cx}{\sqrt{1+x^2}} \text{ into } \sqrt{v_x^2 + v_y^2}$$

$$\text{Doing so gives that the speed is } \sqrt{(\frac{C}{\sqrt{1+x^2}})^2 + (\frac{Cx}{\sqrt{1+x^2}})^2}$$

The expression simplifies to $\sqrt{C^2}$ which is simply C. Clearly, the total speed is constant since its value is C (which is a constant).

Problem 1.0.26 — Source: 2002 AP Physics C Mechanics FRQ (modified)
The velocity of a car is given below.

$$v(t) = \frac{8}{1+5t}$$

a. Assuming an initial position of $x = 0$, determine an expression for the position of the car at time t.

b. Determine an expression for acceleration as a function of time t.

Solution to part a: We know that position is the integral of velocity with respect to time.

$$x(t) = \int v dt = \int [\frac{8}{1+5t}] dt = \frac{8\ln(1+5t)}{5} + C$$

Since we know that $x(0) = 0$, our constant C is simply 0.

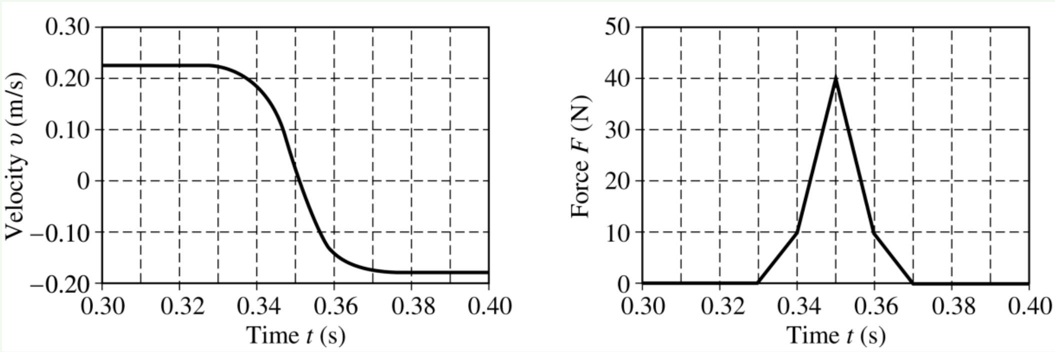
$$\text{Thus, } x(t) = \frac{8\ln(1+5t)}{5}$$

Solution to part b: Acceleration is the derivative of velocity with respect to time.
$a(t) = v'(t)$

$$\text{Since } v(t) = \frac{8}{1+5t}, \text{ we know that } a(t) = -\frac{40}{(1+5t)^2}$$

Problem 1.0.27 — 2001 AP Physics C Mechanics FRQ

A motion sensor and a force sensor record the motion of a cart along a track, as shown above. The cart is given a push so that it moves toward the force sensor and then collides with it. The two sensors record the values shown in the following graphs.

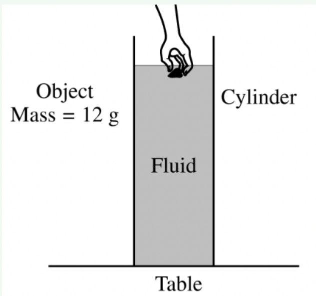


Determine the cart's average acceleration between $t = 0.33$ s and $t = 0.37$ s.

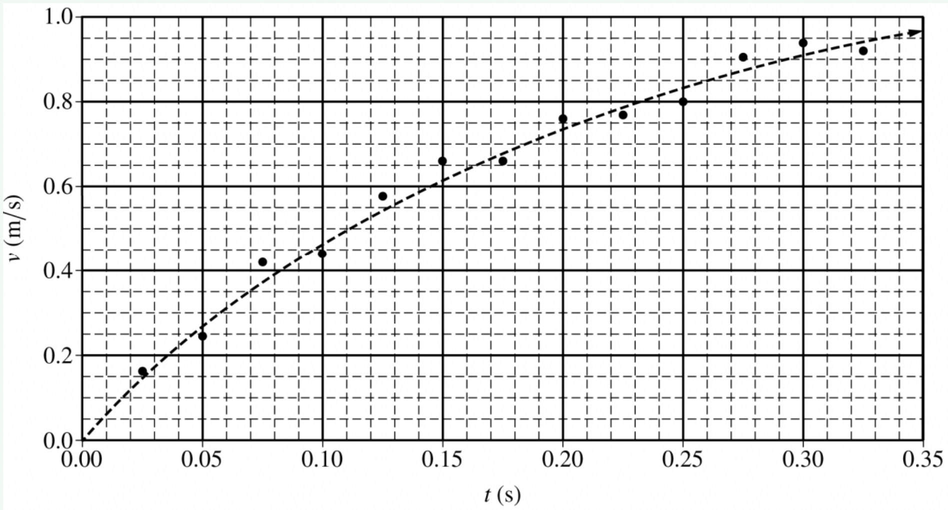
Solution to part a: Average acceleration is the change in velocity over time.
Average acceleration $= \frac{\Delta v}{\Delta t}$
The velocity at $t = 0.37$ s is -0.18 while the velocity at $t = 0.33$ s is 0.22

This means that $\Delta v = -0.18 - 0.22 = -0.40$ while $\Delta t = 0.37 - 0.33 = 0.04$ s.
This means that average acceleration is $\frac{-0.40}{0.04} = -10$

Problem 1.0.28 — 2019 AP Physics C Mechanics FRQ



In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of $m = 12$ g is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.



(a)

i. Does the speed of the object increase, decrease, or remain the same?
______ Increase ______ Decrease ______ Remain the same
ii. In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

iii. Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

The students use the equation $v = A(1 - e^{-Bt})$ to model the spread of the falling object and find the best fit coefficients to be $A = 1.18$ m/s and $B = 5$ s^{-1}.
(b) Use the above equation to:

i. Derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time t.

ii. Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time t.

Solution to part a i: Clearly the speed increases. The reason is that in the speed-time graph, speed goes up as time increases.

Solution to part a ii: The object is falling and the speed increases. We can notice that the rate at which the speed changes also changes. This means that the acceleration isn't constant. In fact, it's decreasing. Initially, the speed increases at a faster rate, but it slows down later. This shows that the magnitude of acceleration decreases over time.

Solution to part a iii: To find the acceleration at that specific time, we have to find the slope at that point. We can use the value of velocity at times $t = 0.225$ and $t = 0.175$ for approximation.

$$\text{Acceleration is } \frac{\Delta v}{\Delta t} = \frac{0.77 - 0.66}{0.225 - 0.175} = 2.2 \text{ m/s}^2$$

Solution to part b i: Since we are given an expression for velocity, we simply have to integrate this to find an expression for position.
$y(t) = \int v(t)dt$

$$y(t) = \int_0^t A(1 - e^{-Bt})dt = A(t + \frac{1}{B}(e^{-Bt} - 1))$$

We can plug in $A = 1.18$ and $B = 5$ to get $y(t) = 1.18(t + \frac{1}{5}(e^{-5t} - 1))$

Solution to part b ii: Although we haven't discussed forces yet, this problem can be solved by simply knowing the formula for force given the acceleration, $F = ma$. This means that force is mass times acceleration. Thus, we must first find an expression for acceleration.

We know that acceleration is the derivative of velocity.

$$a(t) = \frac{dv}{dt} = v'(t)$$

This means that $a(t) = ABe^{-Bt}$
Now, we can pug in $A = 1.18$ and $B = 5$ to get $a(t) = 5.9e^{-5t}$

We know that the mass is $12g$ which is 0.012 kg (we must convert since to find force in the units Newtons, our mass has to be in kilograms). We can plug that in to find an expression for force.

$$F(t) = 0.012 \cdot 5.9e^{-5t} = 0.0708e^{-5t}$$

Problem 1.0.29 — 1982 AP Physics C: Mechanics FRQ
A car of mass M moves with an initial speed v_o on a straight horizontal road. The car is brought to rest by braking in such a way that the speed of the car is given as a function of time t by $v = (v_o^2 - Rt/m)^{1/2}$ where R is a constant.

(a) Determine the time it takes to bring the car to a complete stop.

(b) Develop an equation for the acceleration of the car as a function of time t.

Solution to part a: When the car is at a complete spot, the velocity v will be 0. This means we can set our expression for velocity to 0 and solve for time.

$$(v_o^2 - \frac{Rt}{M})^{\frac{1}{2}} = 0$$

We can square both sides of the equation to get $v_o^2 - \frac{Rt}{M} = 0$
Rearranging the equation and solving for t gives $t = \frac{Mv_o^2}{R}$

Solution to part b: We know that $a = \frac{dv}{dt}$. That means we must differentiate velocity with respect to time to find acceleration.

For those that don't know, we must use the chain rule to integrate our expression for velocity in this situation.

$$a = \frac{dv}{dt} = \frac{1}{2} \cdot (v_o^2 - \frac{Rt}{m})^{-\frac{1}{2}} \cdot -\frac{R}{m}$$

Unit 2 Newton's Laws of Motion

Have you ever tried to push a shopping cart? Have you ever tried to push a car? Certainly, pushing the shopping cart is a lot easier. The reason is that it has less inertia. Inertia is the tendency of objects at rest to stay at rest. Of course a car will have a lot more inertia. It's much heavier when compared to a shopping cart.

In this unit, you will deal with such scenarios and think about the forces that are involved.

Newton's Laws of Motion are a crucial topic on the AP exam. There are three laws that you must be familiar with.

Note 2.0.1 — Newton's First Law
Newton's First Law states that an object at rest always stays at rest unless an external force acts on it. It also states that an object at constant velocity continues to move at that same velocity unless acted by an external force.

If we logically think about Newton's First Law, then it should make sense. Of course an object at rest will always remain at rest unless a certain force acts on it. For example, a shopping cart at a store will always remain at rest. However, it will start moving if a shopper applies a force to push it.

Note 2.0.2 — Newton's Second Law
Newton's Second Law states that a force acted on an object will lead to acceleration. This leads to a crucial formula:

$$F_{net} = ma$$

It states that net force = mass times acceleration.

To conceptually imagine what net force is, think about a tug of war competition. If there is one person on each side, and both are equally strong, then the rope will not move. The reason is that both people will apply the same force. However, if one person is stronger, then they will apply a stronger force. That is when the rope will start to accelerate as the stronger person pulls the rope towards them.

Problem 2.0.3 — If the force applied on a box is 18 N, and the mass of the box is 6 kg, what is the acceleration?

Solution: Since we know that $F = ma$ from Newton's Second Law, we can use that and plug in our given numbers.

$$18 = 6 \cdot a$$

Dividing both sides by 6 gives us an acceleration of 3 $\frac{m}{s^2}$

Note 2.0.4 — Newton's Third Law
Newton's third law states that for every action, there is always a reaction. This means that if you apply a force on something, that object will apply the same exact force back on you.

A Newton's third law pair comprises of specifically two objects interacting, both exerting a force of equal magnitude on each other.

Problem 2.0.5 —
a. If Bob applies a force of 60 N towards the right onto a wall, what force does the wall apply onto Bob? Indicate magnitude and direction.

b. If Bob's mass is 30 kg, what is his acceleration?

Solution to part a: From Newton's Third Law, since Bob applies a force of 60N to the right, we know that the wall must apply that exact same force onto him of 60N. However, it is applied to the left because it is a reaction force.

Solution to b:: Since we know that the force applied onto Bob is 60 N to the left (or -60 N since left is negative), we can use that and Newton's Second Law.

$$F_{net} = m \cdot a$$
$$-60 = 30 \cdot a$$

Dividing both sides by 2 gives $a = -2$ m/s^2

Now let's discuss **friction**.
Friction is resistance that an object might face from another object or a surface. It opposes the relative motion of the two objects.

Note 2.0.6 — Static friction is the friction force on an object that does not slide relative to a surface.

Kinetic friction is the friction force on an object that does slide relative to a surface.

Note 2.0.7 — Friction Analyzed Conceptually
You might still be confused in differentiating these two types of frictions. This short paragraph should clear it up.

When you try to push an object in real life, it does not immediately move. You start by applying a force of 0 Newtons and increase that force. After a few seconds, when your force is high enough, the object will start to accelerate. However, why does it not accelerate until it hits that certain amount? The reason is that static friction has prevented you from doing so. The force that you must apply must overtake the maximum force of static friction. Once that happens, kinetic friction will be the force that opposes your motion.

Static friction force is represented as f_s.
$f_s \leq \mu_s N$ is the relationship that we must know. μ_s represents the coefficient of static friction while N is the normal force.

The maximum static friction is $\mu_s N$. This doesn't mean that the static friction has to have that same magnitude. It can indeed be less than the maximum value.

The numerical value for the kinetic friction force is calculated in the same way by multiplying the friction coefficient (kinetic friction coefficient, not static friction coefficient) by the normal force. In general, the kinetic friction coefficient will be smaller than the static friction coefficient!

Before we move onto free body diagrams, we must learn about a special type of force which will involve calculus. This topic is the main one that sets AP Physics 1 apart from AP Physics C: Mechanics from a conceptual perspective.

Note 2.0.8 — Resistive forces includes the drag force which is a **velocity-dependent** force. This means that the magnitude of the drag/special resistive force depends directly on the velocity.

The resistive force will usually be $F = -bv$ or $F = -bv^2$ where b is a constant.

If the resistive force is the only force on an object in the x-direction, then we know that it's acceleration will be $a = \frac{F_{net}}{m} = -\frac{bv}{m}$ or $-\frac{bv^2}{m}$.

However, this isn't a neat result. We have the acceleration variable on the left and velocity on the right. There's way too many variables to deal with at once. That is why we must use the other way to write acceleration. Remember that acceleration a is also $\frac{dv}{dt}$.

Using $\frac{dv}{dt}$ for acceleration gives that $\frac{dv}{dt} = -\frac{bv}{m}$ or $-\frac{bv^2}{m}$.

Now, we have something promising. We can use separation of variables and integrate both sides to find an expression for velocity.

For example, let's say that $\frac{dv}{dt} = -\frac{bv}{m}$.

Using separation of variables causes us to rearrange the differential equation to get

$$\frac{dv}{v} = -\frac{b}{m}dt$$

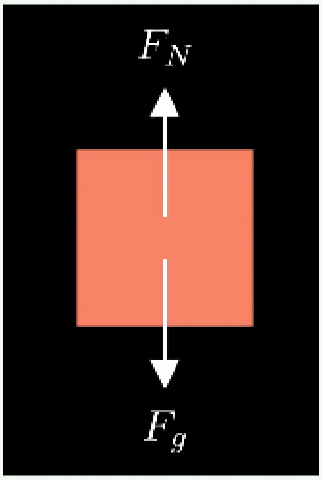
Now, we can integrate both sides to get $\ln|v| = -\frac{bt}{m} + C$
This means that $|v| = e^{-\frac{bt}{m}+C} = Ce^{-\frac{bt}{m}}$

$$\text{Thus, } v(t) = Ce^{-\frac{bt}{m}}$$

Don't worry if you're still confused on how to work with the resistive force. There will be problems in this book that will use such concepts.

Note 2.0.9 — Free Body Diagrams
Free Body Diagrams are one of the most crucial part of this entire course. You must draw a free body diagram to be able to understand a problem and simplify it along with minimizing the number of errors that you make.

You draw a free body diagram by labelling all forces that act on your object and indicate the direction with an arrow. This is extremely important especially when you use Newton's Second Law to find the acceleration.



The above image is a good example of a free-body diagram. There is an object with normal force which points perpendicular to the surface. There is gravitational force which points downwards. You might still be confused, so the best way is to practice a few crucial problems that commonly show up for this unit.

Note 2.0.10 — Atwood's Machines

Atwood's Machines are a type of device that commonly show up on the AP Physics C: Mechanics exam. In this unit, our Atwood's machines will have a **massless pulley**. There will be a lightweight string around it for which **the tension force remains constant throughout the entire string**. Often, the string connects two objects and we work from there using Newton's Laws to analyze the situation.

Important Tips for Solving Atwood's Machines
Since one object goes down while the other goes up, it's important to adapt the proper sign convention.
Sign convention can be something such as positive when a force makes the pulley move clockwise, but negative when it makes it counterclockwise.

We will also analyze each object attached on both sides of the pulley **separately**. We will write out separate equations for them using Newton's Second Law.

Problem 2.0.11 — Atwood's Machines
We have a pulley with two blocks of mass M and m attached. The pulley is frictionless and massless. Find an expression for the acceleration of the blocks once released? Now, find the acceleration when $M = 30\text{kg}$ and $m = 20\text{kg}$.

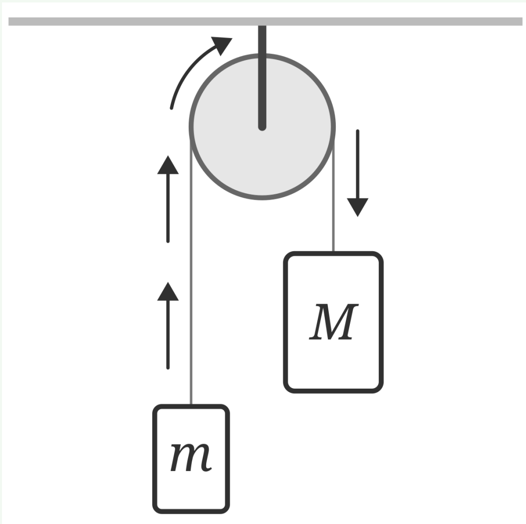
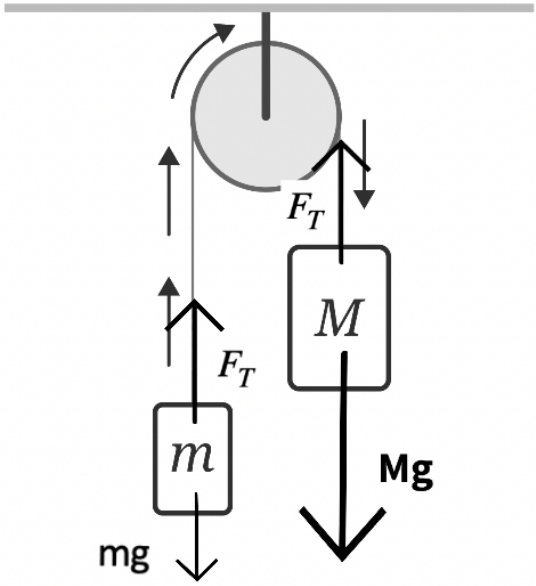


Image Credits: Phyley Website

Solution: In this problem, the first step will be to make a free-body diagram.



Now using our free body diagram, we can write out our two separate equations for each individual object.
Note, that F_T represents the tension force while M represents the mass of the heavier object and m represents the mass of the lighter object.

Newton's Second Law on heavy block: $Mg - T = Ma$
Newton's Second Law on light block: $T - mg = ma$

Now since the tension is a missing variable, we can add both equations since that eliminates the tension variable.
Adding both equations gives $Mg - mg = Ma + ma$

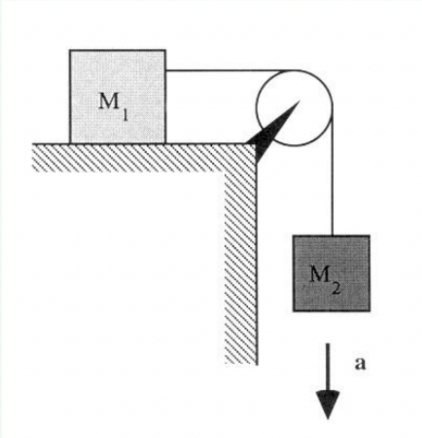
We can factor to get $g(M - m) = a(M + m)$

Now we simply divide both sides by $M + m$ to isolate a (acceleration) and this gives $a = \frac{g(M-m)}{M+m}$
Plugging in our values of M (30) and m (20) gives $\frac{10g}{50}$ which simplifies to $\frac{g}{5}$ which is 1.96 $\frac{\mathrm{m}}{\mathrm{s}^2}$

Bonus Question: What is the value of the tension force in the pulley above?

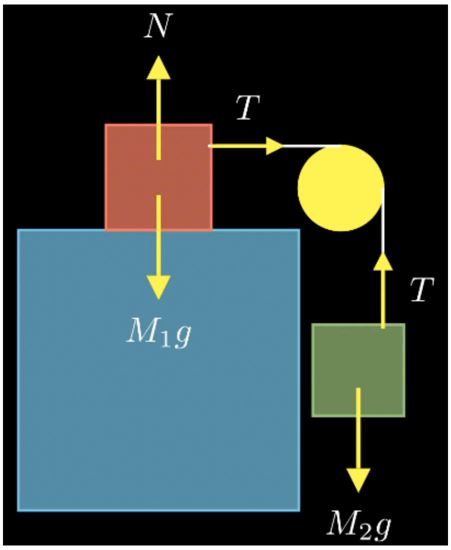
Solution: Using our value of the acceleration, we plug that back into one of the original equations such as $T - mg = ma$
We solve for T by adding mg to both sides and
$T = m(a + g)$
We plug in our values of m (20) and a (1.96) and g (9.8)
$T = 20(1.96 + 9.8) = 235.2$ N

Problem 2.0.12 — Pulley on a Table



Assuming that the table that block M_1 is sitting on is frictionless, and the pulley is massless and frictionless, what is the acceleration of the blocks as M_2 slides down?

Solution: In this problem, we will first draw our free body diagram like always. After that, we'll write out our equations using Newtons Laws. Then, we'll solve them.



We don't need to write an equation for the forces in the vertical direction on M_1 since the normal force balances out the gravitational force.

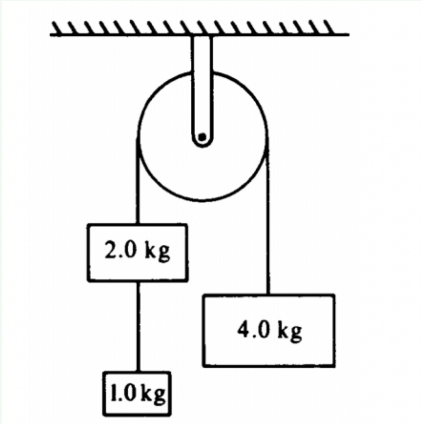
(Forces on M_2:) $M_2g - T = M_2a$
(Forces on M_1:) $T = M_1a$

We don't have our value of tension, so we can add both equations to eliminate tension.

$M_2g = M_2a + M_1a = a(M_2 + M_1)$

We can divide both sides by $M_2 + M_1$ to find acceleration. Doing so gives
a (acceleration) $= \frac{M_2g}{M_2+M_1}$

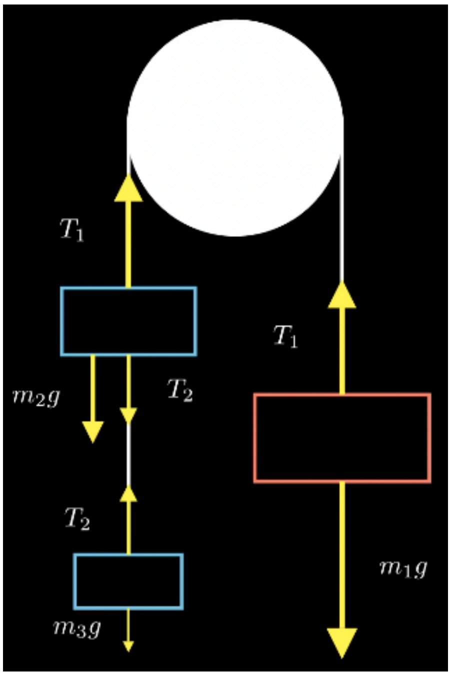
Problem 2.0.13 — Challenging Atwoods Machine



Three blocks of masses 1.0, 2.0, and 4.0 kilograms are connected by massless strings, one of which passes over a frictionless pulley of negligible mass, as shown above. Calculate each of the following.

a. The acceleration of the 4-kilogram block
b. The tension in the string supporting the 4-kilogram block
c. The tension in the string connected to the l-kilogram block

Solution to a: To find the acceleration for the 4kg block, we must draw a free body diagram again.



Obviously the pulley will cause the objects move to the right and downwards because the block of 4 kg is heavier than the other two blocks combined. Thus, we will assume that the clockwise direction is positive.

Before we write out equations using Newtons Laws, we can eliminate T_2 (tension force between the strings connecting the two lighter blocks) because it is an internal force if we consider the two blocks together (the two lighter blocks act like a larger block).

(Forces on m_1:) $m_1g - T_1 = m_1a$
(Forces on m_2 and m_3 as a system): $T_1 - (m_2 + m_3)g = (m_2 + m_3)a$

Since we don't know the value of T_1, we can simply add the two equations to cancel T_1 and get

$$(m_1 - m_2 - m_3)g = (m_2 + m_3)a$$

Dividing both sides by $m_2 + m_3$ gives

$$a = \frac{(m_1 - m_2 - m_3)g}{m_2 + m_3}$$

Substituting our masses ($m_1 = 4$, $m_2 = 2$, $m_3 = 1$) gives that

$$a = \frac{g}{5} = 1.96\frac{\text{m}}{\text{s}^2}$$

Solution to b: The tension in the string supporting the 4-kg block is just T_1. We can substitute our value of acceleration into the equation we found in part a for the forces on m_1 which was $m_1g - T_1 = m_1a$

We can rearrange it to get $T_1 = m_1(g - a)$
Substituting 4 for m_1, 9.8 for g, and 1.96 for a gives 31.36 **N**

Solution to c: The tension in the string supporting the 1-kg block is T_2. To find this, we can't work with both blocks m_3 and m_2 as a system. We have to work separately with the system that only contains block m_3.

The forces on block m_3 are the gravitational force and the tension force.

(Forces on block m_3:) $T_2 - m_3g = m_3a$

$$T_2 = m_3(g + a)$$

Substituting 1 for m_3 and 1.96 for a gives that the tension force on the 1 kg block is $1(9.8 + 1.96)$ which is 11.76 N.

Note 2.0.14 — Apparent Weight
Apparent weight is the weight that the person feels, but it often differs from their actual weight. In most problems, the normal force will be the apparent weight. The reason is that the normal force is the force that will be exerted on a person by something (such as the ground or an elevator), and that's the force that will be felt by the person, causing them to think that it's their actual weight when it's truly their apparent weight.

Most apparent weight problems involve an elevator that accelerates.

Problem 2.0.15 —

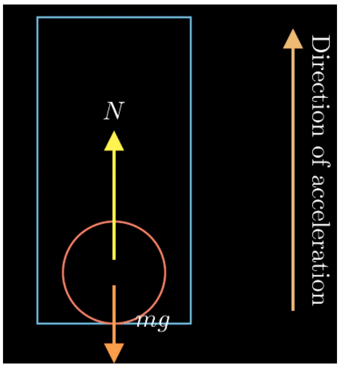
a. If an elevator accelerates **up** with a person standing on a scale, will the person's weight shown by the scale (apparent weight) be less than their actual weight or more?

b. If an elevator accelerates **down** with a person standing on a scale, will the person's weight shown by the scale (apparent weight) be less than their actual weight or more?

c. If an elevator moves at constant speed in any direction, will the person's weight shown by the scale (apparent weight) be less than their actual weight or more?

d. When the elevator is accelerating downwards, when will the person feel complete weightlessness?

Solution to a:

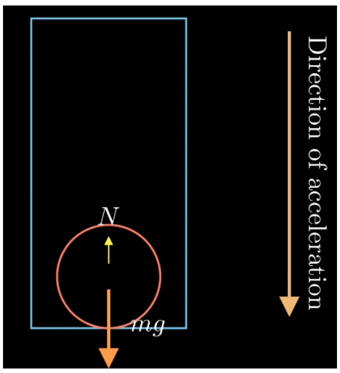


Using the free-body diagram above, we can write an equation for the person. The forces on it are the normal force from the scale (which represents the apparent weight) and the gravitational force. The normal force has a larger magnitude than the gravitational force since the elevator is accelerating up.

$$N - mg = ma$$
$$N = m(g + a)$$

Clearly from the normal force we found, the apparent weight when the elevator accelerates up is greater than the actual weight since the actual weight would be mg, but apparent weight is $m(g + a)$ which is also equal to $mg + ma$.

Solution to b:

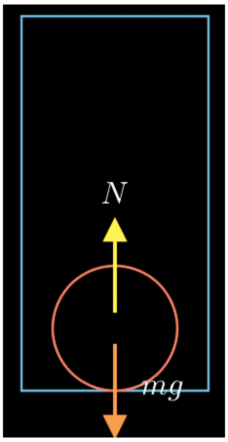


In this problem, we will again write out equation using Newton's Second Law and find the value of N (our normal force which is also the apparent weight). In this situation, the normal force will have a lower magnitude than gravitational force since the elevator is accelerating downwards.

$$mg - N = ma$$
$$N = m(g - a)$$

Clearly for the normal force in this case, the normal force (apparent weight) is less than the actual weight which is mg since we subtract the value of ma from it. Thus, when the elevator accelerates down, the apparent weight is less than the actual weight.

Solution to c:



In this case, we will again write our equation using Newton's Second Law.

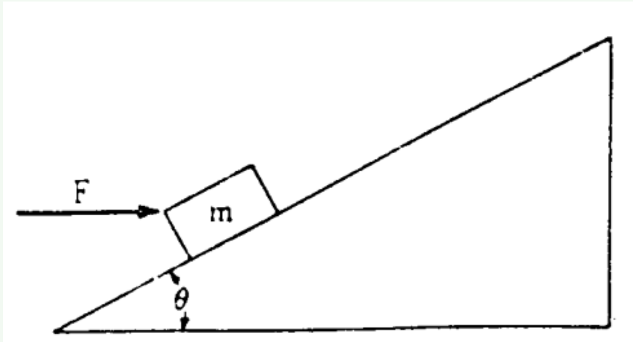
$$mg - N = ma$$

$$\text{Since acceleration is } 0, mg - N = 0$$

Simplifying gives $N = mg$ which means that the normal force (or apparent weight) is equivalent to the actual weight. Thus, when the elevator does not accelerate, the apparent weight is the actual weight.

Solution to d:
Weightlessness is felt when the normal force is 0. In that case, we will now again use the equation $mg - N = ma$.
Plugging in 0 for N gives $mg = ma$ which simplifies to $a = g$.
This means that the elevator must accelerate downwards at g (9.8) which means that it must be in **freefall.** Thus, for a person in an elevator to feel weightlessness, the elevator must be in freefall.

Problem 2.0.16 — 1981 AP Physics C: Mechanics FRQ



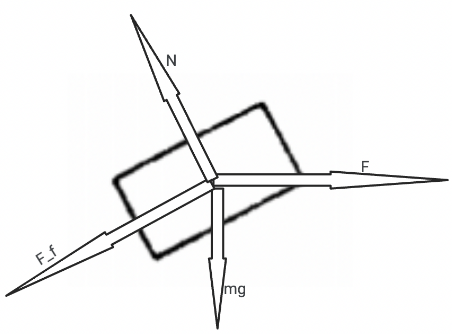
A block of mass m, acted on by a force of magnitude F directed horizontally to the right as shown above, slides up an inclined plane that makes an angle θ with the horizontal. The coefficient of sliding friction between the block and the plane is μ.

(a) Draw and label all the forces that act on the block as it slides up the plane.

(b) Develop an expression in terms of m, θ, F, μ, and g, for the block's acceleration up the plane.

(c) Develop an expression for the magnitude of the force F that will allow the block to slide up the plane with constant velocity. What relation must θ and μ satisfy in order for this solution to be physically meaningful?

Solution to a: Before drawing the free body diagram, we will first identify all forces acting on it. Gravity and normal force are always there. We know that we have an applied force, and there is friction also. Those are the only 4 forces.



Solution to b: Now we will write out our equations.
N represents the normal force
F represents the applied force
mg (F_g) represents the force by gravity
F_f represents force of friction

F_x will be our sum of forces in the x-direction (and in the case of an inclined plane the x-direction is the direction parallel to the inclined plane)
F_y will be the sum of the forces in the y-direction (which is perpendicular to the inclined plane)

We will now calculate the value of our friction force. It is $\mu \cdot$ N. Thus, we have to find the value of N to find friction.

$$F_x : F\cos(\theta) - F_f - mg\sin(\theta) = ma$$
$$F_y : N - F\sin(\theta) - mg\cos(\theta) = 0$$

Using our equation for F_y, since we know that the net force is 0 in that direction, we can find that $N = F\sin(\theta) + mg\cos(\theta)$

We plug this value of N to find F_f. F_f is simply $\mu \cdot$ N which is $\mu(F\sin(\theta) + mg\cos(\theta))$

We plug this into the value of F_f in the equation for F_x to get

$$F_x : F\cos(\theta) - \mu(F\sin(\theta) + mg\cos(\theta)) - mg\sin(\theta) = ma$$

We divide both sides by m to find the acceleration

$$a = \frac{F\cos(\theta) - \mu(F\sin(\theta) + mg\cos(\theta)) - mg\sin(\theta)}{m}$$

Solution to part c: Now since the problem wants the block to slide with constant velocity, this means that the acceleration must be 0 since acceleration is 0 whenever an object is moving at constant velocity. We can set the expresion for acceleration (from part a) to 0

$$\frac{F\cos(\theta) - \mu(F\sin(\theta) + mg\cos(\theta)) - mg\sin(\theta)}{m} = 0$$

Simplifying it by isolating F makes the equation become

$$F(\cos(\theta) - \mu\sin(\theta)) = mg(\sin(\theta) + \mu\cos(\theta)$$

$$\text{Solving for } F \text{ gives } \frac{mg(\sin(\theta) + \mu\cos(\theta)}{\cos(\theta) - \mu\sin(\theta)}$$

Since we know that the force F is towards the right, our value for F must be positive. This only happens when the denominator is positive (meaning that it is greater than 0). We can now write an inequality
$\cos(\theta) > \mu \sin(\theta)$

$$\text{This simply becomes } \tan(\theta) < \frac{1}{\mu}$$

Now, let's transition to **circular motion and gravitation**. This topic involves Newton's Second Law a LOT.

Note 2.0.17 — Circular Motion
Uniform circular motion happens in a circular path. The object that undergoes uniform circular motion (UCM) must do it at constant speed. The velocity of that object is tangent to the circle it makes. The acceleration (called centripetal acceleration) always points towards the center of the circle, which just changes the direction of the object, NOT the speed.

Note 2.0.18 —
The centripetal acceleration always points inward, as stated in the last note. Thus, there must be a force inwards that is causing this inward acceleration. That force is called the centripetal force, and for an object in circular motion with mass m, velocity v, and radius r, the force is

$$F = \frac{mv^2}{r}.$$

Please note that the centripetal force isn't an actual force! It is CAUSED by other forces such as tension, gravity, normal force, etc. Thus, on a free body diagram, you should NEVER draw centripetal force. Centripetal force is the formal label given to the net force that points towards the center.

Problem 2.0.19 — A ball of mass 5 kilograms is swung in a horizontal circle with a constant speed of $v = 15$ meters per second, and an inward force of 4500N is applied. What is the radius of the circle?

Solution: We have

$$\begin{aligned} F &= \frac{mv^2}{r} \\ 4500 &= \frac{5 \times 15^2}{r} \\ \implies r &= \boxed{4 \text{ m}} \end{aligned}$$

Problem 2.0.20 — When a road is dry, a particular car can safely navigate a turn with a 50m radius of curvature at 20m/s without slipping. What is the coefficient of friction if this is the fastest speed the car can take this turn?

Solution: Surprisingly, we don't need to know the mass of the car for this problem. We have

$$\begin{aligned} F &= \frac{mv^2}{r} \\ \mu mg &= \frac{mv^2}{r} \\ \mu g &= \frac{v^2}{r} = \frac{20^2}{50} = 8 \end{aligned}$$

$$\implies \mu = \frac{8}{g} = \boxed{0.8}.$$

In this problem, it's friction force that caused the car to move in a circle. We found an expression for friction force and equated it to $\frac{mv^2}{r}$. This allowed us to cancel out mass.

Problem 2.0.21 — A stone of mass 3 kgs is wrapped around a rope and is swung in a *vertical* circle with radius 6 meters and with a constant velocity of 10 meters per second. What is the tension in the rope at the highest point in the motion? At the lowest point?

Solution: We will still start with

$$F = \frac{mv^2}{r}.$$

But now, notice that at the highest point, $F_{net} = T + mg$. That means that gravity is also contributing to the centripetal force here, not just the tension of the rope.

$$T + mg = \frac{mv^2}{r} = 50$$

$$\implies T = 50 - mg = \boxed{20 \text{ N}}.$$

Now, if we were instead asked the tension at the lowest point, we would use $F_{net} = T - mg$, since mg is then applying a force away from the center.

To find the speed at the lowest point, you will need to use conservation of energy. Don't worry about finding the speed at the lowest point for this problem right now.

Problem 2.0.22 — You are riding a roller coaster going around a vertical loop, on the inside of the loop. If the loop has a radius of 50m, how fast must the cart be moving in order for you to feel three times as heavy at the top of the loop?

Solution: For the person to feel three times as heavy, we need the normal force from the loop to be 3 times the weight of the person. That is $N = 3mg$. Now, we have

$$F = \frac{mv^2}{r}$$

$$N + mg = \frac{mv^2}{r}$$

$$4mg = \frac{mv^2}{r}$$

$$4g = \frac{v^2}{50}$$

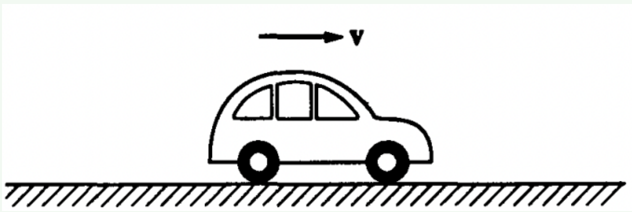
$$\implies v = \boxed{45 \text{ m/s}}$$

Now before we practice more, I want to clarify a few things. There is a common misconception when it comes to uniform circular motion.

As of now, you should know that circular motion can occur both horizontally and vertically. An example of vertical circular motion includes a roller coaster making a 360° turn.

Note 2.0.23 — Vertical circular motion is not uniform circular motion. The reason is that uniform circular motion occurs at constant velocity. However, in vertical circular motion, the velocity is constantly changing due to the force of gravity.

Problem 2.0.24 — 1993 AP Physics C: Mechanics FRQ



A car of mass m, initially at rest at time $t = 0$, is driven to the right, as shown above, along a straight, horizontal road with the engine causing a constant force F_o to be applied. While moving, the car encounters a resistance force equal to $-kv$, where v is the velocity of the car and k is a positive constant.

(a) The dot below represents the center of mass of the car. On this figure, draw and label vectors to represent all the forces acting on the car as it moves with a velocity v to the right.

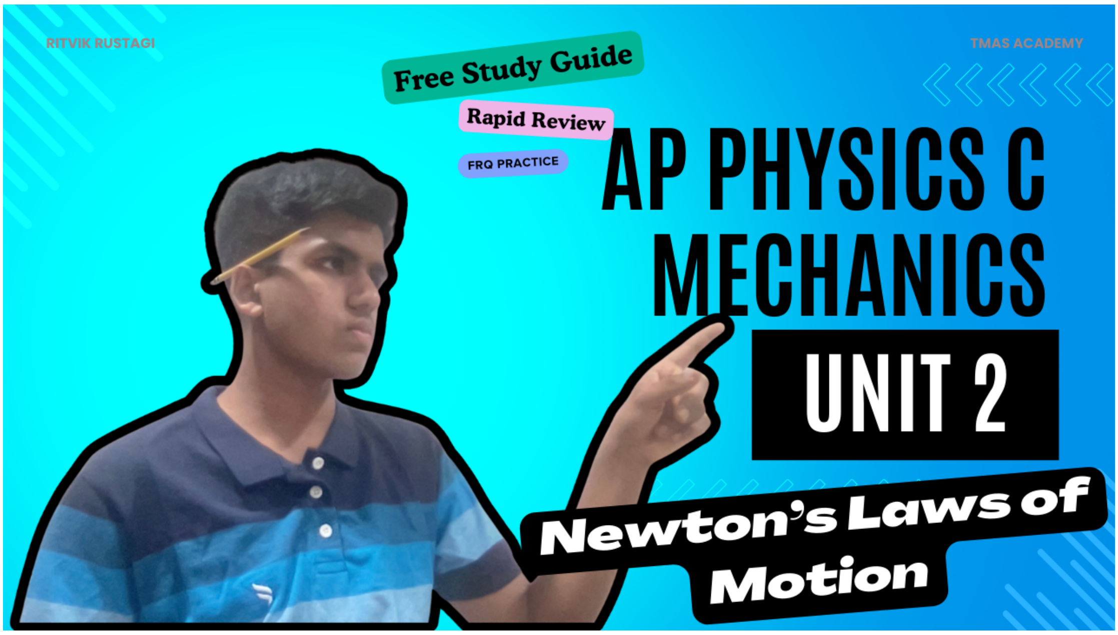
(b) Determine the horizontal acceleration of the car in terms of k, v, F_o, and m.

(c) Derive the equation expressing the velocity of the car as a function of time t in terms of k, v, F_o, and m.

(d) Sketch a graph of the car's velocity v as a function of time t. Label important values on the vertical axis.

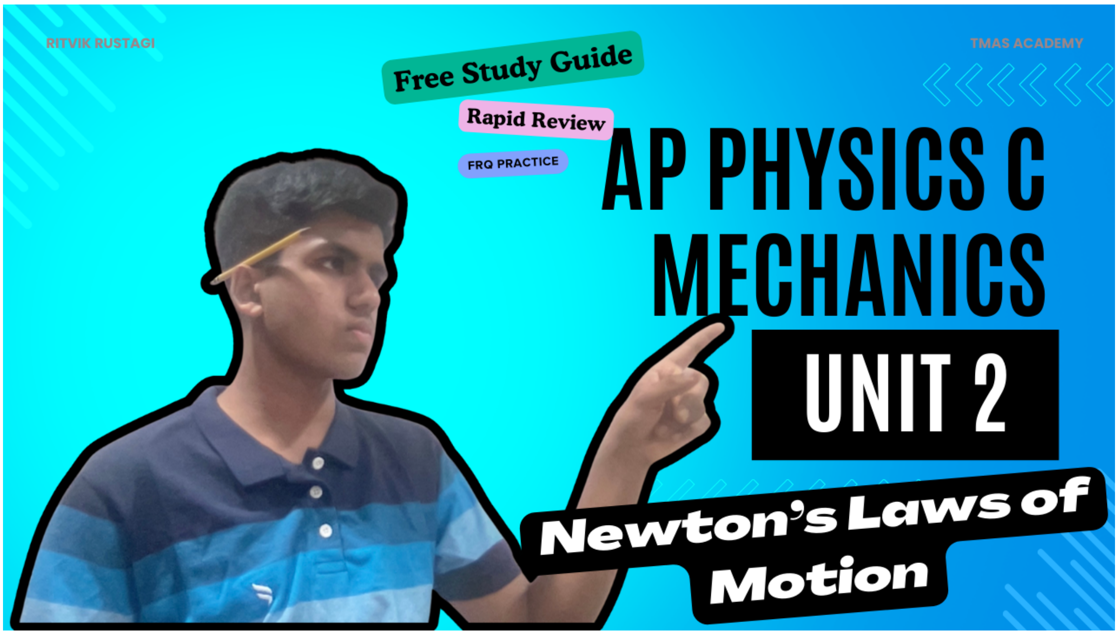
(e) Sketch a graph of the car's acceleration a as a function of time t. Label important values on the vertical axis.

Solution: Video Solution

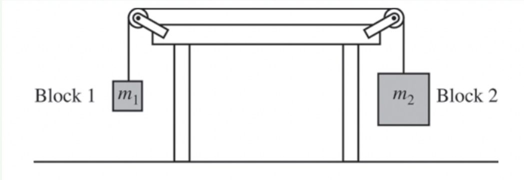


Problem 2.0.25 — 2012 AP Physics C: Mechanics MCQ
The maximum mass that can be hung vertically from a string without breaking the string is 10 kg. A length of this string that is 2 m long is used to rotate a 0.5 kg object in a circle on a frictionless table with the string horizontal. The maximum speed that the mass can attain under these conditions without the tring breaking is most nearly?

Solution: Video Solution



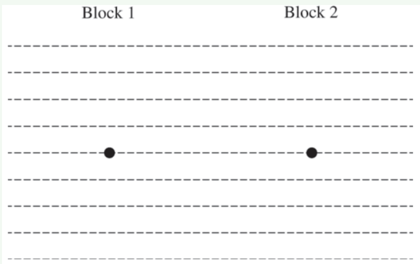
Problem 2.0.26 — 2015 AP Physics 1



Note: Figure not drawn to scale.

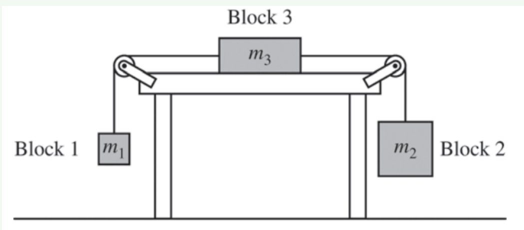
Two blocks are connected by a string of negligible mass that passes over massless pulleys that turn with negligible friction, as shown in the figure above. The mass m_2 of block 2 is greater than the mass m_1 of block 1. The blocks are released from rest.

(a) The dots below represent the two blocks. Draw free-body diagrams showing and labeling the forces (not components) exerted on each block. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces.



(b) Derive the magnitude of the acceleration of block 2. Express your answer in terms of m_1, m_2, and g.

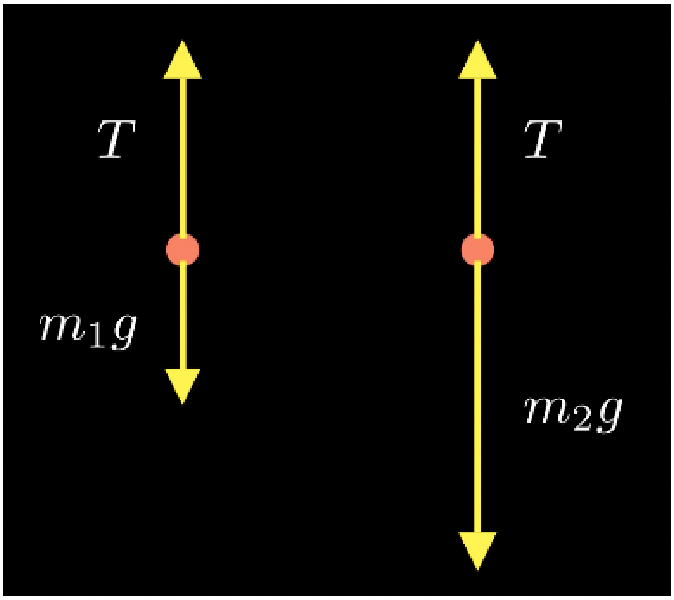
Block 3 of mass m_3 is added to the system, as shown below. There is no friction between block 3 and the table.



Note: Figure not drawn to scale.

(c) Indicate whether the magnitude of the acceleration of block 2 is now larger, smaller, or the same as in the original two-block system. Explain how you arrived at your answer.

Solution to part a: Since m_2 is greater than m_1 in mass, the gravitational force vector for m_2 will be longer than the force vector for m_1. On the other hand, the tension force for both masses will be pointing upwards and will have the same length since tension in a string is the same.



Solution to part b: We can write out an equation using Newton's Second Law ($F = ma$) for each block.

The two blocks will slide towards the direction where the heavier block is.
The equation for mass m_2 is $m_2g - T = m_2a$
Similarly, the equation for block 1 with mass m_1 is $T - m_1g = m_1a$

Now, we can add both of these equations to get

$$g(m_2 - m_1) = (m_1 + m_2)a$$

If we divide both sides by $m_1 + m_2$, then we get that $a = \frac{g(m_2-m_1)}{m_1+m_2}$

Solution to part c: Now, there are technically 2 ropes. One is connecting block 2 and block 3 while the other connects block 1 and block 2.

Now, we can simply consider the forces on the 3 block system as a whole. Tension in both strings will now be an internal force. This means that the net force is simply $g(m_2 - m_1)$
Since we know F_{net}, we simply need to find the mass of this entire system to find acceleration.
The mass of all three blocks combined is simply $m_1 + m_2 + m_3$

$$\text{This means that the acceleration of the 3 block system is } \frac{g(m_2 - m_1)}{m_1 + m_2 + m_3}$$

Since there is an increase of mass in the denominator, the acceleration will now be less on block 2.

Problem 2.0.27 — 1996 AP Physics B FRQ

A 300-kg box rests on a platform attached to a forklift, shown above. Starting from rest at time = 0, the box is lowered with a downward acceleration of 1.5m/s^2

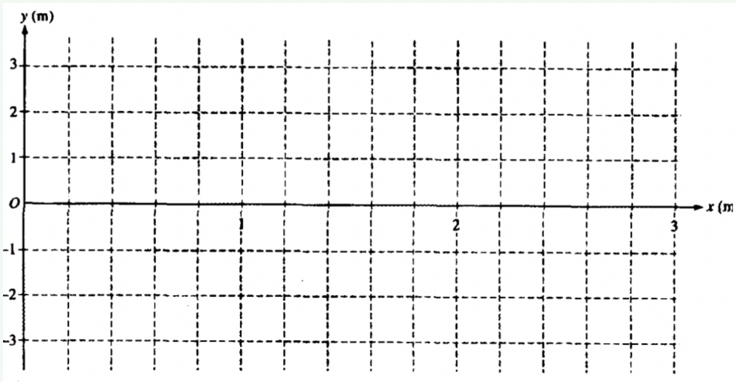
(a) Determine the upward force exerted by the horizontal platform on the box as it is lowered. At time $t = 0$, the forklift also begins to move forward with an acceleration of 2m/s^2 while lowering the box as described above. The box does not slip or tip over.

(b) Determine the frictional force on the box.

(c) Given that the box does not slip, determine the minimum possible coefficient of friction between the box and the platform.

(d) Determine an equation for the path of the box that expresses y as a function of x (and not of t), assuming that, at time $t = 0$, the box has a horizontal position $x = 0$ and a vertical position $y = 2$ m above the ground, with zero velocity.

e. On the axes below sketch the path taken by the box



Solution to part a: The upward force exerted by the horizontal platform is the normal force N.
The only two forces on the box are the gravitational force and normal force. The gravitational force has a larger magnitude since the box accelerates down.

The Newton's Second Law equation is $F_{net} = ma$
We can use this to find $F_{net} = mg - N = ma$
This means that $N = mg - ma = m(g - a)$

In this problem, the downwards direction is taken to be positive.
Since $m = 300$ kg and $a = 1.5\frac{m}{s^2}$, we can find that $N = 300(9.8 - 1.5) = 2490$ N.

Solution to part b: Friction is the only horizontal force on the block. Thus, it must be the force causing it to accelerate with respect to the ground.

This means that $F_f = ma_x$
Since $m = 300$ kg and $a_x = 2\frac{m}{s}$, we know that $F_f = 300 \cdot 2 = 600$ N.

Solution to part c: Since the box doesn't slip, we will use the maximum possible value of static friction. We know that $F_s \leq uN$

We know that $F_s = 600$ from part b.
We can plug in $F_s = 600$ into $F_s = uN$ to find that $uN = 600$

From part a, we already know that $N = 2490$

We can plug this in to find that $u = \frac{600}{2490}$ which is 0.241

Solution to part d: We will first find an equation for position in the x-direction and position in the y-direction with respect to t.
The initial x position is 0. In the x direction, the acceleration is $2\frac{m}{s^2}$ and the initial velocity is 0.

We can use the equation $\Delta x = v_{ix}t + \frac{1}{2}at^2$
We can plug in our variables to find that $\Delta x = \frac{1}{2} \cdot 2 \cdot t^2 = t^2$
We also know that $\Delta x = x_f - x_i$. We already know that $x_i = 0$.
This means $x_f = t^2$
We can take the square root of both sides to find $t = \sqrt{x_f}$

Now we can do something similar for motion in the y-direction.
The acceleration in the y-direction for the block is $-1.5\frac{m}{s^2}$. The initial velocity in the y-direction v_{iy} is 0. The time is $t = \sqrt{x_f}$

We can use the equation $\Delta y = v_{iy}t + \frac{1}{2}at^2$

Plugging in our variables gives $\Delta y = \frac{1}{2} \cdot -1.5 \cdot (\sqrt{x_f})^2$
Solving this gives that $\Delta y = -0.75x_f$
We also know that $\Delta y = y_f - y_i$

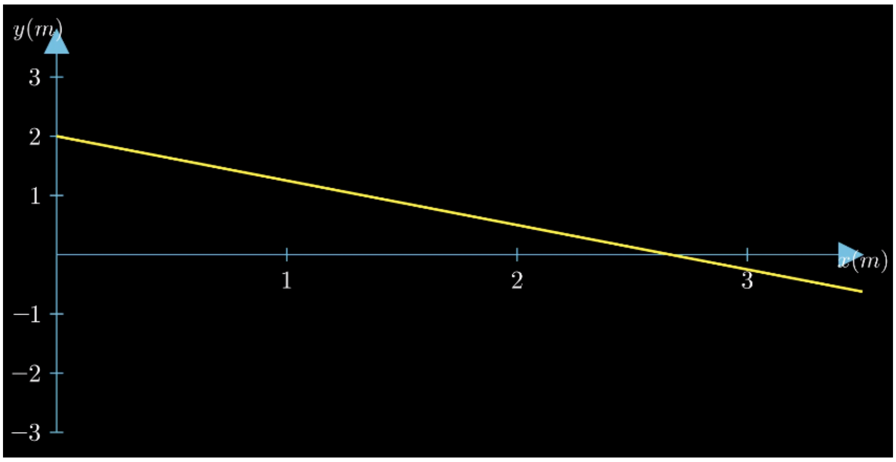
Since $y_i = 2$, we know that $\Delta y = y_f - 2$
We can equate this to $-0.75x_f$ (since we found that $\Delta y = -0.75x_f$ to find that $y_f - 2 = -0.75x_f$

We can add 2 to both sides to get $y_f = -0.75x_f + 2$

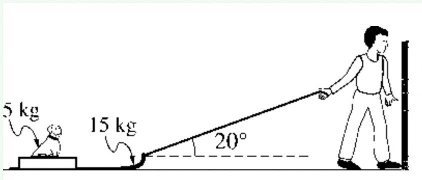
This means the equation of the path of the box in terms of y and x is $y = -0.75x + 2$

Solution to part e: We simply graph $y = -0.75x + 2$ to graph the path.

$y = -0.75x + 2$ is a linear line with y-intercept of 2. The line will have a slope of -0.75

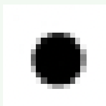


Problem 2.0.28 — 2007 AP Physics 1 FRQ



A child pulls a 15 kg sled containing a 5.0 kg dog along a straight path on a horizontal surface. He exerts a force of 55 N on the sled at an angle of 20° above the horizontal, as shown in the figure. The coefficient of friction between the sled and the surface is 0.22.

a. On the dot below that represents the sled-dog system, draw and label a free-body diagram for the system as it is pulled along the surface.

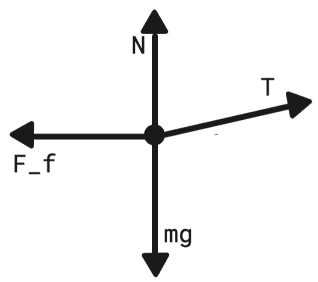


b. Calculate the normal force of the surface on the system.

c. Calculate the acceleration of the system.

d. At some later time, the dog rolls off the side of the sled. The child continues to pull with the same force. On the axes below, sketch a graph of speed v versus time t for the sled. Include both the sled's travel with and without the dog on the sled. Clearly indicate with the symbol t_r the time at which the dog rolls off.

Solution to part a: The forces on the sled-dog system are the tension force, normal force, gravitational force, and friction force.



Solution to part b: We will write Newton's Second Law equations for the forces in the x direction and y direction.

In the y direction, the forces are the normal force, gravitational force, and vertical component of tension.
We must use the equation $F_{net} = ma$ and find the net force.
We get $T\sin(\theta) + N - mg = ma$.
Since acceleration is 0, we get that $T\sin(\theta) + N - mg = 0$
We can rearrange this equation to get that $N = mg - T\sin(\theta)$

We know that $m = 15 + 5 = 20$ kg (since we must sum up the mass of the sled and dog). We also know that $\theta = 20°$ and $T = 55$ N since that is the force the student exerts.
We can plug this in to find that $N = 20 \cdot 9.8 - 55\sin(20) = 177.19$ N.

Solution to part c: There is obviously no acceleration in the y-direction. We must consider the forces in the x-direction to find the acceleration.

In the x direction, the forces are the horizontal component of tension and friction.
Using $F_{net} = ma$, we find that $T\cos(\theta) - \mu N = ma$

We already know that $\theta = 20°$ and $m = 20$ kg. On top of that, we know that the normal force n is 177.19 N from part b. We also know that $\mu = 0.22$. The tension force is 55 N.

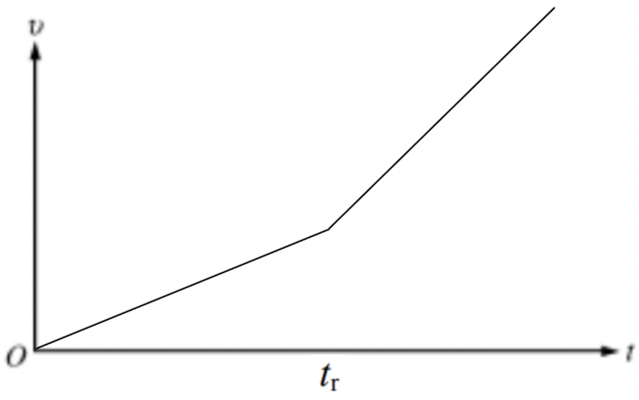
We can plug these variables in to find

$$55\cos(20) - 0.22 \cdot 177.19 = 20 \cdot a$$

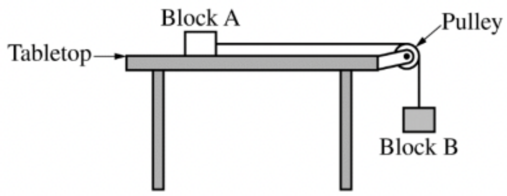
We can solve this equation for a to find that $a = 0.635\frac{m}{s^2}$

Solution to part d: Once the dog rolls off, the acceleration will increase. The reason is that the mass will now be a lot less. Before time t_r, the mass included the dog's mass and sled's mass. However, now it will just include the sled's mass.

The acceleration will rapidly increase as soon as the dog rolls off. It will immediately go up from a lower value. Thus, until time t_r, we will have a linear line. After t_r, we will still have a linear line. However, the linear line will now have a larger slope due to the larger constant acceleration.



Problem 2.0.29 — 2019 AP Physics 1



This problem explores how the relative masses of two blocks affect the acceleration of the blocks. Block A, of mass m_A, rests on a horizontal tabletop. There is negligible friction between block A and the tabletop. Block B, of mass m_B, hangs from a light string that runs over a pulley and attaches to block A, as shown above. The pulley has negligible mass and spins with negligible friction about its axle. The blocks are released from rest.

(a)

i. Suppose the mass of block A is much greater than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.
Briefly explain your reasoning without deriving or using equations.

ii. Suppose the mass of block A is much less than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.
Briefly explain your reasoning without deriving or using equations.

(b) Now suppose neither block's mass is much greater than the other, but that they are not necessarily equal. The dots below represent block A and block B, as indicated by the labels. On each dot, draw and label the forces (not components) exerted on that block after release. Represent each force by a distinct arrow starting on, and pointing away from, the dot.

(c) Derive an equation for the acceleration of the blocks after release in terms of m_A, m_B, and physical constants, as appropriate. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

(d) Consider the scenario from part (a)(ii), where the mass of block A is much less than the mass of block B. Does your equation for the acceleration of the blocks from part (c) agree with your reasoning in part (a)(ii) ?

Yes or **No**

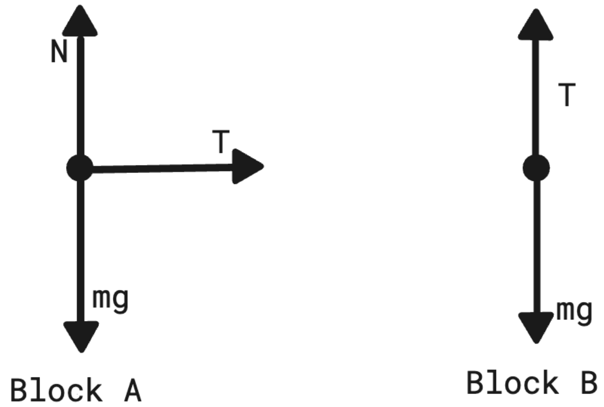
Briefly explain your reasoning by addressing why, according to your equation, the acceleration becomes (or approaches) a certain value when m_A is much less than m_B.

Solution to part a i: The acceleration will be close to 0. The reason is that block B is a lot lighter. It won't be able to pull such a heavy block down due to the fact that block

A has a much greater mass.

Solution to part a ii: The acceleration will be close to g. The reason is that block A is a lot heavier. It will easily be able to pull block B down since block B has a much lower mass.

Solution to part b: The only forces on block B will be tension force and gravitational force. The forces on block A will be tension force, gravitational force, and normal force. Remember that there is **no frictional force** since the problem says that friction is negligible.
Also, the length of the force vectors don't really matter for this problem. We are not told which block might be heavier, so it won't be possible to draw the lengths properly. In such cases, just make the lengths of all force vectors equal.



Solution to part c: We will write an equation using Newton's Second Law for each block.
For block B, the equation is $m_b g - T = ma$
For block A, the equation is $T = m_a a$
Note, for block A we don't need to write an equation for the forces in the y-direction. The reason is that there is no acceleration for block A in that direction when it's on the tabletop. That means the normal force will balance out with block A's weight.

Now, since we know that $m_b g - T = ma$ and $T = m_a a$, we can add both equations to cancel our tension force.
Doing so gives $m_b g = (m_a + m_b)a$
We can divide both sides by $m_a + m_b$ to get that

$$a = \frac{m_b g}{m_a + m_b}$$

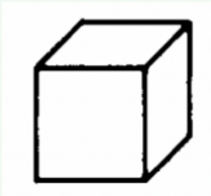
Solution to part d: In part a ii, we said that the acceleration would be close to g when block A's mass would be much less.
Now, we will verify that statement using our equation for acceleration: $a = \frac{m_b g}{m_a + m_b}$

Since block m_a has a very small mass, we can basically say that it is negligible and close to 0.
Plugging in $m_a = 0$ gives $a = \frac{m_b g}{m_b} = g$

This means our answer is **yes**. The equation from part c supports our reasoning for part a ii.

Problem 2.0.30 — 1988 AP Physics B FRQ
A helicopter holding a 70-kilogram package suspended from a rope 5.0 meters long accelerates upward at a rate of 5.2m/s^2. Neglect air resistance on the package.

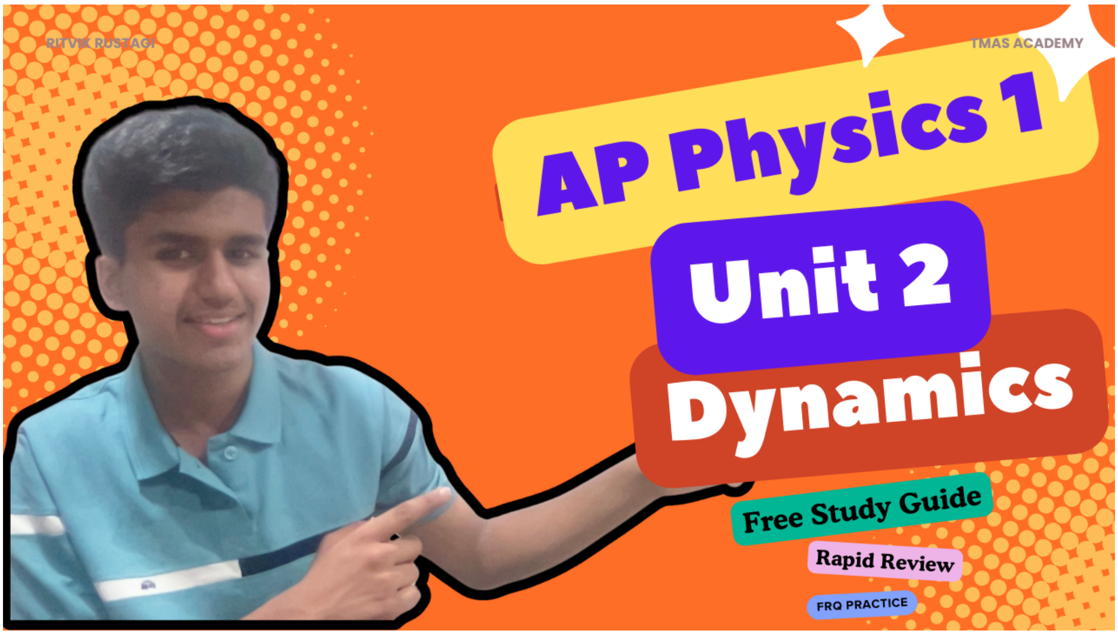
(a) On the diagram below, draw and label all of the forces acting on the package.



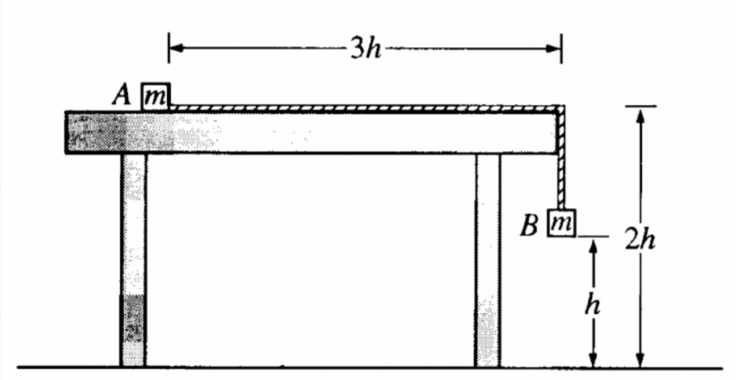
(b) Determine the tension in the rope.

(c) When the upward velocity of the helicopter is 30 meters per second, the rope is cut and the helicopter continues to accelerate upward at 5.2m/s^2. Determine the distance between the helicopter and the package 2.0 seconds after the rope is cut.

Solution: Video Solution



Problem 2.0.31 — 1998 AP Physics B FRQ



Two small blocks, each of mass m, are connected by a string of constant length $4h$ and negligible mass. Block A is placed on a smooth tabletop as shown above, and block B hangs over the edge of the table. The tabletop is a distance $2h$ above the floor. Block B is then released from rest at a distance h above the floor at time $t = 0$. Express all algebraic answers in terms of h, m, and g.

(a) Determine the acceleration of block B as it descends.

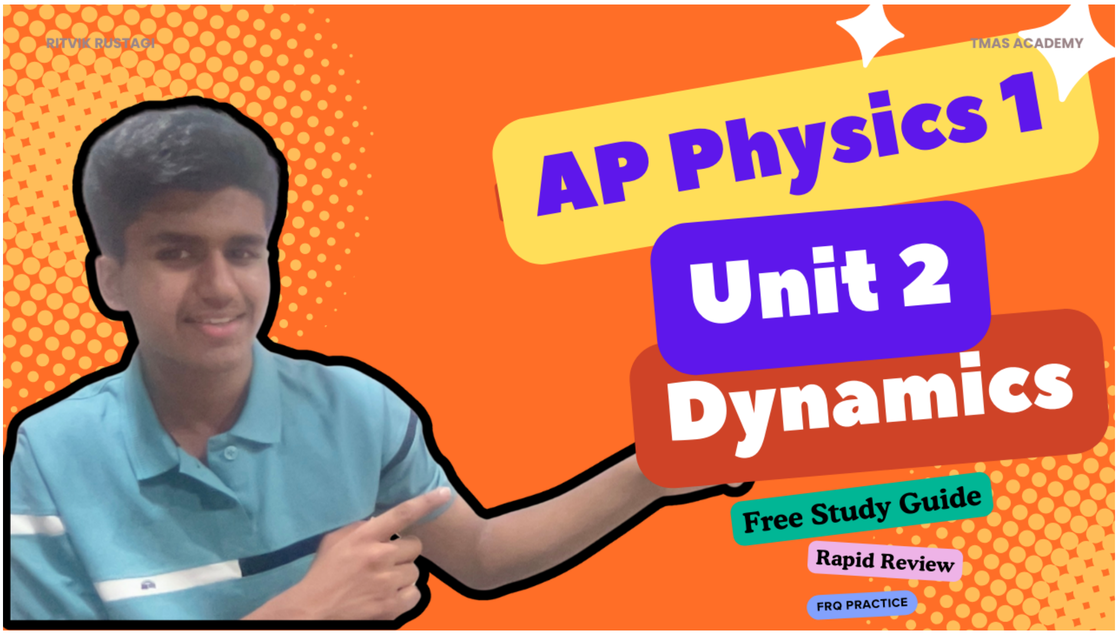
(b) Block B strikes the floor and does not bounce. Determine the time $t = t_1$ at which block B strikes the floor.

(c) Describe the motion of block A from time $t = 0$ to the time when block B strikes the floor.

(d) Describe the motion of block A from the time block B strikes the floor to the time block A leaves the table.

(e) Determine the distance between the landing points of the two blocks

Solution: Video Solution



Problem 2.0.32 — 1977 AP Physics C: Mechanics FRQ
A block of mass m, which has an initial velocity v_o at time $t = 0$, slides on a horizontal surface. If the sliding friction force f exerted on the block by the surface is directly proportional to its velocity (that is, $f = -kv$) determine the following:

(a) The acceleration a of the block in terms of m, k, and v.

(b) The speed v of the block as a function of time t.

(c) The total distance the block slides.

Solution to part a: To find the acceleration, we must find the net force. The reason is that $F_{net} = ma$
The only force in the x-direction is the sliding friction force which is $-kv$
This means that $-kv = ma$
We can divide both sides by m to find that $a = -\frac{kv}{m}$

Solution to part b: To find an expression for speed as a function of time, we must integrate acceleration with respect to time.

$$\text{In part a, we already found that } \frac{dv}{dt} = a(t) = -\frac{kv}{m}$$

$$\text{We can rearrange the equation to get } \frac{dv}{v} = -\frac{k}{m}dt$$

We can integrate both sides to find that $\ln|v| = -\frac{kt}{m} + C$

$$\text{We can cancel the natural log function to get } v = Ce^{-\frac{kt}{m}}$$

Since we know that $v(0) = v_o$ (since it's the initial velocity), we can deduce that the constant $C = v_o$

This means that $v(t) = v_o e^{-\frac{kt}{m}}$

Solution to part c: Now, we must find the distance using the equation we found for velocity in part b.
We know that $v = \frac{dx}{dt}$. This means that $\frac{dx}{dt} = v_o e^{-\frac{kt}{m}}$

We can rearrange this and integrate to get $\int_0^x dx = \int_0^\infty v_o e^{-\frac{kt}{m}} dt$

In the expression above, the tricky part is finding the bounds for integrating with dt. If we wanted to find an expression for position, we would simply integrate velocity from time $t = 0$ to t. However, now we integrate to $t = \infty$ since we want to find the total distance travelled for the entire motion (not the position at a specific point in time).

We can continue integrating with $\int_0^x dx = \int_0^\infty v_o e^{-\frac{kt}{m}} dt$
After integrating and simplifying, we get $x = \frac{mv_o}{k}$
If you struggled to integrate the expression above, make sure to check Ritvik Rustagi's FREE AP Calculus BC book on the TMAS Academy website!

Problem 2.0.33 — 1984 AP Physics C: Mechanics
A small body of mass m located near the Earth's surface falls from rest in the Earth's gravitational field. Acting on the body is a resistive force of magnitude kmv, where k is a constant and v is the speed of the body.

(a) Draw and identify all of the forces acting on the body as it falls.

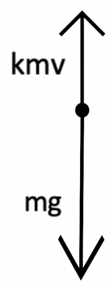
(b) Write the differential equation that represents Newton's second law for this situation.

(c) Determine the terminal speed v_T of the body.

(d) Integrate the differential equation once to obtain an expression for the speed v as a function of time t. Use the condition that $v = 0$ when $t = 0$.

(e) On the axes provided below, draw a graph of the speed v as a function of time t.

Solution to part a: There are only two forces on the mass. There will be gravitational force and the resistive force. Gravitational force points downwards, and the resistive force will oppose the motion.



Solution to part b: Whenever we have a free body diagram, we can write a differential equation by relating acceleration a to $\frac{dv}{dt}$.
We know that $F_{net} = ma$
Our net force is $mg - kmv$

We can equate that to ma and divide both sides by m to find that $a = g - kv$
Since acceleration $a = \frac{dv}{dt}$, our differential equation is $\frac{dv}{dt} = g - kv$

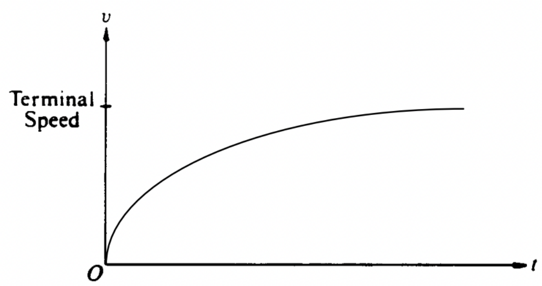
Solution to part c: The terminal speed means that acceleration is 0 at that time. That also means that net force is 0 at that time. This means that $F_{net} = mg - kmv = 0$

We can isolate v to find that $v_T = \frac{g}{k}$ and that is our terminal speed.

Solution to part d: We can rearrange our differential equation to get $\frac{dv}{g-kv} = dt$

$$\text{We can integrate this to get } \int_0^v \frac{dv}{g-kv} = \int_0^t dt$$
$$\text{Integrating gives } -\frac{1}{k}\ln|g-kv||_0^v = t$$
$$\text{This means that } -\frac{1}{k}\ln|g-kv| - (-\frac{1}{k}\ln|g|) = t$$
$$\text{We can rearrange this to get } \ln(\frac{g-kv}{g}) = -kt$$
$$\text{This means that } \frac{g-kv}{g} = e^{-kt}$$
$$\text{We can now multiply both sides by } g \text{ to get } g - kv = ge^{-kt}$$
$$\text{This means that } kv = g - ge^{-kt}$$
$$\text{We can solve for } v \text{ to find that } v = \frac{g(1-e^{-kt})}{k}$$

Solution to part e: Our equation for velocity is a common exponential growth graph. The rate at which velocity changes decreases over time since it's a negative decay (due to the negative exponent for e).



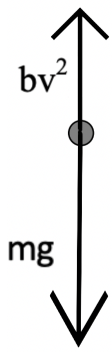
Problem 2.0.34 — 2000 AP Physics C: Mechanics FRQ
A rubber ball of mass m is dropped from a cliff. As the ball falls, it is subject to air drag (a resistive force caused by the air). The drag force on the ball has magnitude bv^2, where b is a constant drag coefficient and v is the instantaneous speed of the ball. The drag coefficient b is directly proportional to the cross-sectional area of the ball and the density of the air and does not depend on the mass of the ball. As the ball falls, its speed approaches a constant value called the terminal speed.

(a) Draw and label all the forces on the ball at some instant before it reaches terminal speed.

(b) State whether the magnitude of the acceleration of the ball of mass m increases, decreases, or remains the same as the ball approaches terminal speed. Explain.
(c) Write, but do NOT solve, a differential equation for the instantaneous speed v of the ball in terms of time t, the given quantities, and fundamental constants.

(d) Determine the terminal speed v_t in terms of the given quantities and fundamental constants.

Solution to part a: There are only two forces on the object. One is the gravitational force which is pointing downward while the other is the drag force which opposes the gravitational force.



Solution to part b: We know that the ball's speed slowly increases. As the speed increases, the magnitude of the drag force goes up. This means that the net force (which is $mg - bv^2$) will go down. Since the net force decreases, the acceleration also decreases as velocity approaches it's terminal value.

Solution to part c: As explained before, to write our differential equation using a free-body diagram, we must find the acceleration using the net force. We can combine that with the fact that the derivative of velocity with respect to time is acceleration.

The net force $F_{net} = mg - bv^2$
We know that $F_{net} = ma$
This means that $a = g - \frac{bv^2}{m}$

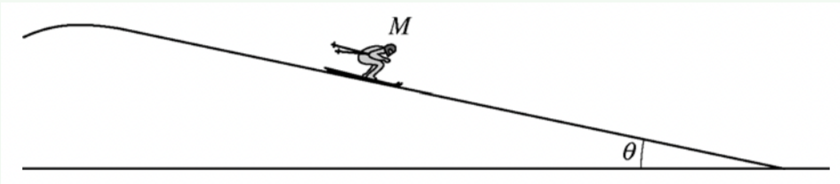
$$\text{Since } a = \frac{dv}{dt}, \text{ we know that } \frac{dv}{dt} = g - \frac{bv^2}{m}$$

Solution to part d: Many people make the mistake of integrating our differential equation to find the terminal velocity. That is not necessary in this situation.

When the object reaches terminal velocity, the net force will be 0. This means that $mg - bv^2 = 0$

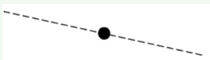
We can solve this to find that $v_t^2 = \dfrac{mg}{b}$ which means that $v_t = \sqrt{\dfrac{mg}{b}}$

Problem 2.0.35 — 2008 AP Physics C: Mechanics FRQ



A skier of mass M is skiing down a frictionless hill that makes an angle θ with the horizontal, as shown in the diagram. The skier starts from rest at time $t = 0$ and is subject to a velocity-dependent drag force due to air resistance of the form $F_b = -b \cdot v$, where v is the velocity of the skier and b is a positive constant. Express all algebraic answers in terms of M, θ, b, and fundamental constants.

(a) On the dot below that represents the skier, draw a free-body diagram indicating and labeling all of the forces that act on the skier while the skier descends the hill.

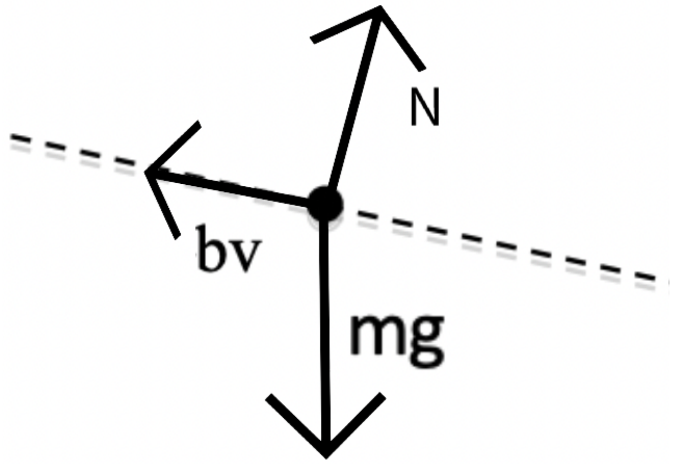


(b) Write a differential equation that can be used to solve for the velocity of the skier as a function of time.

(c) Determine an expression for the terminal velocity v_T of the skier.

(d) Solve the differential equation in part (b) to determine the velocity of the skier as a function of time, showing all your steps.

Solution to part a: There are only three forces on the skier. One is gravitational force which points downwards. There is also the drag force which opposes the skier's motion. The last force is normal force which points perpendicular to the plane.



Solution to part b: To write a differential equation using our free-body diagram, we must know that $a = \frac{dv}{dt}$ (where a represents acceleration)

We find the net force in the direction along the frictionless hill. We must take the component of the gravitational force along the hill. It is $mg\sin(\theta)$. The force opposing this is the resistive force which is bv.

This means that $F_{net} = mg\sin(\theta) - bv = ma$
We can solve for a to get $a = g\sin(\theta) - \frac{bv}{m}$
Since $a = \frac{dv}{dt}$, we know that

$$\frac{dv}{dt} = g\sin(\theta) - \frac{bv}{m}$$

Solution to part c: We know that when the skier reaches terminal velocity, the net force on it is 0. This means that $F_{net} = mg\sin(\theta) - bv = 0$

We can solve the equation for v to find that $v_T = \frac{mg\sin(\theta)}{b}$

Solution to part d: We know that our differential equation is $\frac{dv}{dt} = g\sin(\theta) - \frac{bv}{m}$

$$\text{We can rearrange this to get } \frac{dv}{Mg\sin(\theta) - bv} = \frac{dt}{M}$$

$$\text{Now we can integrate this to get } \int_0^v \frac{dv}{Mg\sin(\theta) - bv} = \int_0^t \frac{dt}{M}$$

$$\text{Integrating gives } -\frac{1}{b}\ln|Mg\sin(\theta) - bv||_0^v = \frac{t}{M}$$

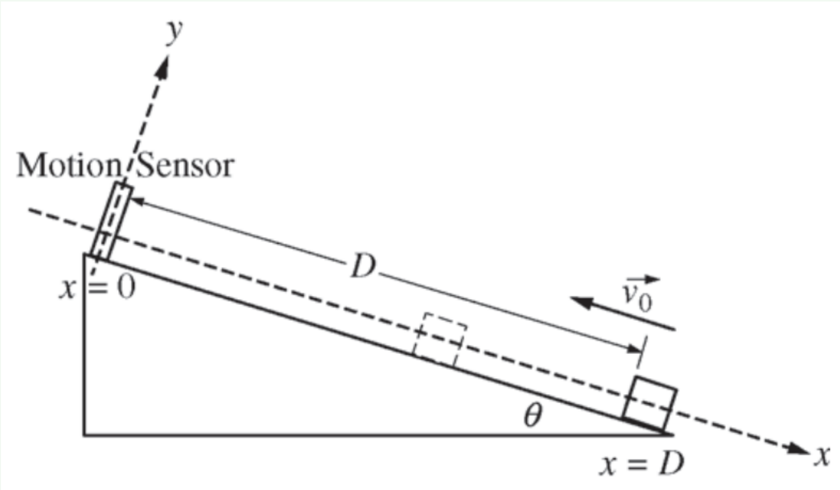
$$\text{We can simplify this to get } \ln\left(\frac{Mg\sin(\theta) - bv}{Mg\sin(\theta)}\right) = -\frac{bt}{M}$$

$$\text{Simplifying further gives } \frac{Mg\sin(\theta) - bv}{Mg\sin(\theta)} = e^{-\frac{bt}{M}}$$

After rearranging and solving for v we can find that $v = \frac{Mg\sin(\theta)}{b}(1 - e^{-\frac{bt}{M}})$

Part d of this problem requires a lot of tedious calculus. This is rare to see on the AP Physics C Mechanics exam now as it has more of a focus on problem solving. However, the types of integration problems seen on the exam is quite repetitive. If you carefully analyze the few problems we solved, then you should be able to attack similar ones on the exam.

Problem 2.0.36 — 2015 AP Physics C Mechanics FRQ



A block of mass m is projected up from the bottom of an inclined ramp with an initial velocity of magnitude v_0. The ramp has negligible friction and makes an angle θ with the horizontal. A motion sensor aimed down the ramp is mounted at the top of the incline so that the positive direction is down the ramp. The block starts a distance D from the motion sensor, as shown above. The block slides partway up the ramp, stops before reaching the sensor, and then slides back down.

(a) Consider the motion of the block at some time t after it has been projected up the ramp. Express your answers in terms of m, D, v_0, t, θ, and physical constants, as appropriate.

i. Determine the acceleration a of the block.

ii. Determine an expression for the velocity v of the block.

iii. Determine an expression for the position x of the block.

(b) Derive an expression for the position x_{min} of the block when it is closest to the motion sensor. Express your answer in terms of m, D, v_0, θ, and physical constants, as appropriate.

Solution to part a i: The acceleration occurs in the direction along the ramp. The only force on it in that direction is one component of gravitational force. The component of gravitational force along the ramp is $-mg\sin(\theta)$. This force points in the opposite direction of the object's velocity (note that down the ramp is defined to be the negative direction).

The acceleration can be found by using the equation $F_{net} = ma$
Since $F_{net} = -mg\sin(\theta)$, we know that $a = -g\sin(\theta)$

Solution to part a ii: We know that velocity is the integral of acceleration with respect to time
$v = \int adt$
Since $a = -g\sin(\theta)$, we know that $v = \int -g\sin(\theta)dt$
This means that $v(t) = -gt\sin(\theta) + C$
Since we know that $v(0) = v_0$ (since it's the initial speed), our constant $C = v_0$

That means our expression for velocity is $v(t) = v_0 - gt\sin(\theta)$

Solution to part a iii: We know that the position of the object is the integral of velocity with respect to time.
$x = \int vdt$

$$\text{Since } v(t) = v_0 - gt\sin(\theta), \text{ we know that } x(t) = \int v(t)dt$$

$$\text{This means that } x(t) = v_0 t - \frac{gt^2\sin(\theta)}{2} + C$$

Since we know that the initial position is D, $x(0) = -D$ (based on our reference). This means that our constant $C = -D$.

That means $x(t) = -D + v_0 t - \frac{gt^2\sin(\theta)}{2}$

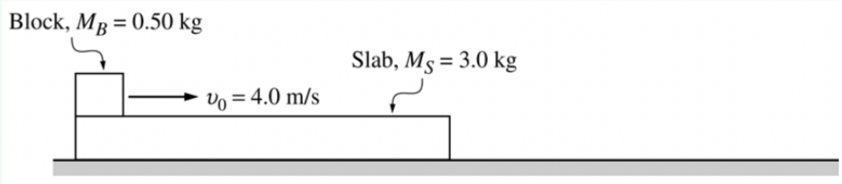
Solution to part b: We will use the kinematics equation $v^2 = v_i^2 + 2ad$
Our distance travelled is $D - x$.
The final velocity will be 0 as it approaches the sensor.

This means that $0 = v_i^2 + 2a(D - x)$
We also know that $a = -g\sin(\theta)$
This means that $v_i^2 = 2g(D - x)\sin(\theta)$

The equation simplifies to $D - x = \frac{v_i^2}{2g\sin(\theta)}$

$$\text{This means that } x_{min} = D - \frac{v_i^2}{2g\sin(\theta)}$$

Problem 2.0.37 — 2006 AP Physics C Mechanics FRQ



A small block of mass $M_B = 0.50$ kg is placed on a long slab of mass $M_S = 3.0$ kg as shown above. Initially, the slab is at rest and the block has a speed v_0 of $4.0m/s$ to the right. The coefficient of kinetic friction between the block and the slab is 0.20, and there is no friction between the slab and the horizontal surface on which it moves.

(a) On the dots below that represent the block and the slab, draw and label vectors to represent the forces acting on each as the block slides on the slab.

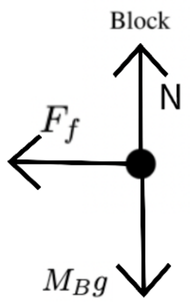
At some moment later, before the block reaches the right end of the slab, both the block and the slab attain identical speeds v_f.

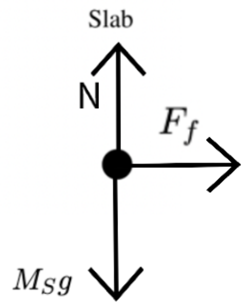
(b) Calculate v_f.

(c) Calculate the distance the slab has traveled at the moment it reaches v_f.

Solution to part a: Both the block and slab will face a gravitational and normal force. The normal force on the block will come from the slab. There will also be a reaction of this normal force that will apply a force on the slab.

There will be a friction force pointing to the left for the block. It will try to slow the block down. There will also be a friction force on the slab (due to the slab and block being in contact). The slab's friction force will point towards the right.





Solution to part b: We must write an equation for the velocities of both the block and the slab overtime.

To do this, we must find their accelerations. We will work with the block first.

The block has an initial velocity of $v_0 = 4$. It has a force of $F_f = -\mu N = \mu M_B g$ on it.

From Newton's Second Law, we know that $F_{net} = ma$. For the block, this means that $-\mu M_B g = M_B a$
This means that $a = -\mu g$.
We can use the kinematics equation $v_f = v_0 + at$
Plugging in our variables gives that $v_f = v_0 - \mu gt$ (this is our expression for velocity of the block over time).

The slab will face the same frictional force of $F_f \mu M_B g$. The only difference is that it will point in the opposite direction.
This means that $\mu M_B g = M_S a$ (due to Newton's Second Law).
We can solve for acceleration to get $a = \frac{\mu M_B g}{M_S}$
We can again use the equation $v_f = v_0 + at$. This time, we will use it for the slab.
The slab's initial velocity is 0. We can substitute its acceleration to get $v_f = \frac{\mu M_B gt}{M_S}$

Now, we have an expression for the velocity for both the block and slab overtime. We can set them both equal to each other (since the problem sates that they attain identical speeds before the block reaches the right end of the lab).

$$\text{Doing so gives: } v_0 - \mu gt = \frac{\mu M_B gt}{M_S}$$

We will now solve for the time at which both have the same common speed. We can plug in all of our variables.

Doing so gives $4 - 0.2 \cdot g \cdot t = \frac{0.2 \cdot 0.5 \cdot g \cdot t}{3}$

This simplifies to $4 - 1.96t = 0.33t$

We can solve for t to get $t = 1.75$ s.

Now, we can plug this back into one of our equations for the final velocity. We will use $v_f = v_0 - \mu gt$.
Plugging in all of our variables gives $v_f = 4 - 0.2 \cdot g \cdot 1.75 = 0.57 m/s$.

Solution to part c: We will use another kinematics equation to find the distance travelled.
We know the slab's initial and final velocity. We also know its acceleration.
This means we can apply the kinematics equation $v_f^2 = v_0^2 + 2a\Delta x$

We can rearrange the equation to get $v_f^2 - v_0^2 = 2a\Delta x$
Now, we divide both sides by $2a$ to get $\Delta x = \frac{v_f^2 - v_0^2}{2a}$

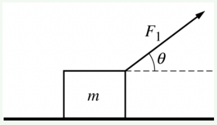
$$\text{The slab's acceleration was found to be } \frac{\mu M_B g}{M_S} = \frac{0.2 \cdot 0.5 \cdot 9.8}{3} = 0.33$$

We also know that its initial velocity is 0 while it's final velocity is $0.57 m/s$.

$$\text{We can plug all of this into } \Delta x = \frac{v_f^2 - v_0^2}{2a}$$

$$\text{Doing so gives } \Delta x = \frac{0.57^2 - 0^2}{2 \cdot 0.33} = 0.49 \text{ m}$$

Problem 2.0.38 — 2007 AP Physics C Mechanics FRQ



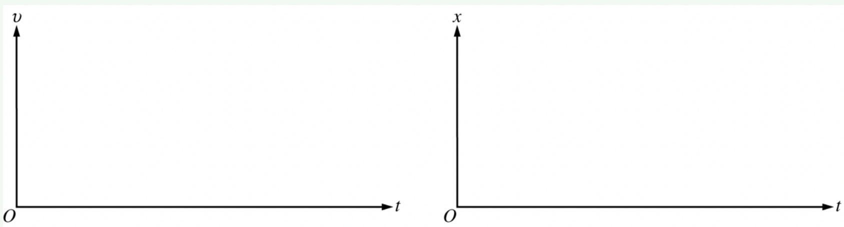
A block of mass m is pulled along a rough horizontal surface by a constant applied force of magnitude F_1 that acts at an angle θ to the horizontal, as indicated above. The acceleration of the block is a_1. Express all algebraic answers in terms of m, F_1, θ, a_1, and fundamental constants.

(a) Draw and label a free-body diagram showing all the forces on the block.

(b) Derive an expression for the normal force exerted by the urface on the block.

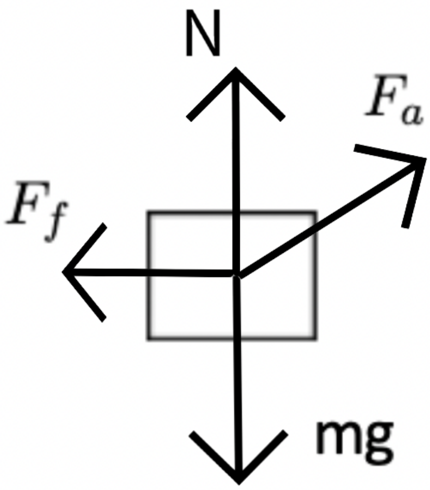
(c) Derive an expression for the coefficient of kinetic friction μ between the block and the surface.

(d) On the axes below, sketch graphs of the peed v and displacement x of the block as functions of time t if the block started from rest at $x = 0$ and $t = 0$.



(e) If the applied force is large enough, the block will lose contact with the surface. Derive an expression for the magnitude of the greatest acceleration a_{max} that the block can have and still maintain contact with the ground.

Solution to part a: There will obviously be gravitational force pointing downwards and normal force pointing upwards. On top of that, we will have the applied force. There will also be a frictional force opposing the block's motion.



Solution to part b: We will write an equation for the forces in the y-direction.
$F_y : N + F_1 \sin(\theta) - mg = 0$
We can solve this equation to find that $N = mg - F_1 \sin(\theta)$ which is our normal force.

Solution to part c: We already wrote an equation for Newton's Second Law in the y-direction. We will do something similar but in the x direction.

$$F_x : F_1 \cos(\theta) - \mu N = ma$$

Since we know that $N = mg - F_1 \sin(\theta)$, we can plug that in.

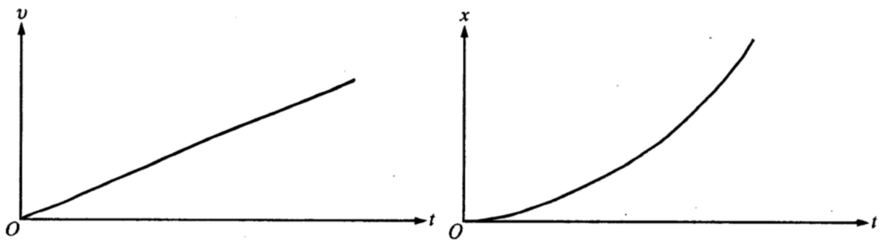
Doing so gives that $F_1 \cos(\theta) - \mu(mg - F_1 \sin(\theta)) = ma_1$

Now, we can rearrange the equation to get $F_1 \cos(\theta) - ma_1 = \mu(mg - F_1 \sin(\theta))$

We can divide both sides by $mg - F_1 \sin(\theta)$to find that $\mu = \dfrac{F_1 \cos(\theta) - ma_1}{mg - F_1 \sin(\theta)}$

Solution to part d: The acceleration of the block is constant and is a_1. This means that the velocity increases at a constant rate. The line for the velocity-time graph should be linear.

However, this means that our graph for position and time will have a parabolic shape.



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Solution to part e: When we are asked for such limiting value problems (for example to maximize acceleration), then our normal force must be 0.

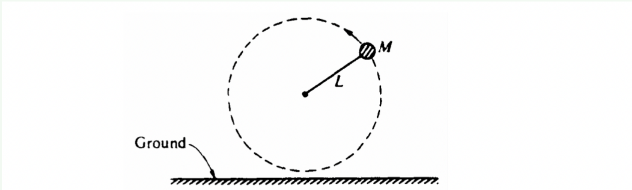
We will again use our two Newton's Second Law equations (one for the x-direction and the other for the y-direction.

$F_x : F_1 \cos(\theta) - \mu N = ma$
$F_y : N + F_1 \sin(\theta) - mg = 0$

We plug in 0 for N to find that our two equations are $F_1 \cos(\theta) = ma$ and $F_1 \sin(\theta) = mg$
The second equation can be rearranged to find that $F_1 = \frac{mg}{\sin(\theta)}$. We can plug this into the first equation to get $\frac{mg}{\sin(\theta)} \cdot \cos(\theta) = ma$

This simplifies to $\frac{mg}{\tan(\theta)} = ma$
We can simplify the equation to find that $a = \frac{g}{\tan(\theta)}$. This is our maximum value of acceleration.

Problem 2.0.39 — 1984 AP Physics B FRQ



A ball of mass M attached to a string of length L moves in a circle in a vertical plane as shown above. At the top of the circular path, the tension in the string is twice the weight of the ball. At the bottom, the ball just clears the ground. Air resistance is negligible. Express all answers in terms of M, L, and g.

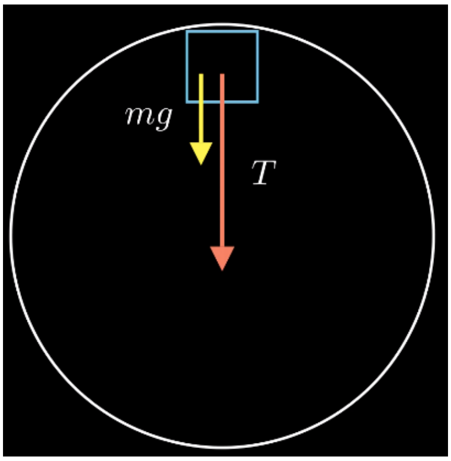
(a) Determine the magnitude and direction of the net force on the ball when it is at the top.

(b) Determine the speed v_o of the ball at the top.

The string is then cut when the ball is at the top.
(c) Determine the time it takes the ball to reach the ground.

(d) Determine the horizontal distance the ball travels before hitting the ground.

Solution to part a: At the top, the net force of the ball points downward since that is also where the centripetal acceleration points (towards the center).
The magnitude of it can be found by drawing a free body diagram first to show all the forces on that ball.



Using the free body diagram, we know that the net force is $T + mg$. Since it's given that the tension at the top is twice the weight, it means $T = 2mg$. We can plug that in to get that the net force is $3mg$ (downwards towards the center).

Solution to part b: We know that the net force pointing towards the center will be causing the centripetal motion.

Since we know our net force as $3mg$, we can equate that to $\frac{mv^2}{r}$

$$3mg = \frac{mv^2}{r}(m \text{ cancels out})$$

$$3g = \frac{v^2}{L}(\text{our radius is equal to } L, \text{ the length of the string})$$

$$3gl = v^2$$

$$v = \sqrt{3gl}$$

Solution to part c: Once the string is cut at the top, the ball will undergo projectile motion. There is an initial horizontal velocity which remains constant for the whole time, and the initial vertical velocity is 0. We can write out all of our variables.

$v_{ox} = \sqrt{3gl}$	$v_{oy} = 0$
$v_x = \sqrt{3gl}$	$v_{oy} =?$
$a_x = 0$	$a_y = g$
$t = ?$	$t= ?$
$\Delta x = ?$	$\Delta y = 2L$

To find the time, we can simply use our variables in the y direction. We will use our kinematics equation that is

$$\Delta y = v_{oy}t + \frac{a_y t^2}{2}$$

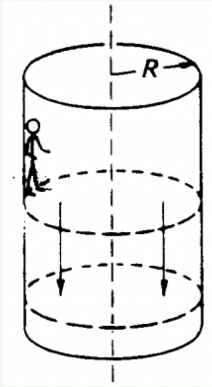
We manipulate this equation to isolate t, and this gives us $t^2 = \frac{2\Delta y}{a_y}$

$$\text{This simplifies to } t^2 = \frac{2 \cdot 2L}{g} \text{ which means } t \text{ is } \sqrt{\frac{4L}{g}}$$

Solution to part d: Since now we know the time and the fact that the horizontal velocity is constant (just like how it is in projectile motion problems that we've solved), we simply multiply the horizontal velocity to the time we found in part C.

$$d = \sqrt{\frac{4L}{g}} \cdot \sqrt{3gL} = 2\sqrt{3L}$$

Problem 2.0.40 — 1984 AP Physics C: Mechanics FRQ



An amusement park ride consists of a rotating vertical cylinder with rough canvas walls. The floor is initially about halfway up the cylinder wall as shown above. After the rider has entered and the cylinder is rotating sufficiently fast, the floor is dropped down, yet the rider does not slide down. The rider has mass of 50 kilograms, the radius R of the cylinder is 5 meters, the angular velocity of the cylinder when rotating is 2 radians per second, and the coefficient of static friction between the rider and the wall of the cylinder is 0.6.

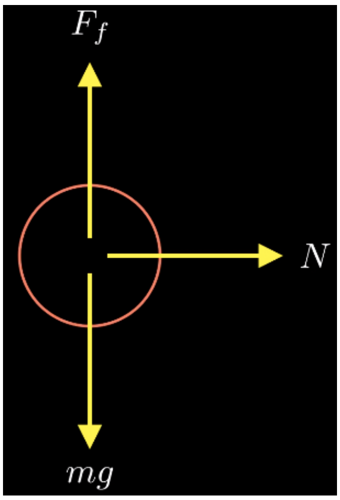
(a) Draw and identify the forces on the rider when the system is rotating and the floor has dropped down.

(b) Calculate the centripetal force on the rider when the cylinder is rotating and state what provides that force.

(c) Calculate the upward force that keeps the rider from falling when the floor is dropped down and state what provides that force.

(d) At the same rotational speed, would a rider of twice the mass slide down the wall? Explain your answer.

Solution to a part a: The forces present are N (normal force), F_f (frictional force), and F_g or mg (gravitational force)



Solution to part b: The centripetal force is clearly caused by the normal force. It is the only force towards the center.

Since we know that the centripetal force equals to $\frac{mv^2}{r}$, we can directly find that value by plugging in our variables.

For that, we need to find v. This one step might be confusing for many of you. It involves a topic that we will learn in a future unit. The relationship in this problem between velocity and angular velocity is $v = \omega r$. Thus, since $\omega = 2$ and $r = 5$, we can find that $v = 2 \cdot 5 = 10$.

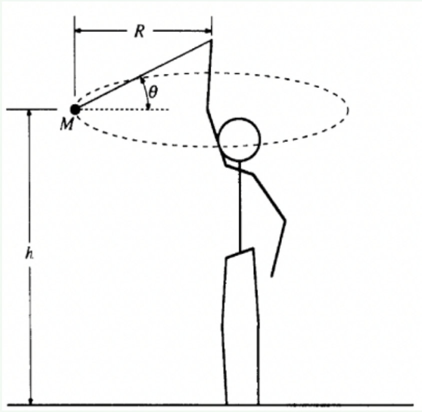
Now we can plug this into $\frac{mv^2}{r}$ to get $\frac{50 \cdot 10^2}{5} = 1000$ N.

Solution to part c: The upward force that prevents the rider from falling is friction force. We know from our free body diagram that $F_f - mg = 0$ since the net force is 0 in the y direction.
We know that $mg = 50 \cdot 9.8 = 490$. This means that the upward friction force will also be 490 N.

Solution to part d: Even if the mass would double, that would mean that the centripetal force would double. The reason is that the radius and rotational speed are both the same. Thus, the centripetal force would simply double in magnitude.

We already found that the force causing the centripetal force would be the normal force since it points towards the center. Since the centripetal force doubles, we know that the normal force doubles. Since friction is proportional to the normal force (because $F_f = \mu N$), the maximum value of friction would also double along with the gravitational force exerted on this heavier mass. However, the mas will cancel out and we're left with the same scenario. Thus, the rider would not slide down even if the mass would double.

Problem 2.0.41 — 1989 AP Physics B FRQ



An object of mass M on a string is whirled with increasing speed in a horizontal circle, as shown above. When the string breaks, the object has speed v_0, and the circular path has radius R and is a height h above the ground. Neglect air friction.

a. Determine the following, expressing all answers in terms of h, v_o, and g.

i. The time required for the object to hit the ground after the string breaks

ii. The horizontal distance the object travels from the time the string breaks until it hits the ground

iii. The speed of the object just before it hits the ground

b. On the figure below, draw and label all the forces acting on the object when it is in the position shown in the diagram above.

c. Determine the tension in the string just before the string breaks. Express your answer in terms of M, R, v_o, and g.

Solution to part a i: After the string breaks, v_{oy} (initial velocity in y-direction) is 0. Also, the distance travelled is simply h (in the y-direction), and the acceleration is g.

We can apply the equation $y = v_{oy}t + \frac{1}{2}a_y t^2$. Plugging in our values gives
$h = \frac{gt^2}{2}$
Solving for t gives $\sqrt{\frac{2h}{g}}$

Solution to part a ii: When the string breaks, the object has speed v_o. This means that is our initial velocity in the x direction. There is also no acceleration in the x-direction which means that this velocity is constant in the x-direction.

Thus, we can just apply the equation $d = vt$ which can also be written as $\Delta x = v_x t$
Plugging in our values gives $\Delta x = v_o t$.

We can substitute our value of t found from part a i.

$$\text{This gives that the horizontal distance travelled is } v_o\sqrt{\frac{2h}{g}}$$

Solution to part a iii: The total velocity of any object is $\sqrt{v_x^2 + v_y^2}$. As long as we know both components of velocity, we can find the total velocity.

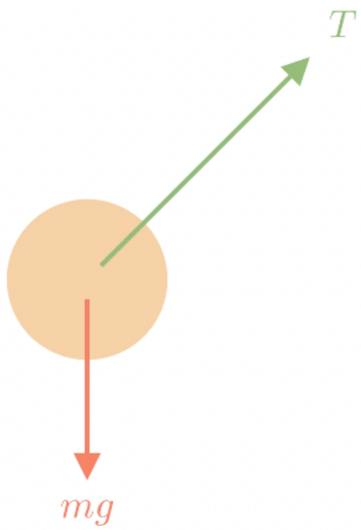
We already know that before the ball hits the ground its velocity in the x direction will simply be v_o.

We can find velocity in the y direction through our kinematics equations.
Since v_{oy} is 0, $a_y = g$, and $t = \sqrt{\frac{2h}{g}}$, we can apply the equation $v_y = v_{oy} + a_y t$

This gives that $v_y = 0 + g\sqrt{\frac{2h}{g}}$ which can be simplified to $v_y = \sqrt{2gh}$.

Since $v_x = v_o$ and $v_y = \sqrt{2gh}$ (our two components of velocity before the object hits the ground), we can plug this into $v = \sqrt{v_x^2 + v_y^2}$ to get that $v = \sqrt{v_o^2 + 2gh}$.

Solution to part b: When the object is in the position shown, the only forces on it are tension and gravity.



Tension simply points along the rope and gravity points downward.

Solution to part c: Right before the string breaks, the object is moving in circular motion.
We can write out the forces in the x and y direction.

The only force in the x direction is the horizontal component of the tension force. This component of tension is what provides the centripetal force. In the y direction, we have the force of gravity and the vertical component of tension. Both equate to each other as there is no motion in the y-direction.

$$F_x : T\cos\theta = Ma = \frac{Mv_o^2}{R}$$

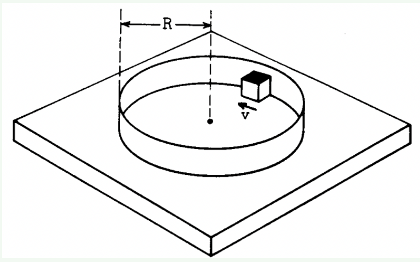
$$F_y : T \sin\theta - Mg = Ma = 0$$

Clearly from the summation of forces in the y-direction, $T\sin\theta = Mg$.
Since we have an expression for both the horizontal and vertical component of the tension force, the total tension force (T) = $\sqrt{(T\cos\theta)^2 + (T\sin\theta)^2}$.

Plugging in our expressions gives that tension is $\sqrt{\frac{M^2 v_o^4}{R^2} + M^2 g^2}$

We can factor out M^2 to get that the tension force is $M\sqrt{\frac{v_o^4}{R^2} + g^2}$

Problem 2.0.42 — 1976 AP Physics C: Mechanics FRQ



A small block of mass m slides on a horizontal frictionless surface as it travels around the inside of a hoop of radius R. The coefficient of friction between the block and the wall is μ; therefore, the speed v of the block decreases. In terms of m, R, μ, and v, find expressions for each of the following.

(a) The frictional force on the block.

(b) The block's tangential acceleration dv/dt

(c) The time required to reduce the speed of the block from an initial value v_0 to $v_0/3$.

Solution to part a: We know that $F_f = \mu N$, where F_f represents frictional force and N represents normal force.
That means we need to find normal force to be able to find frictional force.

Normal force is what causes the block to move in a circle. The normal force drives the centripetal motion. This means that $N = \frac{mv^2}{R}$

We can plug in $N = \frac{mv^2}{R}$ into $F_f = \mu N$ to find that the frictional force F_f is $\frac{\mu m v^2}{R}$

Solution to part b: The frictional force is what drives the tangential acceleration. It is the only force in the tangential direction.

This means that $-F_f = ma$
We can plug in $F_f = \frac{\mu m v^2}{R}$ to get that $-\frac{\mu m v^2}{R} = ma$

We can cancel out the m to find that the acceleration a is $-\frac{\mu v^2}{R}$.

$$\text{Also, } \frac{dv}{dt} \text{ is the same thing as the tangential acceleration so } \frac{dv}{dt} = -\frac{\mu v^2}{R}$$

Solution to part c: Since we know that $\frac{dv}{dt} = -\frac{\mu v^2}{R}$, we can rearrange the equation to integrate it. That will allow us to relate time to velocity.

Rearranging it gives $\frac{dv}{v^2} = -\frac{\mu}{R}dt$

$$\text{Now we can integrate both sides to get } \int_{v_0}^{\frac{v_0}{3}} \frac{dv}{v^2} = \int_0^T -\frac{\mu}{R}dt$$

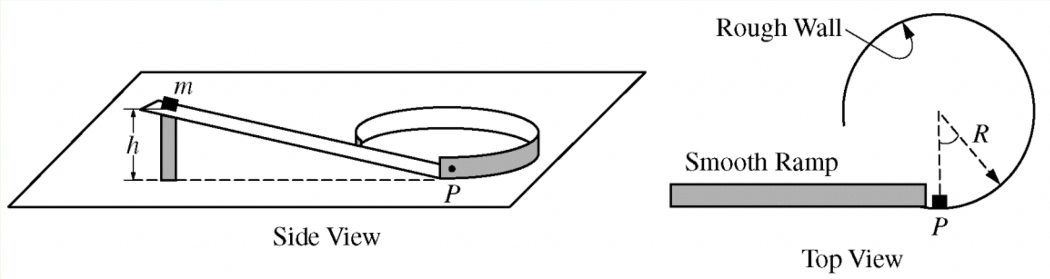
$$\text{Integrating gives } -\frac{1}{v}\Big|_{v_0}^{\frac{v_0}{3}} = -\frac{\mu T}{R}$$

$$\text{This means } -\frac{3}{v_0} - (-\frac{1}{v_0}) = -\frac{\mu T}{R}$$

$$\text{This simplifies to } -\frac{2}{v_0} = -\frac{\mu T}{R}$$

$$\text{We can isolate } T \text{ to find that } T = \frac{2R}{v_0 \mu}$$

Problem 2.0.43 — 2014 AP Physics C Mechanics FRQ



A small block of mass m starts from rest at the top of a frictionless ramp, which is at a height of h above a horizontal tabletop, as shown in the side view above. The block slides down the smooth ramp and reaches point P with a speed v_0. After the block reaches point P at the bottom of the ramp, it slides on the tabletop guided by a circular vertical wall with radius R, as shown in the top view. The tabletop has negligible friction, and the coefficient of kinetic friction between the block and the circular wall is μ.

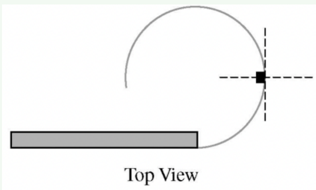
(a) Derive an expression for the height of the ramp h. Express your answer in terms of v_0, m, and fundamental constants, as appropriate.

A short time after passing point P, the block is in contact with the wall and moves with a speed of v.

(b)

i. Is the vertical component of the net force on the block upward, downward, or zero?

ii. On the figure below, draw an arrow starting on the block to indicate the direction of the horizontal component of the net force on the moving block when it is at the position shown.



Justify your answer.

Express your answers to the following in term of v_0, v, m, R, μ, and fundamental constants, as appropriate.

(c) Determine an expression for the magnitude of the normal force N exerted on the block by the circular wall as a function of v.

(d) Derive an expression for the magnitude of the tangential acceleration of the block at the instant the block has attained a speed of v.

(e) Derive an expression for $v(t)$, the peed of the block as a function of time t after passing point P on the track.

Solution to part a: Let's assume that the speed at the top of the ramp is v_i. We know that the speed at the end of the ramp is $v_f = v_o$ (at point P).
Let's assume that the ramp is L units long. If the angle between the ramp and the horizontal plane is θ, then the force along the ramp is $mg\sin(\theta)$. This is what causes the block to accelerate. The acceleration can be found using Newton's Second Law, and it is $g\sin(\theta)$.

We also know that the block travels a displacement of L along the ramp.

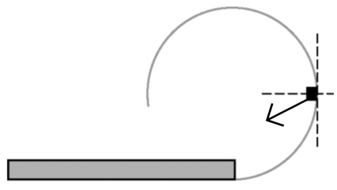
Since we know acceleration, displacement, and final velocity, we can use the kinematics equation $v_f^2 = v_i^2 + 2a\Delta x$

We can plug in our variables to get $v_o^2 = 0 + 2g\sin(\theta) \cdot L$
We know that $\sin(\theta) = \frac{h}{L}$; we can plug that in to find that $v_o^2 = 2g \cdot \frac{h}{L} \cdot L = 2gh$

We can rearrange the equation to find that $h = \frac{v_o^2}{2g}$

Solution to part b i: There is no acceleration in the vertical direction. This means that the vertical component of net force on the block is zero.

Solution to part b ii: We must know that the perspective of the drawing we are given is the top view. We know that the block moves in a circle, so centripetal motion occurs. There will be a net force pointing towards the center; and this net force is what causes the centripetal acceleration.
However, the horizontal component of net force won't directly point towards the center. We must consider that there is a friction force causing the block to slow down. The friction force will oppose the block's motion.



Top View

Solution to part c: The normal force is what drives the circular motion. This means that $N = ma = \frac{mv^2}{R}$.
The centripetal acceleration ($\frac{v^2}{R}$) occurs due to the normal force which drives the circular motion.

Solution to part d: The tangential acceleration is caused by frictional force. We know that $F_f = \mu N$
We also know that $-F_f = ma$
We can plug in $F_f = \mu N$ to find that $a = -\frac{\mu N}{m}$

We know that $N = \frac{mv^2}{R}$ (from part a). We can plug that in to find that $a = -\frac{\mu v^2}{R}$

Solution to part e: For such problems, we must use $a = \frac{dv}{dt}$. This simply means that acceleration is the derivative of velocity with respect to time.

We already know that $a = \frac{dv}{dt} = -\frac{\mu v^2}{R}$.

We use separation of variables to get $\frac{dv}{v^2} = -\frac{\mu}{R}dt$

Now we can integrate the equation to get $\int_{v_0}^{v} \frac{dv}{v^2} = \int_0^t -\frac{\mu}{R}dt$

We can simplify to get $-\frac{1}{v} - (-\frac{1}{v_0}) = -\frac{\mu t}{R}$

We can rearrange the equation and isolate v to find that $v = \frac{Rv_0}{R + \mu v_0 t}$

Unit 3 Work, Energy, Power

Have you ever rode a bicycle down a mountain or a steep road? Have you noticed that the bike speeds up as you move down. Why does this happen? Well the answer to this will be found in this unit. You will learn about different forms of energy to understand how energy is transferred into different forms.

Note 3.0.1 — Energy
Energy can be defined as the capacity to do work. There are two ways energy can change: within a system, or between a system and the external world.

Note 3.0.2 — Types of Energy

1. Kinetic Energy

Kinetic Energy can be thought of as the energy of motion. The equation includes mass (m) and velocity (v).

$$K = \frac{1}{2}mv^2 \tag{3.1}$$

2. Potential Energy

Potential energy can be thought of as stored energy due to position. In AP Physics, there are two types of potential energies: gravitational and spring.

The gravitational potential energy equation includes mass (m), the gravitational acceleration constant (g), and h (vertical displacement).

$$U_g = mgh \tag{3.2}$$

The spring potential energy equation includes k (spring constant) and x, the distance the spring is stretched from equilibrium.

$$U_s = \frac{1}{2}kx^2 \tag{3.3}$$

In general, objects want to move to a point where there is less potential energy. That is the natural tendency of every object.
For example, an object at a certain height will always want to move down.

Note 3.0.3 — Work

Work is defined as a force applied over a displacement. Work done on an object can transfer energy. The unit of work is J (Joules). Energy also has that unit.

Examples:

1. When a force is applied in the direction of displacement,

$$W = Fd.$$

2. When a force is applied in a direction that is an angle θ from the direction of displacement,

$$W = Fd\cos\theta.$$

Note that work is a scalar quantity, not a vector.

Note that only the component of force that is parallel to displacement can do work. Thus, if the force is perpendicular to the displacement vector, then 0 work is done. There is one EXTREMELY important example of this. To learn about it, check out the AP Physics 1 Unit 4 Rapid Review video on the TMAS Academy youtube channel.

Now, let's derive the work energy theorem by using $W_{net} = F_{net}d$. You won't know the work-energy theorem right now. It will be covered soon. Before we cover it, let's derive it.

We know that d is the displacement in this situation. Thus, we can use the kinematics equation $v_f^2 = v_i^2 + 2ad$
We can solve for d to find that $d = \frac{v_f^2 - v_i^2}{2a}$
We also know that $F_{net} = ma$ (from Newton's Second Law).

We can plug this into $W_{net} = F_{net}d$ to find that $W_{net} = ma \cdot \frac{v_f^2 - v_i^2}{2a} = \frac{mv_f^2}{2} - \frac{mv_i^2}{2}$
Since KE $= \frac{1}{2}mv^2$, we know that $K_i = \frac{mv_i^2}{2}$ (initial kinetic energy) and $K_f = \frac{mv_f^2}{2}$ (final kinetic energy).

This means that $W_{net} = \Delta K$ (net work done is the change in kinetic energy)

Work can also be found using calculus (extremely common on the AP exam).
Instead of $W = Fd$, you can also use $W = \int_a^b \vec{F}(r) \cdot d\vec{r}$

Don't forget that work is a **scalar** quantity. It also has a sign (it can be negative, positive, or zero).

Also, if we are given a force-time graph, then the area under the curve is equivalent to the work done by that specific force.

Note 3.0.4 — Work-Energy Theorem
The net-work done on an object is equivalent to the change in kinetic energy.

$$W_{net} = \Delta K$$

Now, it's important to relate potential energy to force.
We already know how kinetic energy relates to force. It relates through the work-energy theorem.

Note 3.0.5 — Conservative vs Non-conservative
When finding work done by a conservative force, only the initial and final points matter. The way you approach the final point doesn't matter (the work done is independent to the path). You only need to consider the final and initial position. For example, in a complete closed path, the work done by a conservative force is zero.

On the other hand, the work done by a non-conservative force depends on the path. The most common example of a non-conservative force is friction.

Now, let's learn how to relate potential energy to force.

Note 3.0.6 — For conservative forces, you can use the formula below to find the change in potential energy.

$$\Delta U = -\int_a^b \vec{F}(r) \cdot d\vec{r}$$

Using the formula above, we can find that

$$F_x = -\frac{dU(x)}{dx}$$

The formula above is extremely useful when we are given an expression for potential energy. Then, we simply have to differentiate it to find the force.

Sometimes, we might be given an expression for $U(x)$ and asked when the force F_x will be 0. We simply differentiate $U(x)$ since we know that the negative of the derivative of potential energy is force.

Now, you should also know the **change in gravitational potential energy**. This is seen all the time on the AP Exam.
The change in gravitational potential energy can be written as $\Delta U_g = mg\Delta h$. Δh represents the change in height.

In general, the change in gravitational potential energy provides much more useful information than just gravitational potential energy. The reason is that gravitational potential energy can be defined by you based on your reference level. For example, if there's an object 2 m above the ground, then you can assume the reference level to be something like the floor or the object's location. Then, if you find $U = mgh$ using both reference levels, you will get a different value for the potential energy

HOWEVER, the change in gravitational potential energy will be constant despite what reference level you use.

Note 3.0.7 — Ideal Spring
An ideal spring is extremely important to know in depth. The force caused by an ideal spring is a conservative force. That means we only care about the initial and final positions when finding the change in potential energy for the spring.

By Hooke's Law, we know that the force on an ideal spring is $F = -k\Delta x$ where Δx is the extension length.

The potential energy can be represented as $U_s = \frac{1}{2}k(\Delta x)^2$

However, a spring is not always ideal. That is when some calculus must be used.

Note 3.0.8 — In a non-ideal spring, the method to finding the change in potential energy is different.

$$U_s = -\int_a^b \vec{F}(r) \cdot d\vec{r}$$

The formula above must be used to find the change in potential energy for a non-ideal spring.

The reason is that the force isn't directly proportional to the distance compressed/extended. That is when we must use calculus. Often, you will be given an equation for force in terms of r. Then, you can simply integrate with respect to r to find the change in potential energy.
You will see many examples with this for the problems from this unit in this book.

Note 3.0.9 — If there are only **internal forces** in a system, then the change in **mechanical energy** is 0. Note that internal forces are forces that two objects apply on each other in the same system. On the other hand, external force comes from an object outside of that system. Remember that if there are external forces, then energy will not be conserved.

This means that $K_i + U_i = K_f + U_f$

For those that don't know, mechanical energy is the energy that can do work. It's different from other factors like heat energy. Just remember that mechanical energy also includes spring potential energy. Many people think that it only includes kinetic and gravitational potential energy.

When a system has **non-conservative forces**, then $W_{nc} = \Delta ME$. This means that the work done by the non-conservative force is equivalent to the change in mechanical energy.

Note 3.0.10 — Total Energy **(E)**
E can be defined as the sum of all different types of energy in the system. Here, we must consider kinetic energy, potential energy, and thermal energy. If a problem asks us to find the total energy, we must account for other forms of energy other than potential and kinetic. However, if we are asked to find the total mechanical energy, then we only include potential (gravitational and spring) and kinetic energy.

Note 3.0.11 — Power
Power is denoted by P and is defined as the rate of change of work. Assuming constant power, that means

$$P = \frac{W}{t},$$

where W is the work done in a time t.

From a calculus perspective, power can be written as $P = \frac{dW}{dt}$. It is the rate at which work is done (the derivative or work with respect to time). When an object is moving at constant velocity, the power is

$$P = \frac{W}{\Delta t} = Fv.$$

Also, we can integrate power if it's written with respect to time. The integral of power with respect to time will give work done!

$$W = \int P dt$$

When an object is moving at constant velocity, the power is

$$P = \frac{W}{\Delta t} = Fv$$

If the angle between the force and velocity vectors is θ, then $P = Fv\cos\theta$
The above is extremely important to know when you want to find the power given the force and velocity.

Problem 3.0.12 — A bodybuilder is in the midst of a an intense training session. He is currently bench pressing a bar with a mass of 250kg. If he does six reps of this mass and his arms are 0.75m long, how much work has been done on the bar between the time the bar was removed from its rack and placed back on the rack?

Solution: This is a tricky problem. We know that the work done is equivalent to the product of the force and displacement. The *net* displacement of the bar 0, so the net work on it is $\boxed{0J}$. The reason is that the bodybuilder brings the bar back to the location where it started. That is what causes the displacement to be 0.

Problem 3.0.13 — A semi-truck carrying a trailer has a total mass of 1500kg. If it is traveling up a slope of 5 degrees to the horizontal at a constant rate of 20m/s, how much power is the truck exerting?

Solution: Note that the force exerted on the truck is equal to the gravitational force parallel to the slope, which is $1500g\cos(5°)$. Since the truck moves at constant velocity, we know that

$$P = Fv = (1500g\cos(5°))(20) = \boxed{26000W}$$

Problem 3.0.14 — An upward force is applied to lift a 20 kg bag a to a height of 5m. The bag is lifted at a constant speed. What is the work done on the bag?

Solution: The work done is equal to the change in potential energy, ΔU, which is equal to

$$mgh = (20)(g)(5) = \boxed{1000 \text{ J}}$$

Problem 3.0.15 — Juri is tugging her wagon behind her. She has a trek ahead of her–five kilometers–and she's pulling with a force of 200 newtons. If she's pulling at an angle of 35 degrees to the horizontal, what work will be exerted on the wagon to get to the repair shop?

Solution:

$$W = Fd\cos\theta = (200)(5)(\cos 35°) = \boxed{819.152 \text{ J}}$$

Problem 3.0.16 — What is the work done in increasing the speed of a 2 kg block from 4 m/s to 9 m/s?

Solution: This is an example of the work-energy theorem. The work done will be equivalent to the change in kinetic energy.

$$W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(2)(9)^2 - \frac{1}{2}(2)(4)^2 = \boxed{65 \text{ J}}$$

Problem 3.0.17 — A 2.5 kg block moving at 5.6 m/s hits a spring with a spring constant of 50 N/m. How much is the spring compressed from its equilibrium position?

Solution: In this problem, the initial kinetic energy will completely convert into spring potential energy.

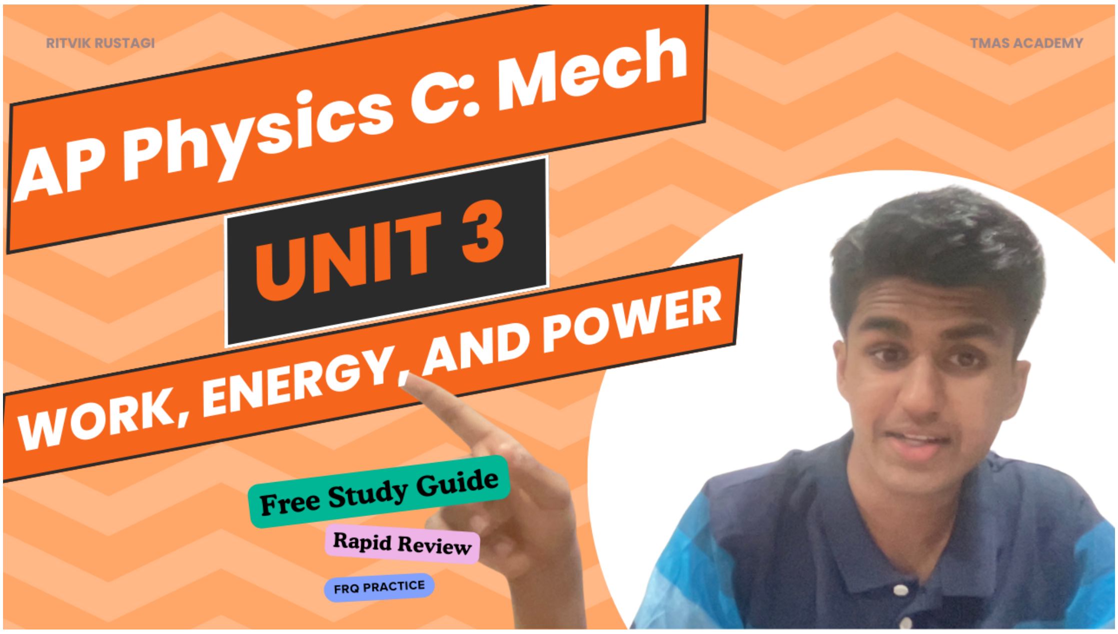
$$\frac{1}{2}mv_i^2 = \frac{1}{2}kx^2$$

We can rearrange the equation to find an expression for x.

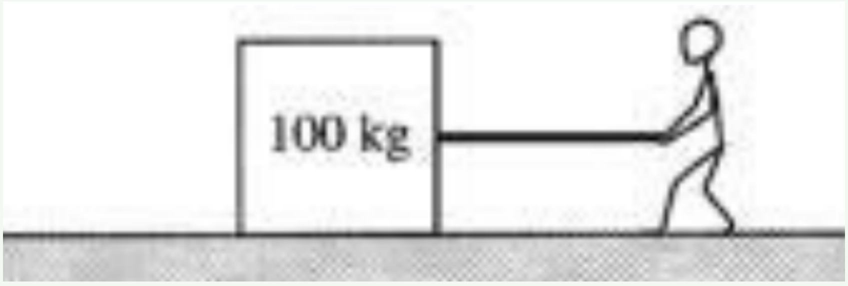
$$x = v_i\sqrt{\frac{m}{k}} = 5.6\sqrt{\frac{2.5}{50}} = \boxed{1.252 \text{ m}}$$

Problem 3.0.18 — 2012 AP Physics C: Mechanics MCQ
A certain one-dimensional conservative force is given as a function of x by the expression $F = -kx^3$, where F is in newtons and x is in meters. A possible potential energy function U for this force is?

Solution: Video Solution



Problem 3.0.19 — 2003 AP Physics C: Mechanics FRQ



The 100 kg box shown above is being pulled along the x-axis by a student. The box slides across a rough surface, and its position x varies with time t according to the equation $x = 0.5t^3 + 2t$, where x is in meters and t is in seconds.

(a) Determine the speed of the box at time $t = 0$.

(b) Determine the following as functions of time t.

i. The kinetic energy of the box

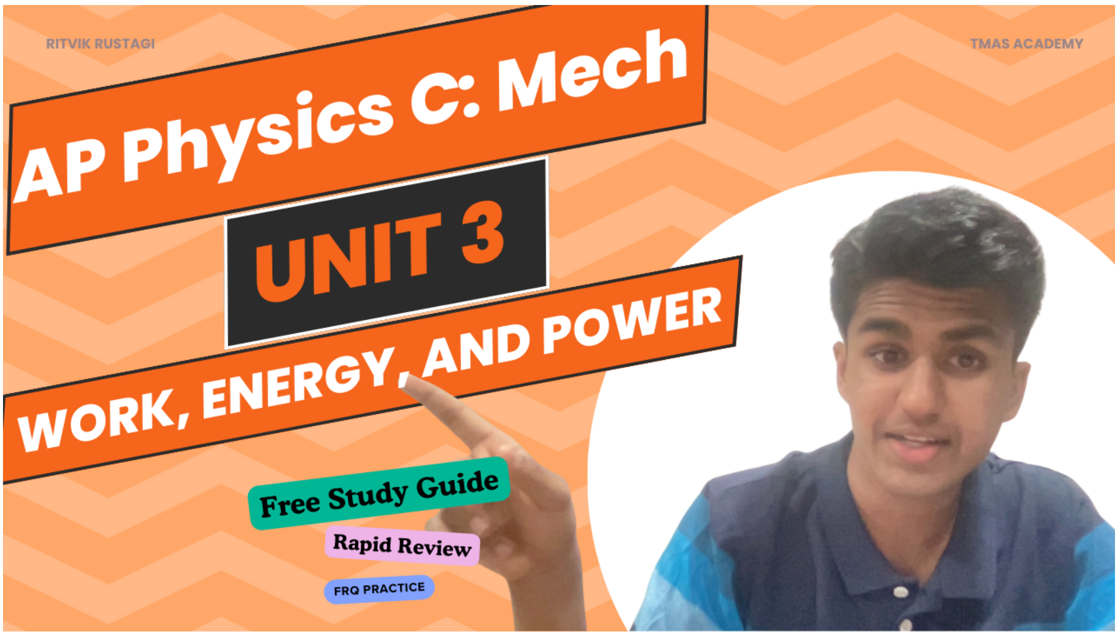
ii. The net force acting on the box

iii. The power being delivered to the box

(c) Calculate the net work done on the box in the interval $t = 0$ to $t = 2$s.

(d) Indicate below whether the work done on the box by the student in the interval $t = 0$ to $t = 2$s would be greater than, less than, or equal to the answer in part (c).

Solution: Video Solution



Solution to part a: We have an equation for the position which is $x = 0.5t^3 + 2t$
Since velocity is the derivative of position with respect to time, we can use the formula $v = \frac{dx}{dt}$

Differentiating our equation for position gives that $v = 1.5t^2 + 2$
At time $t = 0$, the velocity v is clearly $2m/s$.

Solution to part b i: We know that the kinetic energy can be written as $\frac{1}{2}mv^2$
Since we know that velocity v can be represented as $v = 1.5t^2 + 2$, our kinetic energy is $\frac{1}{2}m(1.5t^2 + 2)^2$

We can plug in our value of mass $m = 100$ to find that kinetic energy can be represented as $50(1.5t^2 + 2)^2$

Solution to part b ii: We know that net force can be written as $F_{net} = ma$
That means we need to find an expression for acceleration. We know that acceleration is the derivative of velocity with respect to time.
This means that $a = \frac{dv}{dt}$
That means our net force can be written as $F_{net} = m\frac{dv}{dt}$

Since we know that $v = 1.5t^2 + 2$, we know that $a = \frac{dv}{dt} = 3t$
We can plug that in to find that $F_{net} = m \cdot 3t = 3mt$. Now, we plug in our given mass $m = 100$ to find that the net force is $300t$.

Solution to part b iii: We know that power $P = Fv$
We already have an expression for the net force as $300t$.
We also know that $v = 1.5t^2 + 2$
This means that our power can be written as $P = 300t(1.5t^2 + 2) = 450t^3 + 600t$

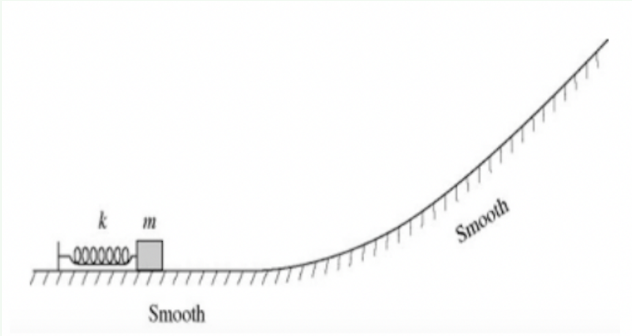
Solution to part c: We know that net work done is simply the change in kinetic energy.
$W = \Delta K$
We must find the initial speed v_i at time $t = 0$. We must also find the speed v_f at time $t = 2$. This will allow us to find the kinetic energies at each point.

Since we know that $v = 1.5t^2 + 2$, we can find the velocity at time $t = 0$ to simply be 2.
At time $t = 2$, the velocity v is $1.5 \cdot 2^2 + 2 = 8$

This means that the change in kinetic energy is $\frac{1}{2} \cdot 100 \cdot (8^2 - 2^2) = 3000$ J. This is also equivalent to the work done since it's the same as ΔK.

Solution to part d: Since our surface is rough, there will always be. an opposing force of friction against the applied force. This means that the student must apply greater work than the net work to account for friction.

Problem 3.0.20 — Physics Bowl MCQ



A box of mass m is pressed against (but is not attached to) an ideal spring of force constant k and negligible mass. The spring is compressed a distance x. After it is released, the box slides up a frictionless incline as shown in the diagram above and eventually stops. If we repeat this experiment with a box of mass $2m$

a. the lighter box will go twice as high up the incline as the heavier box
b. just as it moves free of the spring, the lighter box will be moving twice as fast as the heavier box.
c. both boxes will have the same speed just as they move free of the spring.
d. both boxes will reach the same maximum height on the incline.
e. just as it moves free of the spring, the heavier box will have twice as much kinetic energy as the lighter box.

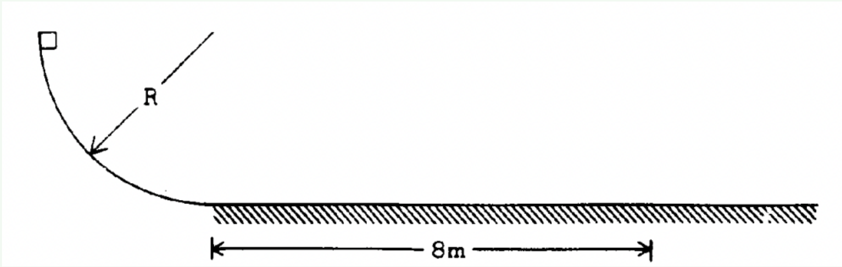
Solution: In this problem, we know that the spring potential energy due to compression will be $\frac{1}{2}kx^2$. Clearly, as long as we compress the same spring the same distance, then the spring potential energy will be the same regardless of what mass is attached to it.

For both the box of mass m and $2m$, th spring potential energy will be the same.
After compression, the box will move to a maximum height where the kinetic energy will be 0.
The spring potential energy will convert to gravitational potential energy.

We can write the equation $\frac{1}{2}kx^2 = mgh$
We can solve for h (the height) to get that $h = \frac{kx^2}{2mg}$

The only variable that changes is mass. Since the variable for mass is in the denominator, the box of mass $2m$ will have half the final height as the box of mass m.
The only answer choice that satisfies this is **A**.

Problem 3.0.21 — 1975 AP Physics B FRQ



A 2-kilogram block is released from rest at the top of a curved incline in the shape of a quarter of a circle of radius R. The block then slides onto a horizontal plane where it finally comes to rest 8 meters from the beginning of the plane. The curved incline is frictionless, but there is an 8-newton force of friction on the block while it slides horizontally. Assume $g = 10$meters per second^2.

(a) Determine the magnitude of the acceleration of the block while it slides along the horizontal plane.

(b) How much time elapses while the block is sliding horizontally?

(c) Calculate the radius of the incline in meters.

Solution to part a: The problem says that there is a 8 N force of friction on the horizontal surface. This force will point leftwards, so the force is -8 N. It also says that the block weighs 2 kg.
We simply apply $F = ma$ and rearrange it to get $a = \frac{F}{m}$. We plug in $F = -8$ and $m = 2$ to find that the acceleration is $-4\frac{m}{s^2}$

Solution to part b: When the block slides horizontally, it covers a distance of 8m with an acceleration of $-4\frac{m}{s^2}$
We also know that v_f (the final velocity) is 0 since friction slows it down.

We can apply the equation $v_f^2 = v_i^2 + 2ad$
We can plug in our known values to get $0^2 = v_i^2 + 2 \cdot -4 \cdot 8$
Simplifying it gives that $v_i = 8\frac{m}{s}$. This is the velocity as soon as the block enters the horizontal part of the track.

Since we know that $v_i = 8$, $v_f = 0$, and $a = -4$, we can apply the equation $v_f = v_i + at$

We can rearrange that equation to get $t = \frac{v_f - v_i}{a}$. We can plug in our variables into this equation to find that $t = 2$ s

Solution to part c: We can find the radius by conserving energy.
We know that $K_i + U_i = K_f + U_f$
Our initial point is the point where the block is released from at rest. The final point we consider is right when the block hits the horizontal part.

The initial kinetic energy is 0. We can assume that the horizontal part of the track is

our reference level which means it has $U_f = 0$
This means our equation simplifies to $U_i = K_f$. All the gravitational potential energy at the top will be converted to kinetic energy.

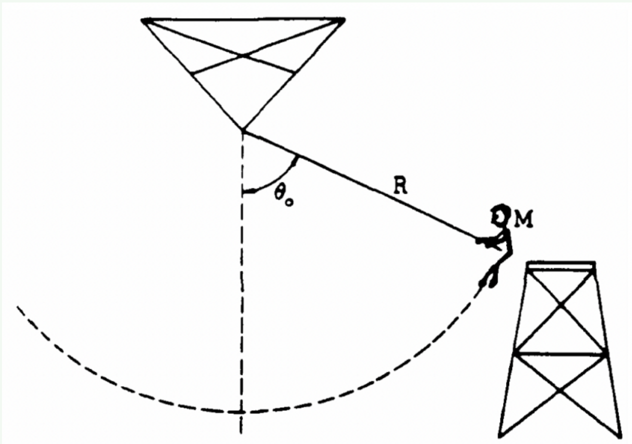
Gravitational potential energy at the top is mgR since R is the height the block is above the horizontal part of the track.
This means $mgR = \frac{1}{2}mv^2$

We can cancel m on both sides to get $gR = \frac{v^2}{2}$.
Now, we divide both sides by g to get $R = \frac{v^2}{2g}$

The velocity that we use is the velocity at the beginning of the horizontal part which was already found to be $8\frac{m}{s}$.

We can plug this in to find that $R = \frac{8^2}{g} = \frac{64}{2g} = 3.265$ m.

Problem 3.0.22 — 1982 AP Physics B FRQ



A child of mass M holds onto a rope and steps off a platform. Assume that the initial speed of the child is zero. The rope has length R and negligible mass. The initial angle of the rope with the vertical is θ_o, as shown in the drawing above.

(a) Using the principle of conservation of energy, develop an expression for the speed of the child at the lowest point in the swing in terms of g, R, and $\cos(\theta_o)$

(b) The tension in the rope at the lowest point is 1.5 times the weight of the child. Determine the value of $\cos(\theta_o)$.

Solution to part a: This problem is an energy conservation problem.
For all energy conservation problems, we should write out our equation.
$K_i + U_i = K_f + U_f$
The initial kinetic energy at the top is 0. We can say that the reference level for gravitational potential energy is the lowest point that the child goes to. This means that $U_f = 0$.

We can simplify the equation to $U_i = K_f$. This means that the initial gravitational potential energy converts to kinetic energy.

The initial gravitational potential energy can be found by first finding the height above the lowest point. Using some trigonometry, it's obvious that the child is a distance $R - R\cos(\theta_o)$ above the lowest point.

This means that the initial gravitational potential energy is $mg(R - R\cos(\theta_o))$
Now, we equate this to the final kinetic energy which can be represented as $\frac{1}{2}mv^2$

After equating both, we get $mg(R - R\cos(\theta_o)) = \frac{1}{2}mv^2$

We can divide both sides by m and then multiply both sides by 2 to get $v^2 = 2g(R - R\cos(\theta_o))$
We can simplify the equation to get $v = \sqrt{2g(R - R\cos(\theta_o))}$

Solution to part b: The two forces on the child are tension force and gravitational force. Both of these forces cause centripetal acceleration.

We can write out our equation as $T - mg = \frac{mv^2}{R}$

Since tension is 1.5 times the weight, we can write this out mathematically to get $T = 1.5mg$
Plugging this into the equation gives $\frac{mg}{2} = \frac{mv^2}{R}$

Now, we can divide both sides by m to get $\frac{g}{2} = \frac{v^2}{R}$

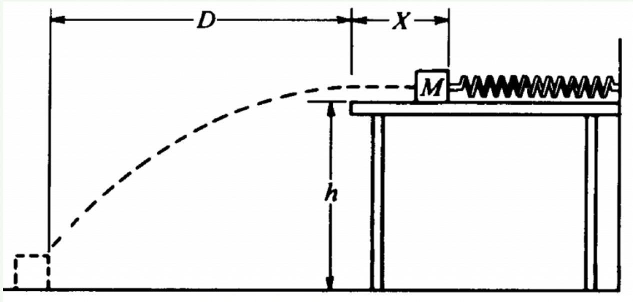
We can plug in our expression for v since we found it in part a ($v = \sqrt{2g(R - R\cos(\theta_o))}$)

$$\text{Plugging it in turns the equation to } \frac{g}{2} = \frac{2g(R - R\cos(\theta_o))}{R} = 2g(1 - \cos(\theta_o))$$

Now, we can divide g from both sides to get $\frac{1}{2} = 2 - 2\cos(\theta_o)$
Multiplying both sides by 2 gives $1 = 4 - 4\cos(\theta_o)$

We can solve the equation to find that $\cos(\theta_o) = \frac{3}{4}$

Problem 3.0.23 — 1986 AP Physics B FRQ



One end of a spring is attached to a solid wall while the other end just reaches to the edge of a horizontal, frictionless tabletop, which is a distance h above the floor. A block of mass M is placed against the end of the spring and pushed toward the wall until the spring has been compressed a distance X, as shown above. The block is released, follows the trajectory shown, and strikes the floor a horizontal distance D from the edge of the table. Air resistance is negligible. Determine expressions for the following quantities in terms of M, X, D, h, and g. Note that these symbols do not include the spring constant.

(a) The time elapsed from the instant the block leaves the table to the instant it strikes the floor

(b) The horizontal component of the velocity of the block just before it hits the floor

(c) The work done on the block by the spring

(d) The spring constant

Solution to part a: The time elapsed from the moment it leaves the table can be found by considering the motion in the y-direction.

The initial velocity in y-direction ($v_o y$) is 0. The acceleration in the y-direction is $a_y = g$. The displacement is $\Delta y = h$

Now, we can use the equation $d = v_o t + \frac{1}{2}at^2$
We can plug in our variables to get $h = \frac{1}{2} \cdot g \cdot t^2$

We can multiply both sides by $\frac{2}{g}$ to get $t^2 = \frac{2h}{g}$.
We can simplify the equation to get $t = \sqrt{\frac{2h}{g}}$.

Solution to part b: There is no acceleration in the x-direction. We should remember this from our projectile motion section in Unit 1.
The initial velocity in x-direction is the same as the final velocity in x-direction.
Since there is no acceleration, we know that $D = vt$ which means $v = \frac{D}{t}$.

We can plug in our expression for t into this to find that $v = D\sqrt{\frac{g}{2h}}$

Solution to part c: The work done on the block by the spring is what caused the block

to gain kinetic energy.
Instead of finding the work done, we can simply find the kinetic energy right at the edge of the table.

We know that the velocity at that point is $v = D\sqrt{\frac{g}{2h}}$.
We can plug this into $K = \frac{1}{2}mv^2$ to get $\frac{MD^2g}{4h}$
The work done is $\frac{MD^2g}{4h}$

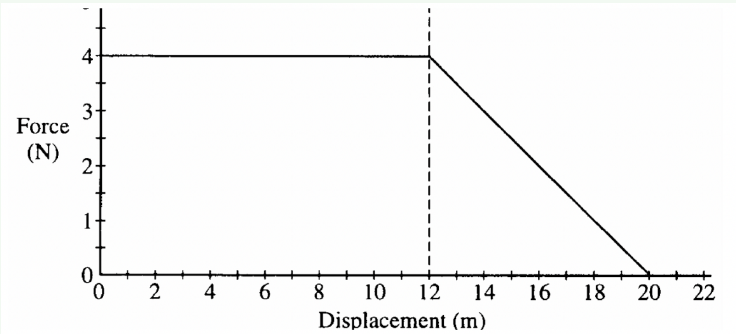
Solution to part d: We know that the work done is equivalent to the kinetic energy. We also know that the kinetic energy was caused by the spring potential energy due to compression.

This means $U_s = K$
We know that $U_s = \frac{1}{2}kx^2$. Since the distance compressed in our case is X, we can plug that in to get $\frac{1}{2}kX^2$
We also know that our kinetic energy is $\frac{MD^2g}{4h}$

We can equate both equations: $\frac{1}{2}kX^2 = \frac{MD^2g}{4h}$

$$\text{We can isolate } k \text{ (spring constant) on one side to find that } k = \frac{MD^2g}{2hX^2}$$

Problem 3.0.24 — 1997 AP Physics B FRQ



A 0.20 kg object moves along a straight line. The net force acting on the object varies with the object's displacement as shown in the graph above. The object starts from rest at displacement $x = 0$ and $time t = 0$ and is displaced a distance of 20 m. Determine each of the following.

(a) The acceleration of the particle when its displacement x is 6 m.
(b) The time taken for the object to be displaced the first 12 m.
(c) The amount of work done by the net force in displacing the object the first 12 m.
(d) The speed of the object at displacement $x = 12$ m.
(e) The final speed of the object at displacement $x = 20$ m.

Solution to part a: When the displacement is 6, the force is 4 N. This can be observed from the graph.

We also know that the mass is 0.2 kg. We can plug in our variables into the equation for Newton's Second Law: $F = ma$.
Doing so gives $4 = 0.2 \cdot a$
This means $a = 20\frac{m}{s^2}$

Solution to part b: Since the force is constant for the first 12 m, we can simply use our kinematic equations because acceleration is constant.
The acceleration for the first 12 m is $20\frac{m}{s^2}$ (we found this in the last part).

We also know that $v_o = 0$ (initial velocity is 0).
We can use the kinematics equation $\Delta x = v_o t + \frac{1}{2}at^2$

We can plug our variables into the equation to get $12 = \frac{1}{2} \cdot 20 \cdot t^2$
This simplifies to $t^2 = 1.2$
We can simplify this to get that $t = 1.095$ s.

Solution to part c: The work done is the area under a force displacement graph.
In this case, we can find the work done for the first 12 m by finding the area under the curve.
it is simply a rectangle with area 48. This means that the work done is 48 N·kg

Solution to part d: To find the speed at displacement $x = 12$ m, we need to know that $W = \Delta K$

The work done is simply the change in kinetic energy.
This means that $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{m}{2}(v_f^2 - v_i^2)$

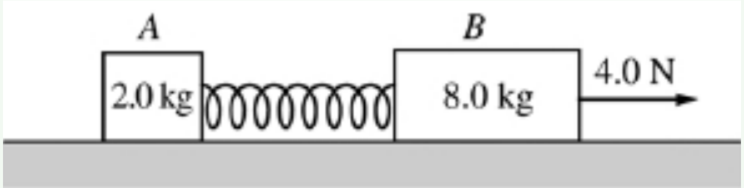
v_i (initial velocity) is just 0. We also know that $W = 48$ (from part c).

We can plug this in along with the mass to find that $48 = \frac{0.2}{2} \cdot v^2$
We can simplify this equation to find that the speed is $21.91\frac{m}{s}$

Solution to part e: Now, we will find the area under the entire curve. We already know that the rectangle has an area of 48. Now, there's a triangle on the right of it with an area of 16.
The total work done is $48 + 16$ which is 64.

We know that the work done is the change in kinetic energy.
This means that $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{m}{2}(v_f^2 - v_i^2)$
v_i (initial velocity) is just 0. We also know that $W = 64$
We can plug this along with the mass to find that $64 = \frac{0.2}{2} \cdot v^2$
Simplifying this equation gives that $v = 25.3\frac{m}{s}$.

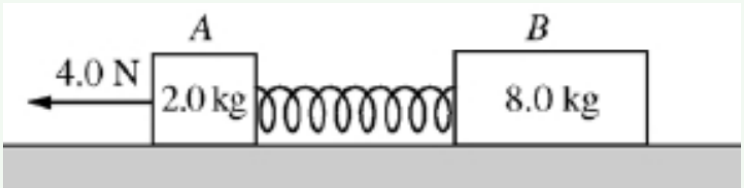
Problem 3.0.25 — 2008 AP Physics B FRQ



Block A of mass 2.0 kg and block B of mass 8.0 kg are connected as shown above by a spring of spring constant 80 N/m and negligible mass. The system is being pulled to the right across a horizontal frictionless surface by a horizontal force of 4.0 N, as shown, with both blocks experiencing equal constant acceleration.

(a) Calculate the force that the spring exerts on the 2.0 kg block.
(b) Calculate the extension of the spring.

The system is now pulled to the left, as shown below, with both blocks again experiencing equal constant acceleration.



(c) Is the magnitude of the acceleration greater than, less than, or the same as before?
______ Greater ______ Less ______ The same
Justify your answer.

(d) Is the amount the spring has stretched greater than, less than, or the same as before? ______ Greater ______ Less ______ The same
Justify your answer.

(e) In a new situation, the blocks and spring are moving together at a constant speed of 0.50 m/s to the left. Block A then hits and sticks to a wall. Calculate the maximum compression of the spring.

Solution to part a: Let's denote the spring force as F_s.
Since the spring force acts in the opposite direction for block B, we can write its Newton's Second Law Equation as $F - F_s = m_b a$ (F is the applied force on block B)
For block A, the spring force causes its acceleration and pulls it to the right. The Newton's Second Law Equation for block A is $F_s = m_a a$

We can add both of the equations we found to get $F = (m_b + m_a)a$
We can substitute our known values to get $4 = (8 + 2)a$
We can solve it to find that $a = 0.4\frac{m}{s^2}$

Now, we can substitute this value of acceleration to the equate for block A which was $F_s = m_a a$. Since we know that $m_a = 2$ kg, we can find that $F_s = 2 \cdot 0.4 = 0.8$ N.

Solution to part b: Since we know that the spring force is 0.8 N, we can use the equation $F_s = k{\cdot}\Delta x$ to find the extension.
We can plug in $F_s = 0.8$ and $k = 80$ to find that $x = 0.01$ m.

Solution to part c: The magnitude of acceleration will be the **same**.
The reason is that the net force on the two block system will still be the same. Since the force i the same and mass is also constant, acceleration will be the same.

Solution to part d: The spring will be stretched **more**.
The reason is that the spring force is causing block B to accelerate. Since block B has a larger mass than block A, it will need a larger force to be able to accelerate at the same rate.
For the spring force to be larger, the distance stretched must be greater for this case.

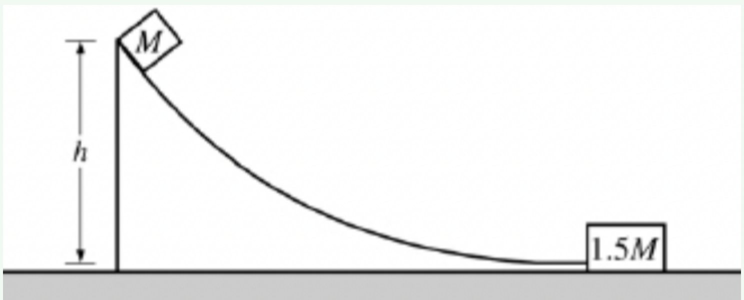
Solution to part e: Right after block A hits and sticks to the wall, block B will continue to move and now compress the spring.
Its kinetic energy will be transferred to spring potential energy.
We can write the equation $K = U_s$ which means $\frac{1}{2}m_b v^2 = \frac{1}{2}kx^2$

We can plug in $m_b = 8$, $v = 0.5$, and $k = 200$ into the equation.

$$\text{Plugging values in gives } \frac{1}{2}\cdot 8 \cdot 0.5^2 = \frac{1}{2}\cdot 200 \cdot x^2$$

We can solve for x (the distance the spring compresses) and find that it equals 0.16 m.

Problem 3.0.26 — 2006 AP Physics B FRQ



A small block of mass M is released from rest at the top of the curved frictionless ramp shown above. The block slides down the ramp and is moving with a speed $3.5v_o$ when it collides with a larger block of mass $1.5M$ at rest at the bottom of the incline. The larger block moves to the right at a speed $2v_o$ immediately after the collision.
Express your answers to the following questions in terms of the given quantities and fundamental constants.

(a) Determine the height h of the ramp from which the small block was released.

(b) The larger block slides a distance D before coming to rest. Determine the value of the coefficient of kinetic friction μ between the larger block and the surface on which it slides.

Solution to part a: We can find the height from which the block was released by using the conservation of energy theorem.
$K_i + U_i = K_f + U_f$
We'll find the initial kinetic energy at the top and the initial potential energy at the top. Then, for the final energy variables we'll use the point right before collision.

Initially, there is no kinetic energy since the block is at rest. Also, we can say that the bottom of the incline is our reference level when finding gravitational potential energy. That means the initial gravitational potential energy is mgh (since it's a distance h above our reference level.

Since $K_i = 0$ and $U_f = 0$, our equation is $U_i = K_f$
This means that the initial gravitational potential energy for the lighter block will convert to kinetic energy.

We know that the light block has a speed $3.5v_o$ at the bottom. That means the final kinetic energy is $\frac{1}{2}M(3.5v_o)^2$

We can now equate our initial gravitational potential energy to the final kinetic energy.

$Mgh = \frac{1}{2}M(3.5v_o)^2$
Cancelling M from both sides gives $gh = \frac{(3.5v_o)^2}{2}$

$$\text{We can isolate } h \text{ in the equation to find that } h = \frac{6.125 \cdot v_o^2}{g}$$

Solution to part b: We know that $W = \Delta K$
This means that the work done is the change in kinetic energy.

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The final speed of the large block is 0 since friction slows it down. The initial speed as already stated in the problem is $2v_o$. The mass of the block is $1.5M$

Also, the work done by the friction force can be found by using the equation $W = F \cdot \Delta x$. This means that the work done is the force times displacement.

We know that the force is $-\mu mg$ (since friction force is the frictional coefficient times normal force). Δx is D since that's the distance the block slides. This means that the work done on the block by the friction force is $-\mu mgD$

Now, we can plug our expression for the work done into $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$$-\mu mgD = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Although we could plug in $1.5M$ for the mass, we don't need to do that since the variable for mass cancels out in this equation.

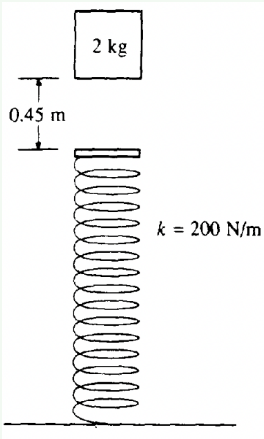
$$\text{It simplifies to } -\mu gD = \frac{v_f^2}{2} - \frac{v_i^2}{2}$$

Now we can plug in $v_i = 2v_o$ and $v_f = 0$ to get

$$-\mu g D = 0 - 2v_o^2$$

We can simplify this equation to find $\mu = \dfrac{2v_o^2}{gD}$

Problem 3.0.27 — 1989 AP Physics C: Mechanics FRQ



A 2-kilogram block is dropped from a height of 0.45 meter above an uncompressed spring, as shown above. The spring has an elastic constant of 200 newtons per meter and negligible mass. The block strikes the end of the spring and sticks to it.

(a) Determine the speed of the block at the instant it hits the end of the spring
(b) Determine the force in the spring when the block reaches the equilibrium position
(c) Determine the distance that the spring is compressed at the equilibrium position
(d) Determine the speed of the block at the equilibrium position
(e) Determine the resulting amplitude of the oscillation that ensues
(f) Is the speed of the block a maximum at the equilibrium position, explain.
(g) Determine the period of the simple harmonic motion that ensues

Solution to part a: We can use conservation of energy to find the speed of the block when it hits the end of the spring.
$K_i + U_i = K_f + U_f$
By the time the block hits the end of the spring, it has moved down a distance of h. That means it loses gravitational potential energy, and the change in gravitational potential energy can be represented as mgh where $h = 0.45$ m.

The initial kinetic energy will be 0 since it is dropped at rest.
This means our equation becomes $\Delta U = K_f$ since the gravitational potentia energy is converted to kinetic energy

We can write this as $mgh = \frac{1}{2}mv^2$

We can cancel out mass m and simplify more to find that $v = \sqrt{2gh}$

$$\text{Since } h = 0.45, \text{ we know that } v = \sqrt{2 \cdot 9.8 \cdot 0.45} = 2.97\frac{m}{s}$$

Solution to part b: At equilibrium, the spring force balances the gravitational force. That means we can just find the gravitational force on the block since that will equal to the magnitude of the spring force.
$F_s = mg = 2 \cdot 9.8 = 19.6$ N

Solution to part c: We can find that distance the spring is compressed by using the equation $F_s = k\Delta x$
Since we know that $F_s = mg$, we also know that $k\Delta x = mg$.

We can rearrange the equation to find that $\Delta x = \frac{mg}{k}$.

$$\text{Since } mg = 19.6 \text{ and } k = 200, \text{ we can plug this in to find that } x = \frac{19.6}{200} = 0.098 \text{ m.}$$

Solution to part d: Now, our new reference level will be the equilibrium point.
We will use the point from where we drop the 2 kg block.

The height of the point where we drop the block from is $h = 0.45 + 0.098 = 0.548$ m above the equilibrium point. The reason is that we must account for both the distance h and the distance that the spring is compressed.

Now, our conservation equation will no longer just be $K_i + U_i = K_f + U_f$
The reason is that now we also have spring potential energy. It will be

$$K_i + U_i + U_{si} = K_f + U_f + U_{sf}$$

Some of the forms of energy will be 0, so we can simplify the equation.

We can use the equation $mgh = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ to find the speed at the equilibrium point. The reason is that at the point where the ball is released initially, it only has gravitational potential energy. However, it will convert to kinetic and spring potential energy by the time it reaches the equilibrium point.

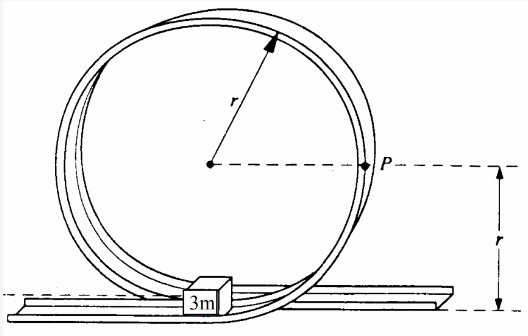
We already know that $h = 0.548$ and $x = 0.098$ (since that's the distance the spring has been compressed)
We can plug in our variables into the equation to get:

$$2 \cdot 9.8 \cdot 0.548 = \frac{1}{2} \cdot 2 \cdot v^2 + \frac{1}{2} \cdot 200 \cdot 0.098^2$$

After evaluating our expression, we can find that $v = 3.13$ m/s.

Solution to part e: Yes. The reason is that this is the point where the gravitational force and spring force balance each other. This means that the acceleration is 0. At any point above or below the equilibrium point, acceleration will oppose the motion causing the object to decelerate (speed will decrease).

Problem 3.0.28 — 1991 AP Physics C: Mechanics FRQ



A small block of mass $3m$ moving at speed $v_o/3$ enters the bottom of the circular, vertical loop-the-loop shown above, which has a radius r. The surface contact between the block and the loop is frictionless. Determine each of the following in terms of m, v_o, r, and g.

(a) The kinetic energy of the block and bullet when they reach point P on the loop

(b) The speed v_{min} of the block at the top of the loop to remain in contact with track at all times

(c) The new required entry speed v_o at the bottom of the loop such that the conditions in part b apply

Solution to part a: We can conserve energy to find the kinetic energy at point P.
$K_i + U_i = K_f + U_f$
We can set our reference level to be the bottom of the loop. This will cause initial potential energy (U_i) to be 0.

We know that kinetic energy $K = \frac{1}{2}mv^2$
The initial kinetic energy at the bottom of the loop is $\frac{1}{2} \cdot 3m \cdot (\frac{v_o}{3})^2$ which is $\frac{mv_o^2}{6}$
We also know that potential energy is mgh. This means that $U_f = 3mgr$ since point P is a distance r above the bottom of the loop and the mass is $3m$.

We can plug these expressions into our conservation of energy formula to get

$$\frac{mv_o^2}{6} = K_f + 3mgr$$

Basically, when the block enters the bottom of the cylinder, it will have kinetic energy. Some of that kinetic energy will be lost when it reaches point P since it will turn into gravitational potential energy.

Subtracting $3mgr$ from both sides gives that $K_f = \frac{mv^2}{6} - 3mgr$

Solution to part b: The block will remain in contact with the loop when a normal force exists. We don't want the normal force to be 0. To find the minimum velocity, we'll set the normal force to 0 (since it allows us to find the "limiting" value).

At the top, the two forces on the block are normal force and gravitational force. Both are pointing downwards.
Using Newton's Second Law, we can write the equation $mg + N = \frac{mv^2}{r}$
We can plug in $3m$ for the mass to get $3mg + N = \frac{3mv^2}{r}$.

Now, we substitute $N = 0$ to get $3mg = \frac{3mv^2}{r}$

We can solve this to find that $v = \sqrt{rg}$

This means that $v_{min} = \sqrt{rg}$

Solution to part c: We will use conservation of energy to find the new required entry speed v_o'

We know that $K_i + U_i = K_f + U_f$. U_i is simply 0 since the bottom of the circular track is our reference level.
This means that $K_i = K_f + U_f$
The final point that we are considering is the top of the loop.

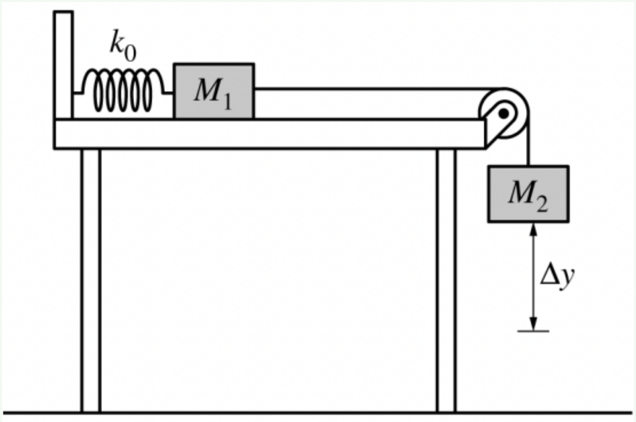
The initial kinetic energy is $\frac{3mv_o'^2}{2}$. The final kinetic energy at the top is $\frac{1}{2} \cdot 3m \cdot (\sqrt{rg})^2$ which is $\frac{3mrg}{2}$
The final potential energy can be found by using the equation mgh. Since the mass is $3m$ and the top of the loop is $2r$ above the bottom of the loop, $U_f = 6mrg$

We can plug in our expressions for the energies to get

$$\frac{3mv_o'^2}{2} = \frac{3mrg}{2} + 6mrg = \frac{15mrg}{2}$$

We can cancel $\frac{3m}{2}$ from both sides to get $v_o'^2 = 5rg$
We can take the square root of both sides to get $v_o' = \sqrt{5rg}$

Problem 3.0.29 — 2022 AP Physics 1 FRQ



Two blocks are connected by a string that passes over a pulley, as shown above. Block 1 is on a horizontal surface and is attached to a spring that is at its unstretched length. Frictional forces are negligible in the pulley's axle and between the block and the surface. Block 2 is released from rest and moves downward before momentarily coming to rest.
k_0 is the spring constant of the spring
M_1 is the mass of block 1
M_2 is the mass of block 2
Δy is the distance block 2 moves before momentarily coming to rest
(a)

i. Block 2 starts from rest and speeds up, then it slows down and momentarily comes to rest at a position below its initial position. In terms of only the forces directly exerted on block 2, explain why block 2 initially speeds up and explain why it slows down to a momentary stop.

ii. Derive an expression for the distance Δy that block 2 travels before momentarily coming to rest. Express your answers in terms of k_0, M_1, M_2, and physical constants, as appropriate.

(b) Indicate whether the total mechanical energy of the blocks-spring-Earth system changes as block 2 moves downward.
_____ Changes _____ Does not change
Briefly explain your reasoning.

Solution to part a i: Block 2 initially speeds up since the gravitational force is stronger than tension. However, it slows down because the magnitude of tension force becomes larger than gravitational force.

Solution to part a ii: We can conserve energy in this problem.
Since energy is conserved, we know $K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}$
There are two potential energies on each side since one is spring potential energy while the other is gravitational potential energy.

The initial and final kinetic energies for each block is 0. The reason is that they both start at rest and momentarily end at rest.

This means the equation can be simplified to $U_{gi} + U_{si} = U_{gf} + U_{sf}$

In this problem, there will be no spring potential energy initially. The reason is that the spring isn't stretched. Eventually, the system of the two blocks will slide causing Block 2 to move down. Thus, gravitational potential energy will be lost. However, spring potential energy will increase since the spring will become stretched. This means that the change in gravitational potential energy will transform to spring potential energy.

Using this, we can know that $\Delta U_g = U_s$

$$\text{Since } M_2 \text{ moves down } \Delta y, U_g = M_2 g \Delta y$$

The spring also gets stretched by a distance Δy since that's the distance block 2 moves down. This means that the spring potential energy at the end is $\frac{1}{2}k_0\Delta y^2$

$$\text{We can substitute this to get } M_2 gy = \frac{1}{2}k_0\Delta y^2$$

$$\text{We can solve for } \Delta y \text{ to find that } \Delta y = \frac{2M_2 g}{k}$$

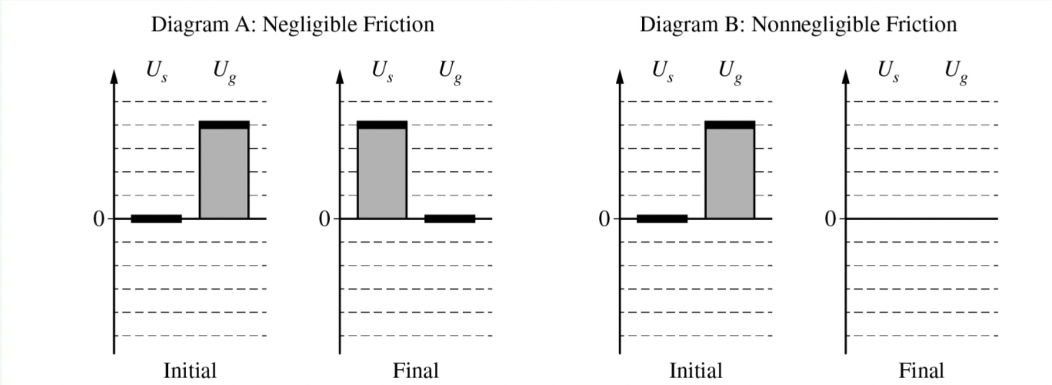
Solution to part b: Mechanical energy **does not change**. The reason is that there are no conservative forces such as friction acting on the system. Thus, mechanical energy stays the same.

Problem 3.0.30 — 2022 AP Physics 1 FRQ Continued

This is a continuation to the FRQ from above. Make sure to refer back to it to solve this part.

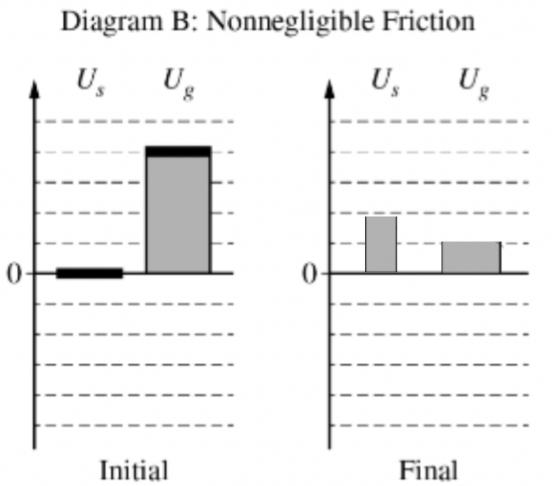
Consider the system that includes the spring, Earth, both blocks, and the string, but not the surface. Let the initial state be when the blocks are at rest just before they start moving, and let the final state be when the blocks first come momentarily to rest. Diagram A at left below is a bar chart that represents the energies in the scenario where there is negligible friction between block 1 and the surface.

The shaded-in bars in the energy bar charts represent the potential energy of the spring and the gravitational potential energy of the blocks-Earth system, U_s and U_g, respectively, in the initial and final states. Positive energy values are above the zero-point line ("0") and negative energy values are below the zero-point line.



(c) Complete diagram B (at right above) for the scenario in which friction is non-negligible. The energies for the initial state are already provided. Shade in the energies in the final state using the same scale as in diagram A.

Solution to part c: Since friction is non-negligible, energy will now be lost. This means that the final sum of U_s and U_g must be less than the initial sum. The initial sum is 4 units. As long as our diagram has a sum of less than 4 units for the final stage, then we will earn the point on the AP exam.



Note that the height of the graph matters. The width doesn't. Don't waste time trying to get equally wide graphs. I made the graph for U_g wider to explain that only the heights of the bars matter.

Problem 3.0.31 — 1986 AP Physics C: Mechanics FRQ
A special spring is constructed in which the restoring force is in the opposite direction to the displacement, but is proportional to the cube of the displacement; i.e., $F = -kx^3$.

This spring is placed on a horizontal frictionless surface. One end of the spring is fixed, and the other end is fastened to a mass M. The mass is moved so that the spring is stretched a distance A and then released. Determine each of the following in terms of k, A, and M.

(a) The potential energy in the spring at the instant the mass is released

(b) The maximum speed of the mass

(c) The displacement of the mass at the point where the potential energy of the spring and the kinetic energy of the mass are equal

Solution to part a: We don't have a linear spring. This means that we must use calculus to find the potential energy.

$$U_s = -\int_a^b \vec{F}(x) \cdot d\vec{x}$$

We can plug in $F = -kx^3$ and integrate. Our bounds are from 0 (when it's at equilibrium) to A (distance stretched)

$$U_s = -\int_0^A -kx^3 dx = -(-\frac{x^4}{4})|_0^A = \frac{A^4}{4}$$

Solution to part b: We know that the potential energy we found in part a is the maximum possibly mechanical energy. We can maximize kinetic energy when potential energy is 0. This means that all of our potential energy in part a (which is $\frac{A^4}{4}$) should convert to kinetic energy. We can equate our expression for potential energy to $\frac{1}{2}Mv^2$

Doing so gives us the equation $\frac{A^4}{4} = \frac{1}{2}Mv^2$

We can isolate v to find that $v_{max} = A^2\sqrt{\frac{k}{2M}}$

Solution to part c: We know that the total energy (potential + kinetic) at any point is $\frac{A^4}{4}$.
We want half of this energy to be in the form of potential energy while the other half to be in the form of kinetic energy.

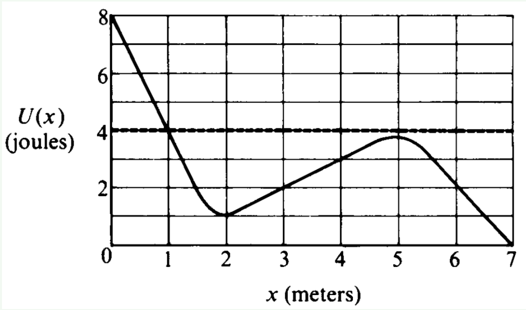
This means we want $\frac{A^4}{8}$ joules of spring potential energy.
If the stretched distance from equilibrium is x, then the spring potential energy is $\frac{1}{4}kx^4$ (using the formula we derived for potential energy in part a)

We set both of these expressions equal to each other and solve for x (the displacement of the mass).

We can write the equation $\frac{A^4}{8} = \frac{1}{4}kx^4$

$$\text{This means that } x = \frac{A}{\sqrt[4]{2}}$$

Problem 3.0.32 — 1987 AP Physics C: Mechanics FRQ



The above graph shows the potential energy $U(x)$ of a particle as a function of its position x.

(a) Identify all points of equilibrium for this particle.

Suppose the particle has a constant total energy of 4.0 joules, as shown by the dashed line on the graph.
(b) Determine the kinetic energy of the particle at the following positions

i. $x = 2.0$ m

ii. $x = 4.0$ m

(c) Can the particle reach the position $x = 0.5$ m? Explain.

(d) Can the particle reach the position $x = 5.0$ m? Explain,

Solution to part a: We know that the derivative of a potential energy and displacement curve gives us force.
$F = -\frac{dU}{dx}$
Equilibrium occurs when the force $F = 0$. This means that we want $-\frac{dU}{dx}$ to be 0. In terms of calculus, this means that we want our slope to be a horizontal line at al points of equilibrium.

Clearly, there is a slope of 0 at $x = 2$ and $x = 5$.

Solution to part b i: We know that the total energy is 4 J. This means that $K + U = 4$ at all points. Although the potential/kinetic energies might decrease/increase overtime, the sum of both quantities should be 4.

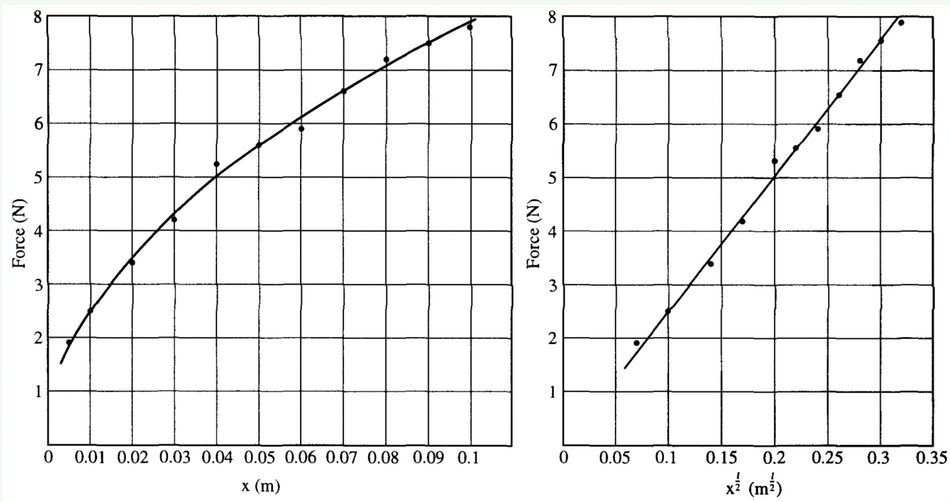
At $x = 2$, the potential energy $U = 1$ J. This means that the kinetic energy K must be 3 J so that the sum of both energies is 4.

Solution to part b ii: Similarly, we want the potential and kinetic energies both to sum to 4.
At $x = 4$ m, the potential energy $U = 3$ J. This means that the kinetic energy K must be 1 J.

Solution to part c: At $x = 0.5$ m, the potential energy is 6 J. However, we know that our total energy is 4 J. Thus, this isn't possible. Our potential energy and kinetic energy must sum to 4. Since our potential energy is 6 at 0.5 m, this exceeds the total energy of 4 J which means it can't occur.

Solution to part d: At $x = 5$ m, the potential energy is slightly less than 4 J. This position is indeed possible since the sum of the potential and kinetic energy can be equal to 4 (since the potential energy doesn't exceed the possible total energy of 4).

Problem 3.0.33 — 1997 AP Physics C: Mechanics FRQ
A nonlinear spring is compressed horizontally. The spring exerts a force that obeys the equation $F(x) = Ax^{1/2}$, where x is the distance from equilibrium that the spring is compressed and A is a constant. A physics student records data on the force exerted by the spring as it is compressed and plots the two graphs below, which include the data and the student's best-fit curves.



(a) From one or both of the given graphs, determine A. Be sure to show your work and specify the units.

(b) Determine an expression for the work done in compressing the spring a distance x.

Solution to part a: We can simply use the left graph and plug in one point into the equation $F = Ax^{\frac{1}{2}}$.
One point that lies on the curve is $(0.01, 2.5)$

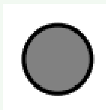
We can plug that in to get $2.5 = A\sqrt{0.01}$
We can solve for A to find that $A = 25$

Solution to part b: We know that work is the integral of force with respect to the displacement. That means we can integrate $F(x) = Ax^{1/2}$.

$$W = \int_0^x F(x) = \int_0^x Ax^{1/2}$$
$$\frac{2}{3}Ax^{3/2}$$

Problem 3.0.34 — A rubber ball of mass m is dropped from a cliff. As the ball falls, it is subject to air drag (a resistive force caused by the air). The drag force on the ball has magnitude bv^2, where b is a constant drag coefficient and v is the instantaneous speed of the ball. The drag coefficient b is directly proportional to the cross-sectional area of the ball and the density of the air and does not depend on the mass of the ball. As the ball falls, its speed approaches a constant value called the terminal speed.

(a) On the figure below, draw and label all the forces on the ball at some instant before it reaches terminal speed.



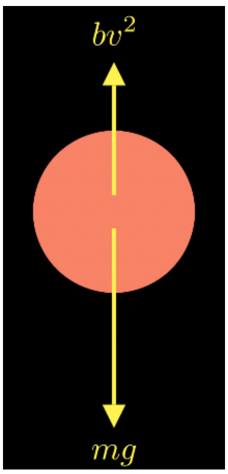
(b) State whether the magnitude of the acceleration of the ball of mass m increases, decreases, or remains the same as the ball approaches terminal speed. Explain.

(c) Write, but do NOT solve, a differential equation for the instantaneous speed v of the ball in terms of time t, the given quantities, and fundamental constants.

(d) Determine the terminal speed v_t in terms of the given quantities and fundamental constants.

(e) Determine the energy dissipated by the drag force during the fall if the ball is released at height h and reaches its terminal speed before hitting the ground, in terms of the given quantities and fundamental constants.

Solution to part a:



Solution to part b: This part should remind you of the topics we discussed in Unit 2. As the ball falls, the speed increases. This causes the drag force to increase in magnitude, opposing the gravitational force. Since the net force now decreases due to the increasing drag force, the acceleration decreases.

Solution to part c: Whenever we write a differential equation for v, we will use the expression $a = \frac{dv}{dt}$.

We can write an expression for acceleration using Newton's Second Law. We will first find the net force, and then use $F_{net} = ma$
We know that the net force is $mg - bv^2$. This means that $a = g - \frac{bv^2}{m}$

$$\text{This means that } \frac{dv}{dt} = g - \frac{bv^2}{m}$$

Solution to part d: The terminal speed v_t is reached when the net force on the ball is 0. When the net force is 0, the acceleration will be 0 causing the ball to reach a terminal velocity.

The net force will be 0 when the drag force cancels out the gravitational force. This occurs when $mg - bv^2 = 0$.
This means that $v^2 = \frac{mg}{b}$ which simplifies to $v_t = \sqrt{\frac{mg}{b}}$. This is the terminal velocity since that is when the net force will be 0.

Solution to part e: Initially when the ball is dropped, it will only have gravitational potential energy. When it reaches terminal velocity, there will be kinetic energy. We can make our reference level be the point where the ball reaches terminal velocity since it will cause the gravitational potential energy at that point to be 0.

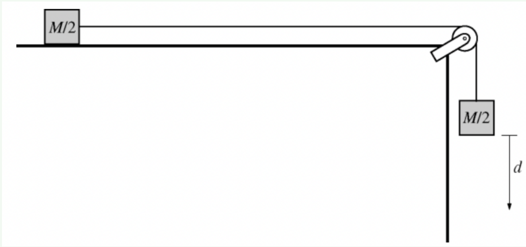
This means that the energy dissipated is $U_i - K_f$

The initial potential energy $U_i = mgh$. The final kinetic energy $K_f = \frac{1}{2}mv_f^2$
We can plug in $v_t = \sqrt{\frac{mg}{b}}$ into $K_f = \frac{1}{2}mv_f^2$ to simplify the expression for kinetic energy.

$$\text{Plugging it in gives } K_f = \frac{1}{2} \cdot m \cdot \frac{mg}{b}$$

$$\text{This means that the dissipated energy is } U_i - K_f = mgh - \frac{m^2g}{2b}$$

Problem 3.0.35 — 2009 AP Physics C: Mechanics FRQ



A block of mass $M/2$ rests on a frictionless horizontal table, as shown above. It is connected to one end of a string that passes over a massless pulley and has another block of mass $M/2$ hanging from its other end. The apparatus is released from rest.

(a) Derive an expression for the speed v_h of the hanging block as a function of the distance d it descends.
Now the block and pulley system is replaced by a uniform rope of length L and mass M, with one end of the rope hanging slightly over the edge of the frictionless table. The rope is released from rest, and at some time later there is a length y of rope hanging over the edge, as shown below. Express your answers to parts (b), (c), and (d) in terms of y, L, M, and fundamental constants.

(b) Determine an expression for the force of gravity on the hanging part of the rope as a function of y.

(c) Derive an expression for the work done by gravity on the rope as a function of y, assuming y is initially zero.

(d) Derive an expression for the speed v_r of the rope as a function of y.

(e) The hanging block and the right end of the rope are each allowed to fall a distance L (the length of the rope). The string is long enough that the sliding block does not hit the pulley. Indicate whether v_h from part (a) or v_r from part (d) is greater after the block and the end of the rope have traveled this distance.
______ v_h is greater.   ______ v_r is greater.   ______ The speeds are equal.
Justify your answer.

Solution to part a: We will conserve energy to find the speed v_h of the hanging block. To find kinetic energy, we must first find the change in potential energy.

When the hanging block descends a distance of d, the change in potential energy ΔU will be $-\frac{Mgd}{2}$. The change in potential energuy of the block on the table will be 0. The reason is that it remains on the table, so there is no change in height unlike the hanging block.

We can now use $\Delta K + \Delta U = 0$

Since $\Delta U = -\frac{Mgd}{2}$, we can plug that in to find that $\Delta K = \frac{mgd}{2}$.

The final kinetic energy will involve accounting for velocities of both of the blocks. The reason is that both are connected and moving at the same speed. The final kinetic energy for one block is $\frac{1}{2} \cdot \frac{M}{2} \cdot v_h^2$
We must multiply this by 2 since we have two blocks with the same kinetic energy. This means that

$$K_f = 2 \cdot \frac{1}{2} \cdot \frac{M}{2} \cdot v_h^2 = \frac{Mv_h^2}{2}$$

We can equate our kinetic energy to change in potential energy to write the equation: $\frac{Mv_h^2}{2} = \frac{Mgd}{2}$
We can rearrange the equation and simplify to find that $v_h = \sqrt{gd}$

Solution to part b: We know $\frac{y}{L}$ fractional part of the rope will be hanging. We can multiply this to the total mass to find the mass of hanging part of the rope. The mass of the hanging part is $\frac{My}{L}$

The force of gravity can be found by multiplying the mass of the hanging rope to g. We find that the force is $\frac{Mgy}{L}$

Solution to part c: We know that the work done by a force can be found using the formula $W = \int F dy$
In this problem, instead of integrating with respect to x, we will integrate with respect to y since that's the variable used to denote the length of the hanging rope.

$$W = \int \frac{Mgy}{L} dy = \frac{Mgy^2}{2L}$$

Solution to part d: We know that the work done will be equivalent to the change of kinetic energy.

$$W = \Delta K$$

We already know that $W = \frac{Mgy^2}{2L}$

We also know that $\Delta K = \frac{1}{2} M v_r^2$

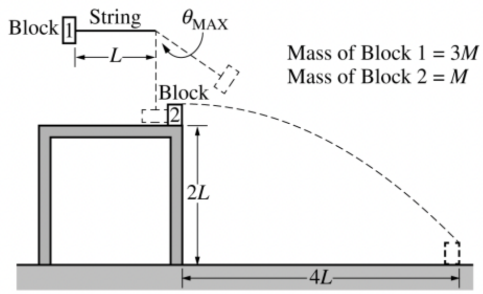
We can equate both expressions to write $\frac{Mgy^2}{2L} = \frac{1}{2} M v_r^2$

We can isolate v_r to find that $v_r = y\sqrt{\frac{g}{L}}$

Solution to part e: In part a, we found that $v_h = \sqrt{gd}$. Since our distane travelled is L, we can plug that in to find that $v_h = \sqrt{gL}$

Similarly, in part d we found that $v_r = y\sqrt{\frac{g}{L}}$. We can plug in $y = L$ (since that's the length of the hanging part) to find that $v_r \sqrt{gL}$
Clearly, the speeds are equal and they are both $\sqrt{gL}$.

Problem 3.0.36 — 2019 AP Physics C Mechanics FRQ

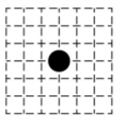


Note: Figure not drawn to scale.

A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{max} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

(b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



(c) Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

For parts (d)–(g), the value for the length of the pendulum is L = 75 cm.

(d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

(e) Calculate the speed of block 2 as it leaves the table.

(f) Calculate the speed of block 1 just after it collides with block 2.

(g) Calculate the angle θ_{max} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.

Note: Do parts f and g after you cover Unit 4. You need to know the topic of momentum to be able to solve both parts. Please come back to this problem once you cover Unit 4.

Solution to part a: This problem speaks conservation of energy since we know the difference in height of block 1 during its motion.

We know that $K_i + U_i = K_f + U_f$

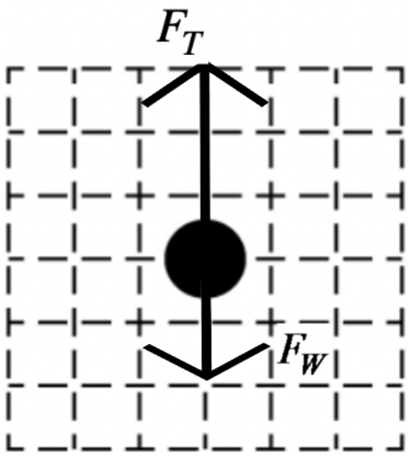
At the top, Block 1 is a distance L above the bottom of its swing. The reason is that it moves down the length of the pendulum during its swing. We can set our reference level to be the table. This will cause U_f to be 0. U_i can be found using the formula $U = mgh$. The height is L, so $U_i = mgL$

At the top, block 1 has no velocity. Thus, it has no kinetic energy initially causing K_i to be 0.
We know that $K_f = \frac{1}{2}mv^2$.

We can plug all of this in to get that $mgL = \frac{1}{2}mv^2$
m cancels out and it leaves us with $gL = \frac{v^2}{2}$
We can isolate v to find that $v = \sqrt{2gL}$

Solution to part b: The only forces on block 1 right before it comes into contact with block 2 are tension force and gravitational force.

Tension force points upwards and gravitational force points downward. Our tension force must be larger since circular motion is occurring. There must be an acceleration pointing towards the center (the centripetal acceleration). For this to be true, tension force must be larger in magnitude.



Solution to part c: We must use Newton's Second Law to relate net force to acceleration.
We know that $F_{net} = ma$
The net force is $F_T - mg$
The block has centripetal acceleration which can be written as $a = \frac{v^2}{r}$. For our problem, the radius r should be written as L (since that's the length of the string).

$$\text{Thus, } F_T - mg = \frac{mv^2}{L}$$

We know that $F_W = mg$
We can plug that in and solve for F_T to find that $F_T = m(g + \frac{v^2}{L})$

Now, we must use $3M$ for the mass (since that's the mass of block 1). In addition, we already found that $v = \sqrt{2gL}$.

$$\text{Plugging all of this in gives that } F_T = 3M(g + \frac{(\sqrt{2gL})^2}{L} = 3M(g + 2g) = 9Mg$$

Solution to part d: This should remind us of our kinematics unit. We must consider the motion in the y-direction. When block 2 leaves the table, it has an initial velocity in the y-direction of 0. It's acceleration is g. We also know that its vertical displacement is $2L$ (the height of the table).

We can use the kinematics equation $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$.
Plugging in our variables gives $2L = 0 + \frac{gt^2}{2}$
We can isolate t to find that $t = \sqrt{\frac{4L}{g}}$. Now, we can plug in our value of L. We must convert it to meters first. This means that $L = 75$ cm $= 0.75$ m.

$$\text{Plugging it in gives } t = \sqrt{\frac{4 \cdot 0.75}{g}} = 0.55 \text{ s}$$

Solution to part e: The velocity that block 2 leave at will remain constant in the x-direction. We should know this from our projectile unit. The horizontal component of velocity remains constant!

$$\text{This means that } v_x t = \Delta x$$

We already know that the time of the motion s 0.55 s. We also know that $\Delta x = 4L = 4 \cdot 0.75 = 3$ m.

This means that $v_x = \frac{3}{0.55} = 5.45 m/s$.

Temporarily skip parts f and g if you haven't covered Unit 4 yet. You need to know conservation of momentum to be able to solve it. Make sure to come back to these parts after you cover that unit!

Solution to part f: Since both blocks collide and there are no external forces, we know we can conserve momentum.

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Right before collision, block 1 has a velocity of $v_{1i} = \sqrt{2gL} = \sqrt{2 \cdot g \cdot 0.75} = 3.83 m/s$. This velocity occurs after it swings down, right before it collides with block 2.

The initial velocity of block 2 at that instant is 0 since it is at rest.

After they both collide, block 2 has a final velocity of $v_{2f} = 5.45$

We can plug all of this into our equation to get

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\text{We isolate } v_{1f} \text{ to get } \frac{m_1 v_{1i} - m_2 v_{2f}}{m_1}$$

We can now plug in our variables to get $v_{1f} = \frac{3M \cdot 3.83 - M \cdot 5.45}{3M} = \frac{3 \cdot 3.83 - 5.45}{3} = 2.01 m/s$.

Solution to part g: After collision, block 2 will move towards the floor. However, since block 1 is tied to the string, it will continue to swing up.
Block 1's kinetic energy after collision will convert to gravitational potential energy.
This means that $K_i = U_f$

We know that the height block 1 gains is $L(1 - \cos\theta)$

This means it gains a gravitational potential energy of $mgL(1 - \cos\theta)$
It's kinetic energy after collision can be found by using the formula $K = \frac{1}{2}mv^2$

We can set both expressions equal to each other: $\frac{1}{2}mv^2 = mgL(1 - \cos\theta)$
Mass cancels out leaving us with $\frac{v^2}{2} = gL(1 - \cos\theta)$

We can now plug in our variables. We know that $v = 2.01$ m/s since this is block 1's final velocity. We also know that $L = 0.75$ m.

$$\text{Plugging this in gives } \frac{2.01^2}{2} = g \cdot 0.75(1 - \cos\theta)$$

We can expand and isolate $\cos\theta$ to get $\cos\theta = 0.725$
This means that $\theta = 43.53°$.

Problem 3.0.37 — 2006 AP Physics C Mechanics FRQ

A nonlinear spring is compressed various distances x, and the force F required to compress it is measured for each distance. The data are shown in the table below.

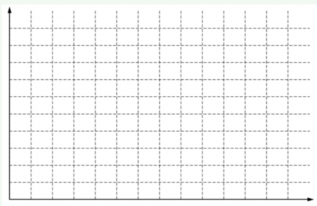
x (m)	F (N)	
0.05	4	
0.10	17	
0.15	38	
0.20	68	
0.25	106	

Assume that the magnitude of the force applied by the spring is of the form $F(x) = Ax^2$.

(a) Which quantities should be graphed in order to yield a straight line whose slope could be used to calculate a numerical value for A?

(b) Calculate values for any of the quantities identified in (a) that are not given in the data, and record these values in the table above. Label the top of the column, including units.

(c) On the axes below, plot the quantities you indicated in (a). Label the axes with the variables and appropriate numbers to indicate the scale.



(d) Using your graph, calculate A.

The spring is then placed horizontally on the floor. One end of the spring is fixed to a wall. A cart of mass 0.50 kg moves on the floor with negligible friction and collides head-on with the free end of the spring, compressing it a maximum distance of 0.10 m.

(e) Calculate the work done by the cart in compressing the spring 0.10 m from its equilibrium length.

(f) Calculate the speed of the cart just before it strikes the spring.

Solution to part a: If we simply graph the force and displacement, then we won't form a linear line. The reason is that $F(x) = Ax^2$. This means that the the force is proportional to the distance squared. Thus, we must graph F vs. x^2. (You can also graph $\sqrt{F}$ vs. x as long as you're consistent with the graph).

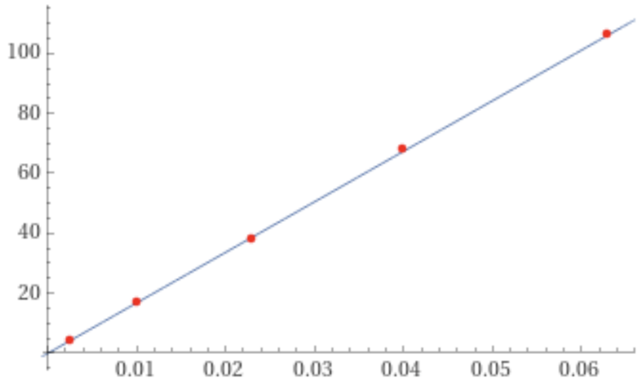
Solution to part b: In the column that we're given, we'll write the various values of

x^2. This will allow us to graph F vs. x^2.

x (m)	F (N)	x^2 (m^2)
0.05	4	0.0025
0.10	17	0.010
0.15	38	0.023
0.20	68	0.040
0.25	106	0.063

Image Credits: College Board website

Solution to part c: We now graph the points from the table. x^2 will go on the x-axis while F will go on the y-axis.



Don't forget to label the units for the graph on the AP exam. For example, the x-axis has a unit of m^2 while the y-axis has a unit of N.

Solution to part d: We know that the slope of our line will be A.
We can use two points from our table to approximate the slope.

$$\text{Doing so gives } A \approx \frac{68 - 38}{0.04 - 0.023} = 1764 \text{ N/m}^2$$

Solution to part e: To find the work done by the spring in the cart, we must integrate force with respect to distance. The reason is that the integral of force with respect to distance is work done.

$$W = \int F(x)dx$$

Since the spring is compressed a maximum distance of 0.10, we must integrate from 0 to 0.10

$$\text{This means that } W = \int_0^{0.10} Ax^2 dx = \frac{Ax^3}{3}|_0^{0.10}$$

$$\text{We can now simplify and plug in } A = 1764 \text{ to find that } W = \frac{1764(0.10)^3}{3} = 0.588 \text{ J.}$$

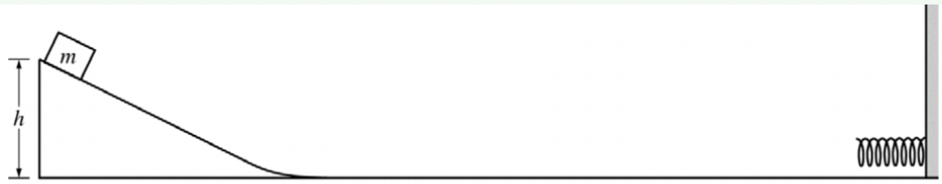
Solution to part f: We know that $W = \Delta K$. This is the work energy theorem and means that the work done will be equivalent to the change in kinetic energy.

Initially, the cart will have kinetic energy before striking the spring. However, after striking the spring and compressing to a maximum distance, the cart will have a kinetic energy of 0. The reason is that the spring will gain the energy in the form of spring potential energy.

This means that $W = \frac{1}{2}mv^2$ since the kinetic energy of the cart before it compresses the spring will be used to compress the spring!

$$\text{We can solve for } v \text{ to get that } v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \cdot 0.588}{0.5}} = 1.533 \text{ m/s}$$

Problem 3.0.38 — 2017 AP Physics C Mechanics FRQ



Note: Figure not drawn to scale.

A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

(a) i. Derive an expression for the speed of the block just before it collides with the spring.
ii. Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?
_____ Greater than _____ Less than _____ Equal to
Justify your answer.

(b) Derive an expression for the maximum compression of the spring.

The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0. Now suppose the block experiences a resistive force as it slides on the horizontal surface. The magnitude of the resistive force $F$is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of kg/m.

(c) i. Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m, h, k, b, v, and physical constants, as appropriate.

ii. Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time $t$can be written in the form $\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$ where v_0 is the speed at $t = 0$.

Solution to part a i: The gravitational potential energy at the top will become kinetic

energy.
We know that $U_i = K_f$
$U_i = mgh$ since the block is a distance h above the ground level. We also know that $K_f = \frac{1}{2}mv_f^2$

We can set both equal to each other to get $mgh = \frac{1}{2}mv_f^2$
Solving for v_f gives that $v_f = \sqrt{2gh}$.

Solution to part a ii: The answer is more than. The reason is that the speed is proportional to the square root of height. When the block move halfway down, then that means the height descended is $\frac{h}{2}$, the speed will be proportional to $\sqrt{\frac{h}{2}}$ which is $\frac{\sqrt{h}}{1.41}$
Clearly, this will be more than half the speed at the bottom of the incline.

Solution to part b: The initial point that we will consider will be the initial point where the block is, a distance h above the ground level.
The final point to consider will be the block at maximum compression. At that point, there will be no gravitational potential energy relative to the ground. There will also be no kinetic energy.

This means that $U_g = U_s$. All of the gravitational potential energy will convert to spring potential energy.
We can find that $mgh = \frac{1}{2}kx_{max}^2$

We can solve for x_{max} to find that $x_{max} = \sqrt{\frac{2mgh}{k}}$.

Solution to part c i: For problems involving a resistive force, the differential equation will often involve $\frac{dv}{dt}$. The reason is that it represents acceleration which can be found by using $F = ma$.

Since $F = -\beta v^2$ (the force is negative because it opposes the velocity), we can find that $a = \frac{\beta v^2}{m}$.

$$\text{Since } a = \frac{dv}{dt}, \text{ we can use that to write the differential equation } \frac{dv}{dt} = -\frac{\beta v^2}{m}$$

Solution to part c ii: For those that don't know how to solve differential equations nor the technique of separation of variables, please check out TMAS Academy's AP Calculus BC book. Unit 7 of the book will cover many problems that will allow you to learn differential equations and how to solve them.

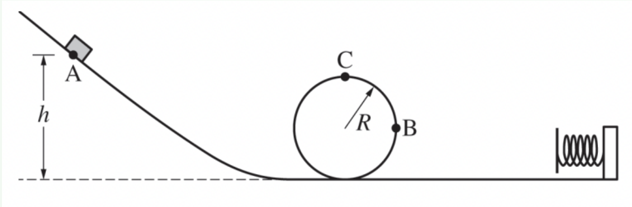
$$\text{Using separation of variables gives } \frac{dv}{v^2} = -\frac{\beta}{m}dt$$

$$\text{Integrating both sides gives } \int_{v_0}^{v(t)} \frac{1}{v^2}dv = -\frac{\beta}{m}\int_0^t dt$$

$$\text{We can solve the integrals to get } -\frac{1}{v(t)} + \frac{1}{v_0} = -\frac{\beta t}{m}$$

$$\text{We can rearrange the equation to get } \frac{1}{v(t)} = \frac{\beta t}{m} + \frac{1}{v_0}$$

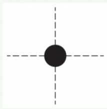
Problem 3.0.39 — 2021 AP Physics C Mechanics FRQ



Note: Figure not drawn to scale.

A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \frac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

(a) On the dot below, which represents the block, draw an arrow that represents the direction of the acceleration of the block at point B in the figure above. The arrow must start on and point away from the dot.



Justify your answer.

(b)

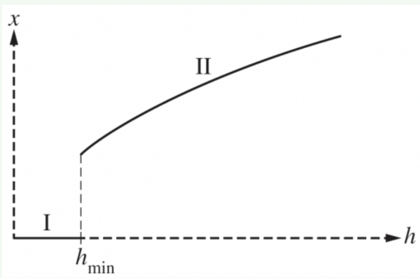
i. Derive an expression for the speed v of the block at point B.

ii. Derive an expression for the magnitude of the net force F on the block at point B.

(c) In terms of R, derive an expression for the minimum height h_{min} necessary for the block to maintain contact with the track through point C.

(d) It is determined that $h = 0.30$ m and $R = 0.10$ m. If the block is released from a height greater than that found in part c, what would be the maximum compression x_{MAX} of the spring?

(e) A graph of the maximum compression of the spring as a function of height is shown below. The height h_{min} is the height calculated in part c.

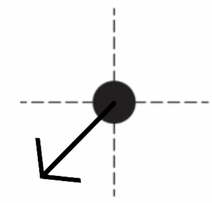


i. Explain why section I appears as a horizontal line segment on the horizontal axis.

ii. Explain the reason for the shape of section II on the graph.

Solution to part a: The block will have centripetal acceleration on the loop. The acceleration vector must point radially (towards the center).

There will also be acceleration in the y-direction caused by gravitational force. That will be pointing downwards. That means the resultant will be a diagonal line in the southwest direction.



Solution to part b i: To find the speed at point B, we should use conservation of energy.
We can use point A our initial point and B as the final point.
Conservation of energy tells us $K_i + U_i = K_f + U_f$

Relative to the ground, the gravitational potential energy at A is mgh. Similarly, the gravitational potential energy at point B is mgR.
The initial velocity is 0 at point A. Thus, $K_i = 0$. The kinetic energy at point B can be represented as $K_f = \frac{1}{2}mv^2$

We can plug all of this in to write $mgh = \frac{1}{2}mv^2 + mgR$
We can cancel m from both sides to get $gh = \frac{v^2}{2} + gR$
We can subtract gR from both sides to get $\frac{v^2}{2} = gh - gR = g(h - R)$
Now, we can multiply both sides by 2 and then square root both sides to get

$$v = \sqrt{2g(h - R)}$$

Solution to part b ii: At point B, there will be a normal force F_N towards the center that causes the centripetal acceleration. There will also be a gravitational force F_g downwards.

This means that the net force is $F_{net} = \sqrt{F_N^2 + F_g^2}$
Now, we can apply Newton's Second Law with the normal force.
We know that $F_N = ma = \frac{mv^2}{R}$ (since the normal force contributes to the centripetal force)

We can plug this in along with $F_g = mg$ to get $F_{net} = \sqrt{(\frac{mv^2}{R})^2 + (mg)^2}$

Now, we can substitute the velocity that we found in part b i. Substituting $v = \sqrt{2g(h - R)}$ gives

$$F_{net} = \sqrt{(\frac{2mg}{R}(h - R))^2 + (mg)^2}$$

Solution to part c: We first use conservation of energy to find an expression using h_{min}.
We know that $K_i + U_i = K_f + U_f$
Our initial point is A and final is now C. That means $U_i = mgh$ and $U_f = mg \cdot 2R = 2mgR$ (since $2R$ is the height of point C above the ground).

We also know that $K_i = 0$. Assuming that the velocity at point C is v_c, we know that $K_f = \frac{1}{2}mv_c^2$

We can substitute all of that into our equation for conservation of energy to get

$$mgh = 2mgR + \frac{1}{2}mv_c^2$$

We can isolate v_c to find that $v_c = \sqrt{2g(h-2R)}$

Now, we write an equation using Newton's Second Law at point C. We know that normal force and gravitational force both point downwards.

$$\text{This means that } F_N + mg = \frac{mv_C^2}{R}$$

Whenever we are asked to find the minimum height necessary, we have a limiting problem. That means the normal force must be the lowest possible. Thus, the normal force at point C should be 0.

We can plug in $F_N = 0$ to get $mg = \frac{mv_C^2}{R}$.
We can isolate v_C in the equation above to find that $v_C = \sqrt{gR}$

We used conservation of energy and Newton's Second Law to find v_C two times. Now, we can set both expressions we found for v_C equal to each other.

This gives us the equation: $\sqrt{2g(h-2R)} = \sqrt{gR}$

We can square both sides to get $2g(h-2R) = gR$

We can divide both sides by g and expand to get $2h - 4R = R$ which means that $2h = 5R$

A final simplification gives that $h = \frac{5R}{2}$. This means that $h_{min} = \frac{5R}{2}$

Solution to part d: We will conserve energy again. Whenever a problem asks you to find the maximum compression, you should immediately think of energy conservation. It's simple to conserve energy since at maximum compression, velocity is 0. This means that our kinetic energy at that point will be 0.

At point A, there is only gravitational potential energy. We will make the reference level the ground. This will cause the gravitational potential energy at the level of the spring to be 0.
All of the gravitational potential energy at point A will be converted to spring potential energy. The gravitational potential energy at point A is mgh.

Spring potential energy can be represented as $U_s = \frac{1}{2}kx_{max}^2$
Since $U_g = U_s$, we can write the equation $mgh = \frac{1}{2}kx_{max}^2$

We can isolate x_{max} to find that $x_{max} = \sqrt{\frac{2mgh}{k}}$
We know that $k = \frac{mg}{2R}$. We can plug this in to find that

$$x_{max} = \sqrt{2mgh \cdot \frac{2R}{mg}} = \sqrt{4hR} = \sqrt{4 \cdot 0.3 \cdot 0.1} = 0.35 \text{ m}$$

Solution to part e i: When the height is lower than h_{min}, then there won't be enough gravitational potential energy to reach the spring. That is why section I is horizontal since there is no compression that occurs for the spring.

Solution to part e ii: As the height increases, the gravitational potential energy goes up. Thus, there will be a greater amount of energy that will convert to spring potential energy. This will cause the distance of compression to go up. However, the rate at which the spring compresses decreases. The reason is that height is proportional to the square of the distance compressed.

Unit 4 Systems of Particles and Linear Momentum

Have you ever wondered what makes collisions so dangerous? If a car collides with a wall, then why is it so dangerous?

To answer such questions, you must dive deep into this unit and learn about momentum.

Note 4.0.1 — Momentum
The momentum (p) of an object is it's mass times velocity (m · v).
The momentum of a *system* is defined as the sum of each object's momentum in that system

Clearly, since momentum is proportional to mass, an object with a larger mass will have a larger momentum. A lighter mass will have a smaller momentum.

Similarly, an object moving at a high speed will have a greater magnitude of momentum in contrast to a slow object.

To conceptually think about this scenario, try to picture a photon of light coming towards you. Light is the fastest thing. However, when it strikes you, you won't feel anything.
The reason is that a photon of light has negligible mass. It's mass is so so so small, then it's momentum is negligible. However, if a car runs straight into you, then the momentum will be great. Even though the car has a lower velocity than light, the mass of a car is much larger. That is why a car colliding with a person can have disastrous effects.

Note 4.0.2 — Impulse
On an object, if there is a force F applied for a time t, then we say that the impulse on the object is

$$J = F\Delta t.$$

Another important thing is that the change in momentum of an object is the impulse applied on it:

$$J = \Delta p = m\Delta v.$$

The same equation for impulse that involves force can be written with calculus!

$$\vec{J} = \Delta\vec{p} = \int \vec{F}dt$$

$\vec{J}$ represents impulse. $\Delta\vec{p}$ represents the change in momentum. $\int \vec{F}dt$ represents the integral of force with respect to time, which is also impulse.

In addition, if we know the average force on an object, then we can also say that the impulse is $F_{avg}t$

If you are given a force-time graph, then the area under the curve is the impulse.

If you know your change in momentum, then you can divide it by time to find the average force.

$$F_{\text{avg}} = \frac{\Delta p}{t}$$

Now, we know how to find the average force given the impulse. Why does this make colliding with a wall so dangerous?

In an accident, a car rapidly comes to rest. This causes the average force on it to be extremely high. This average force can shatter the car and cause the passengers many serious health problems. The large impulse occurs over such a small period of time, and this maximizes the average force.

Remember that the derivative of momentum with respect to time is force!

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Sometimes you will be given an expression for momentum in terms of t. Then, you can simply differentiate it to find the force.

Note 4.0.3 —
The Law of Conservation of Momentum states that as long as there are no external forces, the momentum of a system will be conserved.

In AP Physics conservation of momentum problems, you should make an equation which equates the initial and final momentum, such as the following:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$

Some values will be given while some will be unknown. You can use such an equation to find the unknown values.

Note 4.0.4 — Elastic Vs. Inelastic Collisions
In an elastic collision, not only is momentum conserved, but kinetic energy is also conserved.
However, in an inelastic collision, although momentum is conserved, kinetic energy is NOT conserved.

In an inelastic collision, some energy is lost through heat or sound. In a **completely inelastic collision**, the objects stick together and move at the same velocity after colliding.

If a problem tells you that two objects stick together after collision, then you can automatically say that it's an inelastic collision. Momentum will be conserved, but kinetic energy will not.

If you are not told whether a collision is inelastic or elastic, you should find the initial kinetic energy of the system and the final kinetic energy. If both quantities equate, then it's an elastic collision. However, if they don't equate, then it's an inelastic collision because kinetic energy wasn't conserved.

Problem 4.0.5 — Two sumo wrestlers are in a match. At the start of the match, they both lunge at each other. They hit and miraculously come to a stand still. One wrestler was 200kg and traveling at a velocity of 2.3ms at the instance of collision. If the other wrestler was traveling at 2.9ms, what is his mass?

Solution: In this case, we can apply the law of conservation of momentum. Since they come to a "stand still" after the collision, the final momentum is zero. Hence, the initial momentum must also be zero. That is,

$$m_1 v_{1i} + m_2 v_{2i} = 0$$
$$\implies m_1 = \frac{m_2 v_{2i}}{v_1} = \frac{(200)(2.3)}{2.9} = \boxed{159\text{kg}}.$$

Problem 4.0.6 — Initially, a car of mass m_1 is moving at a speed v_1 towards another car of mass m_2 at rest. Eventually, the two cars collide, and a completely inelastic collision occurs. What is the speed of both cars after they collide.

Solution: Since this is a collision, momentum is conserved. However, energy is not because it is given that the collision is inelastic. After colliding, both cars move at the same speed which we assume is v_f.

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$

m_2 is at rest initially, so $m_1 v_1 = m_1 v_f + m_2 v_f$

Dividing both sides by $m_1 + m_2$ gives that $v_f = \dfrac{m_1 v_1}{m_1 + m_2}$

Problem 4.0.7 — A 0.15kg baseball is thrown with a speed of 40m/s. If it takes 0.7s for the baseball to come to rest in the catcher's glove, what is the average force the catcher experiences due to the ball?

Recall that $J = Ft$. In this case, F varies, so the equation is

$$J = F_{\text{avg}} t$$

$$\implies F_{\text{avg}} = \frac{J}{t} = \frac{\Delta mv}{t} = \frac{(0.15)(40)}{0.7} = \boxed{8.57\text{N}}.$$

Note 4.0.8 — Center Of Mass Center Of Mass (COM) is the point where the mass is balanced in a gravitational field.

$$x_{\text{cm}} = \frac{\sum_{i=1}^{N} m_i x_i}{M}$$

To find the center of mass in a two (or even three) dimensional coordinate system, we can find the center of mass in each dimension using the one dimensional formula above. This will give us a final coordinate (x_{cm}, y_{cm}, z_{cm}), which is our center of mass in the 3 dimensional space.

An important idea to know is that if the net force is 0, then the acceleration of the COM is 0.

It is very unlikely that you will need to use the formula for the center of mass on the AP Exam. However, you should conceptually understand what the center of mass is and the idea behind it.

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Many of you might be wondering the reason regarding why momentum is conserved in a collision.

We know that when two objects collide, they will each apply a force on each other. However, that force is an **internal force** when we consider both objects to be part of the same system. Although that force changes the velocity of each individual object, the velocity of the **center of mass** remains constant. That is why momentum is conserved in a collision.

Problem 4.0.9 — 2008 AP Physics B FRQ

A 70 kg woman and her 35 kg son are standing at rest on an ice rink, as shown above. They push against each other for a time of 0.60 s, causing them to glide apart. The speed of the woman immediately after they separate is 0.55 m/s. Assume that during the push, friction is negligible compared with the forces the people exert on each other.

(a) Calculate the initial speed of the son after the push.

(b) Calculate the magnitude of the average force exerted on the son by the mother during the push.

(c) How do the magnitude and direction of the average force exerted on the mother by the son during the push compare with those of the average force exerted on the son by the mother? Justify your answer.

(d) After the initial push, the friction that the ice exerts cannot be considered negligible, and the mother comes to rest after moving a distance of 7.0 m across the ice. If their coefficients of friction are the same, how far does the son move after the push?

Solution to part a: Before they separate from each other, the momenta of both people is simply 0. The reason is that both aren't moving which means velocity is 0.

However, after they separate from each other, the woman and her son move in opposite directions. We can conserve momentum in this problem.

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

m_1 is the mass of the woman while m_2 is the mass of the son.

Since initial velocity for both is 0, the equation becomes

$$0 = m_1 v_{1f} + m_2 v_{2f}$$

From here, we are already given that v_{1f} is 0.55 m/s. We can use this value to find the final velocity for the son.

Plugging in our values gives $0 = 70 \cdot 0.55 + 35 \cdot v_{2f}$

We can solve this to get that v_{2f} is -1.1. Since the problem asks for the speed, we can simply take the absolute value of -1.1 to get 1.1 m/s as our final answer.

Solution to part b: Whenever a problem related to a collision asks us to find the force, we should think about the impulse-momentum theorem since it relates force to the change in momentum.

$$F\Delta t = m(v_f - v_i)$$

We already know that our Δt is 0.6 s since that's the amount of time the collision lasted.

The son's final speed was 1.1 and their initial speed was 0. We can plug these values in to find the average force.

$$F \cdot 0.6 = 35(1.1 - 0)$$

We can find that $F = 64.17$ N.

Solution to part c: The force exerted by the mom on the son and the force exerted by the son on the mom satisfy the Newton's third law pair. This means that the forces are equal and in opposite directions.

Solution to part d: Since the mother comes to a stop because of friction, her final velocity is 0. We already know that her initial velocity (after separating) was 0.55m/s. On top of that, we know the displacement of the motion to be 7m.
Thus, we can apply the kinematics equation $v_f^2 = v_i^2 + 2 \cdot a \cdot \Delta x$

Plugging in our values gives $0^2 = 0.55^2 + 2 \cdot a \cdot 7$

Solving for acceleration (a) gives 0.02161 m/s^2.

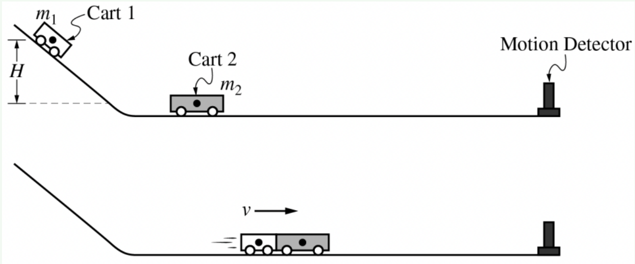
Since acceleration is only caused by the frictional force, we can see that the net force in the x direction is μmg (the frictional force). We can equate this to ma to find the acceleration to be μg.
This means that acceleration is **not** dependent on mass. It will be the same for both the mom and the son. That means we can use this acceleration value to see how far the son moves.

We know that the son's initial velocity (velocity after separating) is 1.1m/s. We also know that his final velocity is simply 0. We can again use the equation $v_f^2 = v_i^2 + 2 \cdot a \cdot \Delta x$.

Plugging in our values gives $0^2 = 1.1^2 + 2 \cdot 0.02161 \cdot \Delta x$

Solving for Δx gives us an answer of 28m.

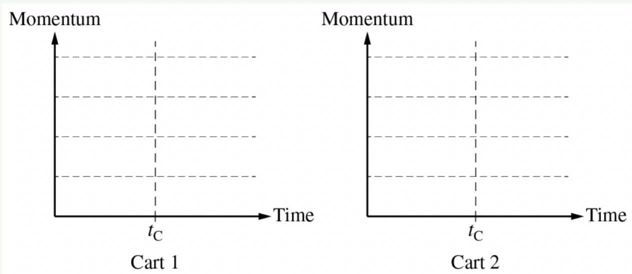
Problem 4.0.10 — 2022 AP Physics C: Mechanics FRQ



Cart 1 of mass m_1 is held at rest above the bottom of an incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and smoothly transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance of h, as shown. At time t_C, Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(a) During the collision, is the impulse on Cart 1 from Cart 2 greater than, less than, or equal to the magnitude of the impulse on Cart 2 from Cart 1?

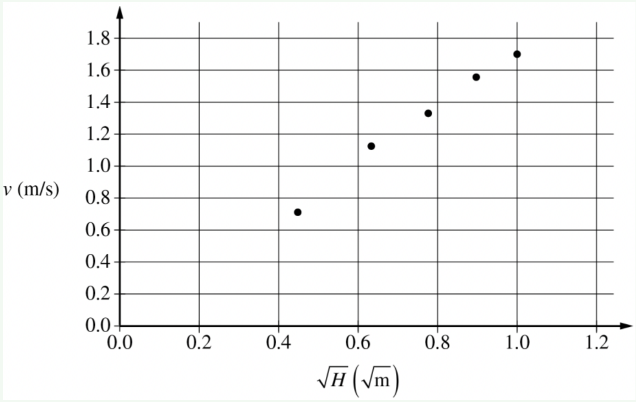
(b) On the following axes, draw graphs of the magnitude of the momentum of each cart as a function of time t, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.



(c) Show that the velocity v of the two-cart system after the collision is given by the equation $v = \sqrt{2g}\left(\frac{m_1}{m_1+m_2}\right)\sqrt{H}$

Problem 4.0.11 — 2022 AP Physics C: Mechanics FRQ Continued
This is the more experimental part of the free response question from above. It causes you to use a given graph to find a key relationship by first finding the slope.

(d) A group of students use the setup to perform an experiment. They measure the mass of Cart 1 to be $m_1 = 0.250$ kg. The mass of Cart 2 is unknown. The students perform several trials and in each trial, Cart 1 is released from a different height H and the final velocity of the two-cart system is measured. The students graph v as a function of $\sqrt{H}$, as shown below.



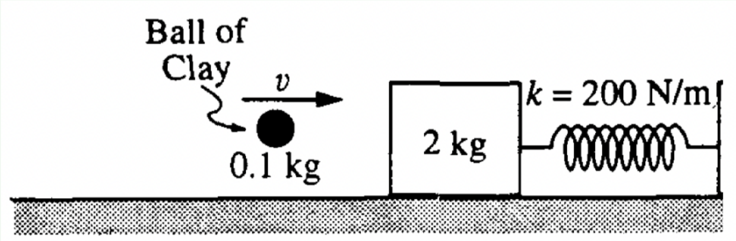
i. Draw a line that represents the best fit to the data points shown.

ii. Use the best-fit line to calculate the mass of Cart 2.

(e) After the experiment, the students use a balance to measure the mass of Cart 2 and find it to be less than what was determined in part (d). To explain this discrepancy, one of the students proposes that the mass of Cart 1 was incorrectly measured at the beginning of the experiment. The students measure the mass of Cart 1 again and record a new value, m_1'.

Should the students expect that m_1' will be greater than 0.250 kg, less than 0.250 kg, or equal to 0.250 kg?

Problem 4.0.12 — 1994 AP Physics C: Mechanics FRQ



A 2-kilogram block is attached to an ideal spring (for which $k = 200$ N/m) and initially at rest on a horizontal frictionless surface, as shown in the diagram above.

In an initial experiment, a 100-gram (0.1 kg) ball of clay is thrown at the 2-kilogram block. The clay is moving horizontally with speed v when it hits and sticks to the block. The spring is attached to a wall as shown. As a result, the spring compresses a maximum distance of 0.4 meters.

(a) Calculate the energy stored in the spring at maximum compression.

(b) Calculate the speed of the clay ball and 2-kilogram block immediately after the clay sticks to the block but before the spring compresses significantly.

(c) Calculate the initial speed v of the clay.

Solution to part a: The energy stored in a spring is $\frac{1}{2}kx^2$ where k is the spring constant and x is the distance the spring has been compressed.

Clearly, $k = 200$ as already given in the problem statement. On top of that, the maximum distance compressed x is already given as 0.4
We can plug these in to find that the energy stored in the spring is $\frac{1}{2} \cdot 200 \cdot 0.4^2 = 16$ J.

Solution to part b: As soon as the ball collides with the box, both move at the same velocity. Their kinetic energy is converted to spring potential energy.
The kinetic energy of the ball is $\frac{1}{2}mv^2$ while the kinetic energy of the box is $\frac{1}{2}Mv^2$
This means that the combined kinetic energy is $\frac{1}{2}(m + M)v^2$
This combined kinetic energy converts into spring potential energy which was already found to be 16J in part a.

Since $KE = U_s$, $\frac{1}{2}(m + M)v^2 = 16$.
We can multiply both sides by 2 to get $(m + M)v^2 = 32$
Now we can divide both sides by $m + m$ to get $v^2 = \frac{32}{m+M}$

Since we know that $m = 0.1$ kg and $M = 2$ kg, we can plug this in to get that $v^2 = \frac{32}{0.1+2}$

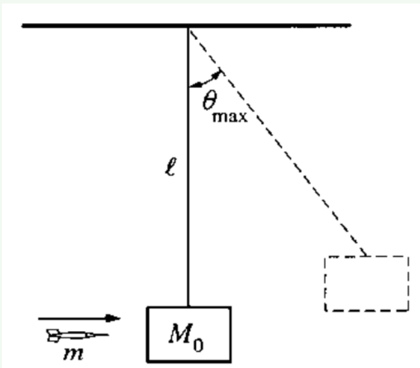
Simplifying gives that $v = 3.9$ m/s

Solution to part c: Momentum is conserved in this motion. If the initial speed of the clay is v, then the initial momentum of the system is mv (the box is at rest at this moment).

The final momentum is $(m + M)v_f$
We know that $m = 0.1$kg and $M = 2$kg and $v_f = 3.9\frac{m}{s}$ from part b. Note, that 3.9 is the velocity for both the clay and box right at the moment when the clay collides with the box. Plugging these variables in gives that the final momentum is $2.1 \cdot 3.9$ which is 8.19

Since $p_i = p_f$ (initial momentum = final momentum), we can write the equation $mv = 8.19$
Since $m = 0.1$, we can divide both sides by 0.1 to get that v is $81.9\frac{m}{s}$.
Thus, The initial velocity of the clay is 81.9 m/s.

Problem 4.0.13 — 1999 AP Physics C: Mechanics FRQ



In a laboratory experiment, you wish to determine the initial speed of a dart just after it leaves a dart gun. The dart, of mass m, is fired with the gun very close to a wooden block of mass M0 which hangs from a cord of length l and negligible mass, as shown above. Assume the size of the block is negligible compared to l, and the dart is moving horizontally when it hits the left side of the block at its center and becomes embedded in it. The block swings up to a maximum angle from the vertical. Express your answers to the following in terms of m, M_0, l, θ_max, and g.

(a) Determine the speed v_0 of the dart immediately before it strikes the block.

(b) The dart and block subsequently swing as a pendulum. Determine the tension in the cord when it returns to the lowest point of the swing.

Solution to part a: Since the dart sticks with the block after collision, we know that it's an inelastic collision.
In an inelastic collision, both objects move with the same velocity after they collide.

Let's assume the speed that the dart and block combined move with is v_f right after they collide.

The initial speed of the dart is v_o. Now, let's conserve momentum.
$p_i = p_f$
Since the initial speed of the block is 0 (only the dart is moving before the collision), the initial momentum of the system is just mv_o

The final momentum is the sum of both masses multiplied by their common velocity which is v_f.
Equating momentum gives us the equation $mv_o = (m + M)v_f$

Now, we can conserve energy using θ_{max}. When the combined dart and block reach the angle θ_{max}, there will be no kinetic energy at that moment. All the kinetic energy will turn into potential energy.

Let's say that the place where the dart and wooden block collide are at height 0 (our reference level). This means that the height of θ_{max} will be $l - l\cos(\theta_{max})$

We know that $KE_i + U_i = KE_f + U_f$ since energy is conserved.
The initial gravitational potential energy is 0. The final kinetic energy is also 0.

We can simplify the equation to $KE_i = U_f$

The initial kinetic energy is $KE_i = \frac{1}{2}(m + M)v_f^2$ (we use v_f since this is the variable for our combined velocity after collision).

The final gravitational potential energy is $(m + M)gl(1 - \cos(\theta_{max}))$

Setting both expressions equal gives

$$\frac{1}{2}(m + M)v_f^2 = (m + M)gl(1 - \cos(\theta_{max}))$$

We can cancel out $m+M$ and then multiply both sides by 2 to get $v_f^2 = 2gl(1-\cos(\theta_{max})$
This simplifies to $v_f = \sqrt{2gl(1 - \cos(\theta_{max})}$

Since we now find the speed of the dart and block as soon as they collide, we can now plug this into our equation for conservation of momentum.
From conserving momentum, we know $mv_o = (m + M)v_f$
We can divide both side by m to get $v_o = \frac{m+M}{m}v_f$

Now, we simply plug in $v_f = \sqrt{2gl(1 - \cos(\theta_{max})}$ to get $v_o = \dfrac{m + M}{m}\sqrt{2gl(1 - \cos(\theta_{max}))}$

In summary, we were able to solve this problem by conserving momentum and energy.

Solution to part b: At the bottom, we know that there will be a centripetal force created from the tension force and gravitational force.
We can write a Newton's Law equation: $T - (m + M)g = (m + M)a = \frac{(m+M)v^2}{l}$

We can add $(m + M)g$ to both sides to get $T = (m + M)(g + \frac{v^2}{l})$
Note, that in this problem the radius has a length of l, the length of the cord. We need to use l instead of r.

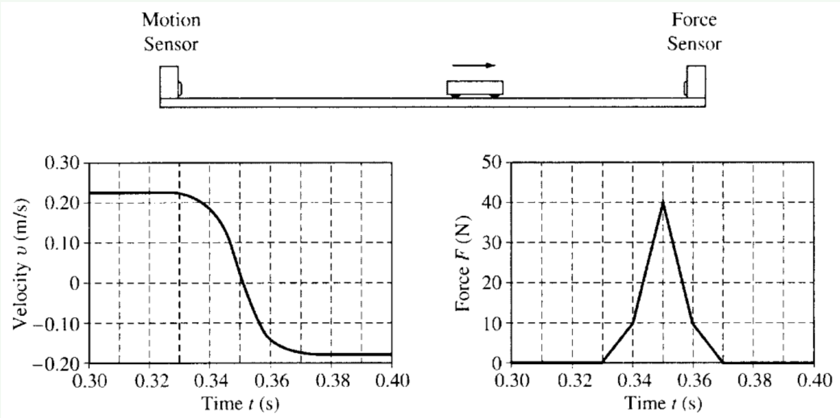
Now, the question is what value of velocity should we use. Should we use the velocity before collision or after collision?

The answer is that we use the velocity right after collision which is $v_f = \sqrt{2gl(1 - \cos(\theta_{max}))}$.
The reason is that we want the velocity for the COMBINED mass.

We can plug this value of velocity into $T = (m + M)(g + \frac{v^2}{l})$
Doing so gives that $T = (m + M)(g + 2g - 2g\cos(\theta_{max}))$ which simplifies to

$$T = (m + M)(3g - 2g\cos(\theta_{max}))$$

Problem 4.0.14 — 2001 AP Physics C: Mechanics FRQ



A motion sensor and a force sensor record the motion of a cart along a track, as shown above. The cart is given a push so that it moves toward the force sensor and then collides with it. The two sensors record the values shown in the above graphs.

(a) Determine the cart's average acceleration between $t = 0.33$ s and $t = 0.37$ s.

(b) Determine the magnitude of the change in the cart's momentum during the collision.

(c) Determine the mass of the cart.

(d) Determine the energy lost in the collision between the force sensor and the cart.

Solution to part a: To find the average acceleration in that time interval, we need to use the graph.
The average acceleration is $\frac{v_f - v_i}{\Delta t}$

The velocity at $t = 0.37$s can be estimated to be $-0.18\frac{m}{s}$.
Similarly, the velocity at $t = 0.33$s can be estimated to be $0.22\frac{m}{s}$.
We also know that Δt is jut 0.04

Plugging these variables in gives that average acceleration is $\frac{-0.18-0.22}{0.04} = 10\frac{m}{s^2}$

Solution to part b: We know that the area under a force-time graph is impulse which is the same thing as change in momentum.

We can use the graph on the right and try to find the area under it. We can break the graph up into simple shapes such as triangles and rectangles to find that the area is 0.6 This means that the change in momentum is 0.6kg $\cdot$ m/s

Solution to part c: The change in momentum is mass times change in velocity.
$\Delta p = m\Delta v$
Our change in velocity is $-0.18 - 0.22 = -0.4\frac{m}{s}$

We also already know that Δp is 0.6 from part b.

We can rearrange the equation $\Delta p = m\Delta v$ to $m = \frac{\Delta p}{\Delta v}$

$$\text{Plugging our values in gives that } m = \frac{0.6}{0.4} = 1.5 \text{ kg}$$

Solution to part d: The energy lost would be the difference in kinetic energy before and after.
We need to find ΔK which is change in change in kinetic energy.

$$\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{m}{2}(v_f^2 - v_i^2)$$

We know that $m = 1.5$ kg. We can plug in $v_f = -0.18$ and $v_i = 0.22$ to find ΔK

$$\Delta K = \frac{1.5}{2}((-0.18)^2 - (0.22)^2) = -0.012 \text{ J}$$

This means that the energy lost in the collision between the force sensor and the cart is 0.012 J.

Problem 4.0.15 — 1992 AP Physics B FRQ
A 30-kilogram child moving at 4.0 meters per second jumps onto a 50-kilogram sled that is initially at rest on a long, frictionless, horizontal sheet of ice.

(a) Determine the speed of the child-sled system after the child jumps onto the sled.
(b) Determine the kinetic energy of the child-sled system after the child jumps onto the sled.

After coasting at constant speed for a short time, the child jumps off the sled in such a way that she is at rest with respect to the ice.
(c) Determine the speed of the sled after the child jumps off it.
(d) Determine the kinetic energy of the child-sled system when the child is at rest on the ice.
(e) Compare the kinetic energies that were determined in parts (b) and (d). If the energy is greater in (d) than it is in (b), where did the increase come from? If the energy is less in (d) than it is in (b), where did the energy go?

Solution to part a: We will conserve momentum in this problem since we have an inelastic collision.

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

m_1 represents the mass of the child while m_2 represents the sled.

Initially, the sled is at rest which means $v_{2i} = 0$.
This means our equation becomes $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$

In addition, since this is an inelastic collision, both objects will move together after. This means that their final speed will be the same. We can denote the common final speed as v_f

This further simplifies the equation to $m_1 v_{1i} = (m_1 + m_2)v_f$
We know that $m_1 = 30$, $m_2 = 50$, and $v_{1i} = 4$.

We can plug this in to get $30 \cdot 4 = (30 + 50)v_f$
We can solve this to find that $v_f = 1.5$ m/s

Solution to part b: After the child jumps onto the sled, the kinetic energy of the child-sled system will be $\frac{1}{2}(30 + 50) \cdot 1.5^2 = 90$ J
In this problem, we are able to combine the masses and treat them as one because they are moving together.

Solution to part c: We can again use conservation of momentum.

$$(m_1 + m_2)v_i = m_1 v_{1f} + m_2 v_{2f}$$

$m_1 = 30$, $m_2 = 50$, and $v_i = 1.5$

Initially, both the sled and person are moving at the same speed.

After collision, $v_{1f} = 0$ since the child is now at rest.

We can plug these variables in to get $(30 + 50)1.5 = 0 + 50v_{2f}$
We can solve this to get $v_{2f} = 2.4$ m/s.

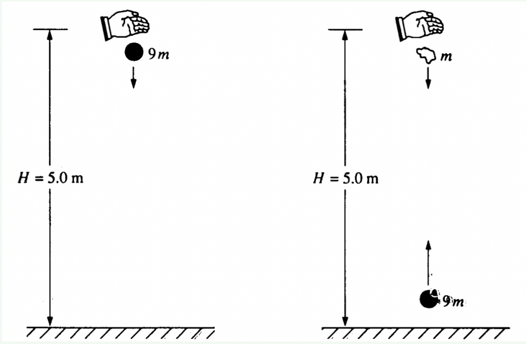
Solution to part d: The kinetic energy of the system can be found by summing up the individual kinetic energies for the child and sled.
Since the child is at rest, the kinetic energy is 0.

The sled's kinetic energy can be found through the equation $K = \frac{1}{2}mv^2$. Since it has a velocity of 2.4 and mass of 50, its kinetic energy is $\frac{1}{2} \cdot 50 \cdot 2.4^2 = 144$ J.

The sum of both kinetic energies is $0 + 144$ which is just 144 J.

Solution to part e: Clearly the kinetic energy is now greater than part *b*. 144 J is greater than 90 J. The reason for this can be due to the child doing work on the sled. The work that is done on the sled by the child causes kinetic energy to be greater.

Problem 4.0.16 — 1992 AP Physics C: Mechanics FRQ



A ball of mass $9m$ is dropped from rest from a height $H = 5.0$ meters above the ground, as shown above on the left. It undergoes a perfectly elastic collision with the ground and rebounds. At the instant that the ball rebounds, a small blob of clay of mass m is released from rest from the original height H, directly above the ball, as shown above on the right. The clay blob, which is descending, collides with the ball 0.5 seconds later, which is ascending. Assume that $g = 10\text{m/s}^2$, that air resistance is negligible, and that the collision process takes negligible time.

(a) Determine the speed of the ball immediately before it hits the ground.
(b) Determine the rebound speed of the ball immediately after it collides with the ground, justify your answer.
(c) Determine the height above the ground at which the clay-ball collision takes place.
(d) Determine the speeds of the ball and the clay blob immediately before the collision.
(e) If the ball and the clay blob stick together on impact, what is the magnitude and direction of their velocity immediately after the collision?

Solution to part a: The speed of the ball immediately before it hits the ground can be found by conserving energy. You can also apply kinematics if you want to.
The balls potential energy will all convert to kinetic energy by the time it hits the ground.

Conservation of energy says $K_i + U_i = K_f + U_f$

We can make the ground our reference level. The potential energy at the top can be represented as mgh where mass is $9m$ and h is 5 m.

Our initial kinetic energy is obviously 0 since the ball is dropped from rest. The final potential energy is also 0 since our reference level is the ground.

Our equation just becomes $U_i = K_f$

We can rewrite this as $mgh = \frac{1}{2}mv^2$

m cancels out so we are left with $gh = \frac{v^2}{2}$

We can multiply both sides by 2 and then take the square root of both sides to get $v = \sqrt{2gh}$

Since our height is 5, we can plug that in to get that $v = \sqrt{2 \cdot 9.8 \cdot 5}$ which is 9.9 m/s.

Solution to part b: Since the ball undergoes an elastic collision, the rebound speed will be the same as the speed it collides with. The speed it collides with is 9.9 m/s (found in part a)
This means that the rebound speed will also be 9.9 m/s.

The reason regarding why the rebound speed is the same is that the ground is super "heavy." If you think logically, of course a ball bouncing on the ground will not cause the ground to move. Earth is too heavy. The ball is way too light in comparison. Also, many people might try to think logically about this problem. For example, you may have experience bouncing a tennis ball or even a basketball. You have probably noticed that unless you keep applying force with your hand, both of the balls will stop. Their rebound height will not be the same. The reason is that the ground absorbs some of the energy. HOWEVER, in this problem it says that the ground makes perfectly elastic collisions with the ground. That means no energy is lost which is why the rebound speed is the same.

Solution to part c: We can find the height above the ground by observing the clay blob.
We know that they collide in 0.5 s (it says that in the problem statement), so we can use that information to find the height.

The clay blob's initial velocity $v_i = 0$ and $t = 0.5$ s. On top of that, $a = g = 9.8$

We can use the kinematics equation $\Delta y = v_i t + \frac{1}{2}at^2$
We can plug in our variables to find that $\Delta y = 0 + \frac{1}{2} \cdot g \cdot 0.5^2 = 1.225$ m.

Since this is the distance the clay ball displaces from the top, it is $5 - 1.225 = 3.775$ m above the ground.

Solution to part d: We will use kinematics equations two times to find the speeds for each object.
For the ball, $v_f = v_i + at$. We know that $t = 0.5$ and $a = g$ and $v_i = 9.9$.
Using these values, we can find that $v_f = 9.9 - 9.8 \cdot 0.5 = 5$ m/s.

Similarly, for the clay, we can use the same equation $v_f = v_i + at$. We know that $t = 0.5$ and $a = g$ and $v_i = 0$ since it is dropped from rest.
Using these values, we can find that the clay is moving at $v_f = 0 - 9.8 \cdot 0.5 = -4.9$ m/s.

Solution to part e: Since both objects stick together after impact, we know that an inelastic collision happens. Their common velocity will be the same after impact, and we can denote it as v_f

We will conserve momentum since we have a collision.

$$p_i = 9m \cdot 5 - m \cdot 4.9 = 40.1m \text{ Expression for initial momentum)}$$

At the same time, $p_f = (9m + m)v_f$

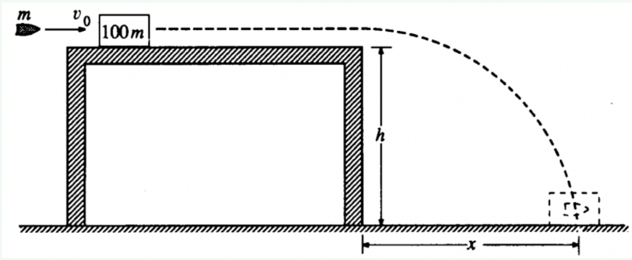
Since $p_i = p_f$, we can plug in both expressions to get

$$40.1m = (9m + m)v_f$$

We can cancel out m from both sides to get $40.1 = 10v_f$

After dividing 10 from both sides, we get that $v_f = 4.01$ m/s.

Problem 4.0.17 — 1990 AP Physics B FRQ



A bullet of mass m is moving horizontally with speed v_o when it hits a block of mass $100m$ that is at rest on a horizontal frictionless table, as shown above. The surface of the table is a height h above the floor. After the impact, the bullet and the block slide off the table and hit the floor a distance x from the edge of the table. Derive expressions for the following quantities in terms of$m, h, v)o$, and appropriate constants:

(a) the speed of the block as it leaves the table
(b) the change in kinetic energy of the bullet-block system during impact
(c) the distance x

Suppose that the bullet passes through the block instead of remaining in it.
(d) State whether the time required for the block to reach the floor from the edge of the table would now be greater, less, or the same. Justify your answer.

(e) State whether the distance x for the block would now be greater, less, or the same. Justify your answer.

Solution to part a: After collision, the bullet and block travel together. This means that we have an inelastic collision.
Momentum will be conserved. The initial momentum is mv_o (since only the bullet is moving).
The final momentum will be $(m + 100m)v_f$ since both masses will move with the same speed.

We can set both expressions equal to each other and write $mv_o = (m + 100m)v_f$
We can divide m from both sides to get $v_o = 101v_f$

After dividing 101 from both sides, we get $v_f = \dfrac{v_o}{101}$

Solution to part b: Change in kinetic energy can be represented as $\Delta K = K_f - K_i$

We can first find the initial kinetic energy. It is simply $\frac{1}{2}mv_o^2$ since only the bullet moves.
The final kinetic energy can again be found using the equation $K = \frac{1}{2}mv^2$. Our mass will be $m + 100m$ which is $101m$. We combine the masses since both objects move together. The velocity of both will be $\frac{v_o}{101}$.

$$\text{We can plug this in to find that } K_f = \frac{1}{2} \cdot 101m \cdot (\frac{v_o}{101})^2$$

$$\text{We can simplify the expression to find that } K_f = \frac{mv_o^2}{202}$$

Now, we can compute $\Delta K = K_f - K_i$. We can plug in our expressions to find that

$$\Delta K = \frac{mv_o^2}{202} - \frac{mv_o^2}{2} = -\frac{50mv_o^2}{101}$$

Solution to part c: The distance x can be found using kinematics equations.
After the bullet and block leave the table, they undergo projectile motion.
We must remember that the vertical motion is independent from the horizontal motion. Since the initial vertical speed is 0, the vertical displacement is h, and the acceleration is g, we can find the time it takes to fall using the equation $\Delta y = v_i t + \frac{1}{2}at^2$

After we plug in our variable, we get $h = 0 + \frac{1}{2} \cdot g \cdot t^2$
We can solve for t to find that it is $\sqrt{\frac{2h}{g}}$

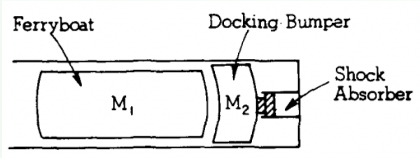
Since our horizontal velocity remains the same in projectile motion, we can simply multiply the horizontal velocity by the time taken to fall to find the horizontal distance travelled.
This means that $x = v_x t$. We already know that $v_x = \frac{v_o}{101}$ from part a.

$$\text{This means } x = \frac{v_o}{101}\sqrt{\frac{2h}{g}}$$

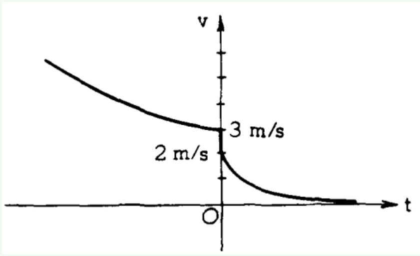
Solution to part d: The time would be the same. We can figure this out by considering motion in the y-direction. The vertical displacement, acceleration in vertical direction, and initial vertical velocity will all remain the same. Even though the horizontal velocity will differ, the motion in the vertical direction will remain the same. This will cause time to be the same.

Solution to part e: Previously, all of the bullet's momentum would be transferred to the block. However, now only part of it will be transferred. The reason is that the bullet will preserve some of its momentum to move through the block. This causes the momentum that the block receives to be lower. This will lead to a lower velocity in the x-direction.
Since the horizontal velocity is now lower for the block, the distance x travelled will also be lower.

Problem 4.0.18 — 1979 AP Physics C: Mechanics FRQ



A ferryboat of mass $M_1 = 2.010^5$ kilograms moves toward a docking bumper of mass M_2 that is attached to a shock absorbed. Shown below is a speed v vs. time t graph of the ferryboat from the time it cuts off its engines to the time it first comes to rest after colliding with the bumper. At the instant it hits the bumper, $t = 0$ and $v = 3$ meters per second.



(a) After colliding inelastically with the bumper, the ferryboat and bumper move together with an initial speed of 2 meters per second. Calculate the mass of the bumper M_2.

(b) After colliding, the ferryboat and bumper move with a speed given by the expression $v = 2e^{-4t}$. Although the boat never comes precisely to rest, it travels only a finite distance. Calculate that distance.

(c) While the ferryboat was being slowed by water resistance before hitting the bumper, its speed was given by $\frac{1}{v} = \frac{1}{3} + \beta t$, where E is a constant. Find an expression for the retarding force of the water on the boat as a function of speed.

Solution to part a: We can conserve momentum to find the mass of the bumper.
The initial momentum of the mass M_1 will cause both M_1 and M_2 to move together with the same velocity after.
This means that $M_1 v_i = (M_1 + M_2)v_f$ (since it's an inelastic collision).

We know that $v_i = 3$ since it's the initial speed before collision. However, $v_f = 2$ since that's the speed at which both masses move together with. We also know that $M_1 = 2 \cdot 10^5$

We can plug all of that in to get $2 \cdot 10^5 \cdot 3 = (2 \cdot 10^5 + M_2) \cdot 2$
We can solve the equation to find that $M_2 = 10^5$ kg.

Solution to part b: We must use the idea that displacement is an integral of velocity.

$$x = \int v dt$$

We can plug in $v = 2e^{-4t}$ with our bounds from 0 to ∞.

$$\text{Doing so gives that } x = \int_0^\infty 2e^{-4t}dt = -\frac{1}{2}e^{-4t}|_0^\infty = 0.5 \text{ m}$$

This means that the distance travelled is 0.5 m.

Solution to part c: We know that $F = ma$ from Newton's Second Law.
Since we have an expression for velocity v, we can use the formula $a = \frac{dv}{dt}$ to find the acceleration.

$$\text{Differentiating velocity gives that } a = -\beta(\frac{1}{3} + \beta t)^{-2}$$

$$\text{Since we know that } \frac{1}{v} = \frac{1}{3} + \beta t, \text{ we can plug that into our expression for } a$$

$$\text{In our expression for acceleration, we replace } \frac{1}{3} + \beta t \text{ with } \frac{1}{v}$$

Doing so gives that $a = -\beta v^2$.

Since $F = ma$, we can plug in our expression for acceleration to find an expression for our force: $-m\beta v^2$

Problem 4.0.19 — 2009 AP Physics C Mechanics FRQ
A 3.0 kg object is moving along the x-axis in a region where its potential energy as a function of x is given as $U(x) = 4.0x^2$, where U is in joules and x is in meters. When the object passes the point $x = -0.50$ m, its velocity is $+2.0$ m/s. All forces acting on the object are conservative.

(a) Calculate the total mechanical energy of the object.

(b) Calculate the x-coordinate of any points at which the object has zero kinetic energy.

(c) Calculate the magnitude of the momentum of the object at $x = 0.60$ m.

(d) Calculate the magnitude of the acceleration of the object as it passes $x = 0.60$ m.

Solution to part a: The mechanical energy is the sum of the kinetic and potential energy. We know that the mechanical energy will remain constant in this problem. Thus, we can just find it at the point $x = -0.5$ since we know the velocity at that point.

The velocity at $x = -0.5$ is 2.0 which means that the kinetic energy is $\frac{1}{2}mv^2 = \frac{1}{2} \cdot 3 \cdot 2^2 = 6$ J.
The potential energy is $U(-0.5) = 4(-0.5)^2 = 1$ J.

Thus, the total mechanical energy is $K + U = 6 + 1 = 7$ J.

Solution to part b: We know that the total mechanical energy is 7 J.
The kinetic and potential energy at any point should sum to 7. For the kinetic energy to be 0, the potential energy must be 7. All of the mechanical energy will be part of potential energy.

This means that $U(x) = 4x^2 = 7$.
We can find that $x = \pm\sqrt{\frac{7}{4}} = \pm 1.32$ m.

Solution to part c: We know that momentum is $p = mv$. Thus we must find the velocity at $x = 0.6$.
We know that $K + U = 7$ at all points.
We can find that $U(0.6) = 4(0.6)^2 = 1.44$ J.

This means that $K = 7 - 1.44 = 5.56$ J at $x = 0.6$
Since $K = \frac{1}{2}mv^2$, we can set it equal to 5.56 to find the velocity at that point.

Doing so gives $\frac{1}{2}(3)v^2 = 5.56$ which means $v = 1.92$ m/s.
We can plug this into the formula for momentum to find that

$$p = mv = 3 \cdot 1.92 = 5.76 \text{ kg} \cdot \text{ m/s}$$

Solution to part d: To find the acceleration, we will use our expression for the potential energy.

$$\text{We know that } F = -\frac{dU}{dx}$$

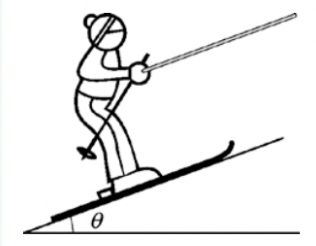
$$\text{Using that, we can find that } F = -\frac{d}{dx}[4x^2] = -8x$$

Using the formula $F = ma$, we can find that the acceleration a is $\frac{F}{m} = \frac{-8x}{3}$. Since we want the acceleration at $x = 0.6$ m, we can plug that in to get

$$a = \frac{-8 \cdot 0.6}{3} = -1.6$$

However, we only want the **magnitude** of acceleration so the answer is 1.6 m/s^2.

Problem 4.0.20 — 2010 AP Physics C: Mechanics FRQ



A skier of mass m will be pulled up a hill by a rope, as shown above. The magnitude of the acceleration of the skier as a function of time t can be modeled by the equations

$$a(t) = \begin{cases} a_{\max} \sin\left(\frac{\pi t}{T}\right) & \text{if } 0 < t < T \\ 0 & \text{if } t \geq T \end{cases}$$

where a_{max} and T are constants. The hill is inclined at an angle θ above the horizontal, and friction between the skis and the snow is negligible. Express your answers in terms of given quantities and fundamental constants.

(a) Derive an expression for the velocity of the skier as a function of time during the acceleration. Assume the skier starts from rest.

(b) Derive an expression for the work done by the net force on the skier from rest until terminal speed is reached.

(c) Determine the magnitude of the force exerted by the rope on the skier at terminal speed.

(d) Derive an expression for the total impulse imparted to the skier during the acceleration.

Solution to part a: We can find an expression for velocity by integrating our expression for acceleration with respect to time.

$$v = \int a dt$$

$$v = \int_0^t a_{max} \sin(\frac{\pi t}{T}) dt = -\frac{a_{max} T}{\pi} \cos(\frac{\pi t}{T})|_0^t = \frac{a_{max} T}{\pi}(1 - \cos(\frac{\pi t}{T}))$$

Solution to part b: We know that the work done is equivalent to the change of kinetic energy from the Work-Energy theorem.

We can find the change in kinetic energy by finding the initial and final velocities. We know that the initial velocity is 0. The final velocity occurs at time $t = T$. We can plug in T instead of t into $v = \frac{a_{max} T}{\pi}(1 - \cos(\frac{\pi t}{T}))$ to find that

$$v_f = \frac{2a_{max} T}{\pi}$$

This means that the final kinetic energy is $K_f = \frac{1}{2} \cdot m \cdot (\frac{2a_{max} T}{\pi})^2 = \frac{2ma_{max}^2 T^2}{\pi^2}$

Since the initial kinetic energy is 0 due to the initial velocity being 0, we know that the work done is

$$\frac{2ma_{max}^2T^2}{\pi^2}$$

Solution to part c: To find the force exerted by the rope, we should use Newton's Second Law.
We know that the acceleration will be 0 at terminal speed.
This means that $F_{net} = ma = 0$
The forces along the ramp are one component of gravity and the tension force from the rope.

The net force F_{net} is $T - mg\sin(\theta)$
Since we know that the net force is 0 at terminal speed, we can find that the tension force is $T = mg\sin(\theta)$

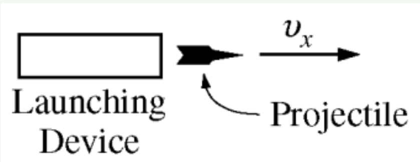
Solution to part d: Impulse is simply the integral of force with respect to time.

$$J = \int F dt$$

Since we know that the acceleration a is $a_{max}\sin(\frac{\pi t}{T})$, the force F is $ma = ma_{max}\sin(\frac{\pi t}{T})$

$$\text{This means that the impulse is } J = \int_0^T ma_{max}\sin(\frac{\pi t}{T})dt = \frac{2ma_{max}T}{\pi}$$

Problem 4.0.21 — 2011 AP Physics C Mechanics FRQ



A projectile is fired horizontally from a launching device, exiting with a speed v_x. While the projectile is in the launching device, the impulse imparted to it is J_p, and the average force on it is F_{avg}. Assume the force becomes zero just as the projectile reaches the end of the launching device. Express your answers to parts (a) and (b) in terms of v_x, J_p, F_{avg}, and fundamental constants, as appropriate.

(a) Determine an expression for the time required for the projectile to travel the length of the launching device.
(b) Determine an expression for the mass of the projectile.

The projectile is fired horizontally into a block of wood that is clamped to a tabletop so that it cannot move. The projectile travels a distance d into the block before it stops. Express all algebraic answers to the following in terms of d and the given quantities previously indicated, as appropriate.

(c) Derive an expression for the work done in stopping the projectile.
(d) Derive an expression for the average force F_b exerted on the projectile as it comes to rest in the block.

Solution to part a: In this problem, we are given the impulse and average force. This should remind us of our formula relating force and impulse.

$$J = \int F dt$$

Impulse is the integral of force with respect to time.
In this problem, $J_p = F_{avg} \cdot t$. We simply multiply average force to time to find the change in momentum, which is the impulse
We can divide both sides by F_{avg} to find that

$$t = \frac{J_p}{F_{avg}}$$

Solution to part b: From the impulse momentum theorem, we know that the impulse equals to the change of momentum.

$$J = \Delta p = m\Delta v$$

The initial speed of the projectile was 0. The final speed is v_x since that's the speed it exited with.
This means that $J_p = mv_x$
We can divide both sides by v_x to find that the mass is

$$\frac{J_p}{v_x}$$

Solution to part c: To find the work done, you should always remember the work-energy theorem.
The work energy theorem says that the work done is the change of kinetic energy.

$$W = \Delta K$$

In this problem, the final kinetic energy of the projectile is 0 since it comes to rest. However, the initial kinetic energy can be found using the formula $K = \frac{1}{2}mv^2$. We know that the velocity is v_x. The mass is $\frac{J_p}{v_x}$ (we found this in part b).

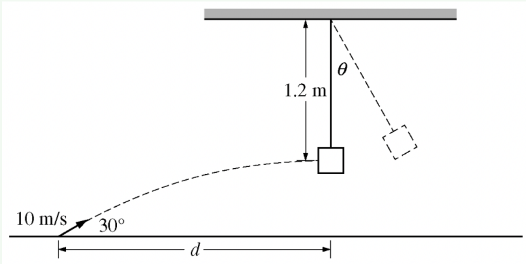
$$\text{Plugging all of this in gives that } K = \frac{1}{2}mv^2 = \frac{1}{2} \cdot \frac{J_p}{v_x} \cdot v_x^2 = \frac{J_p v_x}{2}$$

Since the final kinetic energy is 0, we can find that $\Delta K = 0 - \frac{J_p v_x}{2} = -\frac{J_p v_x}{2}$ which is also the work done.

Solution to part d: We know that the integral of force with respect to displacement is work done.

In this problem, we can approximate the work done to $F_{avg} \cdot d$.
Since we know that the work done by the force is $\frac{J_p v_x}{2}$, we can divide this by d to find that F_b (the average force) is $\frac{J_p v_x}{2d}$

Problem 4.0.22 — 2015 AP Physics C Mechanics FRQ



A small dart of mass 0.020 kg is launched at an angle of 30° above the horizontal with an initial speed of 10 m/s. At the moment it reaches the highest point in its path and is moving horizontally, it collides with and sticks to a wooden block of mass 0.10 kg that is suspended at the end of a massless string. The center of mass of the block is 1.2 m below the pivot point of the string. The block and dart then swing up until the string makes an angle θ with the vertical, as shown above. Air resistance is negligible.

(a) Determine the speed of the dart just before it strikes the block.

(b) Calculate the horizontal distance d between the launching point of the dart and a point on the floor directly below the block.

(c) Calculate the speed of the block just after the dart strikes.

(d) Calculate the angle θ through which the dart and block on the string will rise before coming momentarily to rest.

Solution to part a: At the top, the dart's vertical velocity will be 0. However, there will be horizontal velocity.
The dart's horizontal velocity at the top will remain constant through its motion (as we learned during the projectile motion section).
This means we can find the dart's initial horizontal velocity as it will remain constant throughout its trajectory.

The dart's initial horizontal velocity is $10\cos(30)$ which is 8.7 m/s.

Solution to part b: This part of the problem is asking us for the horizontal displacement during that time interval.
We must first find the time it takes for the object to reach the top. We can do this by considering the motion in the y-direction.

v_{0y} (the initial velocity in y-direction) is $v\sin(30) = 10\sin(30) = 5$.
The final velocity in the y-direction is $v_y = 0$. The acceleration in the y-direction is $-g$.

We can use the kinematics equation $v_f = v_i + at$ to model this motion.
Plugging in our variables for the y-direction gives $0 = 5 - gt$

$$\text{Solving for } t \text{ gives } t = \frac{5}{g} = \frac{5}{9.81} = 0.51 \text{ s}$$

Now, we must use the formula $\Delta x = v_x t$ to find the horizontal displacement in the

x-direction. Since $v_x = 8.7$, we can find that $\Delta x = 8.7 \cdot 0.51 = 4.44$ m.

Solution to part c: This problem speaks out conservation of momentum.
We know that momentum is conserved in a collision. The dart's initial momentum at the top is $m_d v_x$ where m_d represents its mass and v_x is its velocity at the top.

Since we have an inelastic collision, we can represent the final momentum of the dart and block right after the collision as $(m_d + m_b)v_f$ since they move with the same common velocity.

By conservation of momentum, we know that $m_d v_x = (m_d + m_b)v_f$

We can plug in our known variables. We know that $m_d = 0.02$ and $m_b = 0.1$

We also know that $v_x = 8.7$. Plugging all of this in gives $0.02 \cdot 8.7 = (0.02 + 0.1)v_f$

We can solve for v_f to find that $v_f = 1.45$ m/s.

Solution to part d: We know that the dart and block will move at the same speed. Their kinetic energy right after they collide will convert to gravitational potential energy.

Their kinetic energy is $\frac{1}{2} \cdot (m_d + m_b) \cdot v^2$

The change in potential energy will be mgh. We must find the height h that they go up. We should recognize the gain in height from our traditional $L(1 - \cos(\theta))$ which is used for pendulums all the time.

In this case, $L = 1.2$ so the gain in height is $1.2(1 - \cos(\theta))$.

This means that the potential energy at the top is $(m_d + m_b) \cdot g \cdot 1.2(1 - \cos(\theta))$

We can set the kinetic energy at the bottom and gravitational potential energy at the top equal to each other.

$$\frac{1}{2} \cdot (m_d + m_b) \cdot v^2 = (m_d + m_b) \cdot g \cdot 1.2(1 - \cos(\theta))$$

$m_d + m_b$ cancels out from both sides. We can cancel them out to get

$$\frac{v^2}{2} = g \cdot 1.2(1 - \cos(\theta))$$

We know that $v = 1.45$ (since that's their common velocity at the bottom). We can plug that in and solve for $\cos(\theta)$ to find that $\cos(\theta) = 0.91$

This means that θ is around $24.5°$

Problem 4.0.23 — 2016 AP Physics C Mechanics FRQ

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(a) On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

(b) Derive an expression for the speed of the blocks immediately after the collision.

(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.

(d) Derive an expression for the maximum distance D that the spring is compressed.

(e)

i. In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

______ Left ______ Right ______ The net force on the block of mass 2M is zero.

Justify your answer.

ii. Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

______ The magnitude of the net force is greater on the block of mass 2M.

______ The magnitude of the net force is greater on the block of mass 3M.

______ The magnitude of the net force on each block has the same nonzero value.

______ The magnitude of the net force on each block is zero.

Justify your answer.

Solution to part a: Initially, the only forces on each block are the normal force and gravitational force.

Credits: Full credit goes to College Board for the image above.
Solution to part b: In a collision, momentum will be conserved. In this problem, the initial momentum comes from the block of mass $3M$.
It's initial momentum is $p_i = 3M \cdot -v_0 = -3Mv_0$.
After it collides with the block of mass $2M$, both of them will have the same velocity since they move together.

The final momentum can be represented as $p_f = (2M + 3M)v_f$
We can set p_i and p_f equal to each other to write the equation $-3Mv_0 = (2M + 3M)v_f$
Solving the equation gives that $v_f = -\frac{3Mv_0}{5}$
Since we are asked to find the speed, we can ignore the sign. The speed is $\frac{3Mv_0}{5}$

Solution to part c: Since the two blocks act like one larger mass (due to them moving together), their kinetic energy after the collision can be written as $K = \frac{1}{2}(2M + 3M)v_f^2$.

$$\text{We can plug in } v_f = -\frac{3Mv_0}{5} \text{ to find that } K = \frac{9Mv_0^2}{10}$$

Solution to part d: All of the kinetic energy right after the collision will convert to spring potential energy as the spring is compressed.

We already know that the kinetic energy is $K = \frac{9Mv_0^2}{10}$.
The spring potential energy for a linear spring can be represented as $\frac{1}{2}kx^2$. However, we have a nonlinear spring. That means our spring potential energy will be represented as

$$U_s = -\int F(x)dx$$

We can integrate $F(x) = -Bx^3$ from 0 to D to find that $U_s = \frac{BD^4}{4}$

We can set KE and U_s equal to each other to get $\frac{9Mv_0^2}{10} = \frac{BD^4}{4}$

We can solve this equation for D to find that $D = \sqrt[4]{\frac{18Mv_0^2}{5B}}$

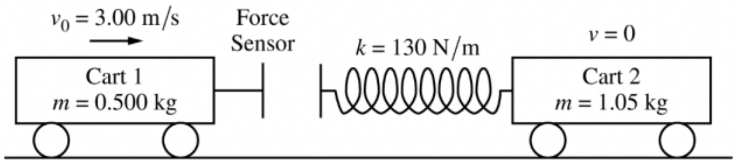
Solution to part e i: At maximum compression, the two blocks are at rest at that instant. The spring force is the only external force on the block of mass $2M$ and will apply a force towards the right to oppose the spring's compression.

Solution to part e ii: Since both of the blocks stick together, they will have the same acceleration since they move together. This means that the net force on the heavier

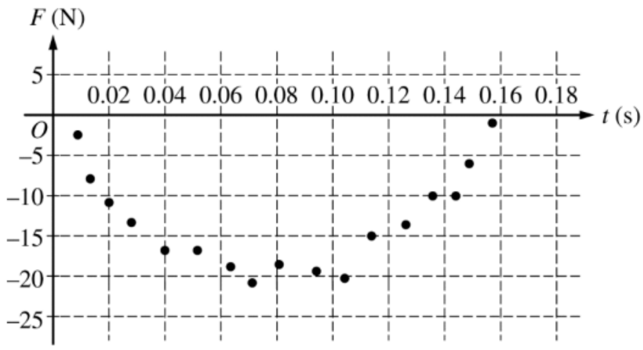
mass will be larger.
We can use Newton's Second Law to figure this out. We can apply $F_{net} = ma$ for both blocks. Although acceleration is the same, the block of mass $3M$ has a larger mass. This will cause it to have a larger net force on it.

Thus, the answer is $\boxed{\text{The magnitude of the net force is greater on the block of mass } 3M}$

Problem 4.0.24 — 2018 AP Physics C Mechanics FRQ



Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00m/s$ toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130N/m$. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = (3200N/s^2)t^2 - (500N/s)t$



(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)

i. Calculate the speed of cart 1 after the collision.

ii. In which direction does cart 1 move after the collision?

_____ Left _____ Right

_____ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)

i. Calculate the speed of cart 2 after the collision.

ii. Show that the collision between the two carts is elastic.

(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

ii. Calculate the maximum elastic potential energy stored in the spring.

Solution to part a: We know that impulse is the integral of force with respect to time.
$J = \int F dt$
For our problem, the integral bounds are from $t = 0$ to $t = 0.16$ since that's the time

interval in which a force exists (as seen from the graph).

$$J = \int_0^{0.16} [3200t^2 - 500t]dt = \frac{3200}{3}t^3 - 250t^2|_0^{0.16} = -2.03 \text{ N} \cdot \text{ s}$$

Solution to part b i: We know that the impulse is the change in momentum. $J = \Delta p$

Since this is the impulse on cart 1, we know that $J = m_1(v_f - v_0)$
We already know that $m_1 = 0.5$ and $v_0 = 3$. We also know that J (the impulse) is -2.03

We can plug this in to write the equation: $-2.03 = 0.5(v_f - 3)$

We can multiply both sides by 2 to get $-4.06 = v_f - 3$

We can now add 3 to both sides to find that $v_f = -1.06$ m/s. This is the velocity, and speed is simply the magnitude of velocity (since it has no direction). Thus, the speed is $1.06m/s$.

Solution to part b ii: We found the velocity of cart 1 after the collision to be -1.06 m/s. Since the velocity is negative, the cart moves to the left.

Solution to part c i: The impulse we found in part a was the amount that was delivered to cart 1. The amount of impulse that cart 2 faces is $-J$.

We know that impulse is the change in momentum.
Thus, for cart 2, $-J = \Delta p$

$$\text{We know that } \Delta p = p_f - p_i = m_2(v_f - v_i)$$

Cart 2's initial velocity was 0 since it was at rest.
This means that $-J = m_2 v_f$

We can divide both sides by m_2 to get $v_f = -\frac{J}{m_2}$

$$\text{We can plug in } J = -2.03 \text{ and } m_2 = 1.05 \text{ kg to get } v_f = -\frac{-2.03}{1.05} = 1.93 \text{ m/s}$$

Solution to part c ii: In an elastic collision, kinetic energy is conserved.

$$K_{1i} = K_{if} + K_{2f}$$

Note, that K_{1i} represents the kinetic energy of cart 1 before collision. K_{1f} represents the kinetic energy of cart 1 right after the collision while K_{2f} is the kinetic energy of cart 2 right after the collision. We can ignore K_{2i} since we know that cart 2 was at rest before the collision, so its kinetic energy was simply 0.

$$K_{1i} = \frac{1}{2} \cdot 0.5 \cdot 3^2 = 2.25$$

$$K_{1f} = \frac{1}{2} \cdot 0.5 \cdot 1.06^2 = 0.2809$$

$$K_{2f} = \frac{1}{2} \cdot 1.05 \cdot 1.93^2 = 1.96$$

We can plug these individual kinetic energies into $K_{1i} = K_{if} + K_{2f}$. Doing so tells us that both sides are around the same. Thus, since kinetic energy is conserved, our collision is elastic.

Solution to part d i: We can conserve momentum and use our method to find the speed of the center of mass.
We know that the initial momentum p_i is $m_1 v_0$

The final momentum p_f can be written as $(m_1 + m_2)v_{cm}$
We can set p_i and p_f equal to each other. Doing so gives us the equation

$$m_1 v_0 = (m_1 + m_2)v_{cm}$$

We can divide both sides by $m_1 + m_2$ to find that $v_{cm} = \dfrac{m_1 v_0}{m_1 + m_2}$

Substituting our values gives that $v_{cm} = \dfrac{0.5 \cdot 3}{0.5 + 1.05} = 0.97$ m/s

Solution to part d ii: The maximum elastic potential energy occurs when the spring is compressed to its maximum possible distance. Initially, there is only kinetic energy. However, eventually there will be both kinetic energy and spring potential energy. Thus, some of the initial kinetic energy must convert to spring potential energy. Note that maximum elastic potential energy will occur after the spring has been compressed so much to the point that both carts move together with the same velocity. The final velocity of both carts will be v_{cm}, which we already found in part d i.

We know that $K_i = K_f + U_s$

$$K_i = \frac{1}{2} m_1 v_0^2$$

$$K_f = \frac{1}{2}(m_1 + m_2)v_{cm}^2$$

This means that $U_s = \dfrac{1}{2} m_1 v_0^2 - \dfrac{1}{2}(m_1 + m_2)v_{cm}^2$

We can substitute our values to get $U_s = \dfrac{1}{2} \cdot 0.5 \cdot 3^2 - \dfrac{1}{2} \cdot (0.5 + 1.05) \cdot 0.97^2 = 1.52$ J

Thus, the maximum elastic potential energy stored in the spring is 1.52 J.

Unit 5 Rotation

Have you ever been on a seesaw in the park near your house? Maybe you went with multiple friends. Did you notice that sometimes the seesaw would be balanced, so neither your friend nor you would move up and down. Then, you might tell your friend to move further away from you (further from the pivot). After that, you would start to rotate. Now to truly understand the famous seesaw, you must learn about torque and rotational motion.

Before you proceed to learn about torque and rotational motion, I want to define the term axis of rotation. Axis of rotation is basically a straight line that some body rotates around. For example, a carousel will have an axis of rotation since the entire ride spins around an imaginary axis.

Now let's dive into torque.

Note 5.0.1 — Torque
Torque is the rotational analogue of force. If a rigid body rotates around an origin O, torque is defined as

$$\tau = FR\sin\theta,$$

where F is the magnitude of the force applied at a point at distance R from O, and the smaller angle between the line of force and the line between R and O is θ.

In simple words, you can simply use the formula $\tau = FR$ as long as the distance R you find is the perpendicular distance between the pivot and the force. This distance R is also known as the lever arm.

In addition, be very careful about the **direction** that you denote in a problem. Traditionally, the **counterclockwise** direction is defined as positive. Thus, if a torque is causing an object to spin in the counterclockwise direction, write it with a positive sign. However, if it's causing it to move in the clockwise direction, write it with a negative sign. You will see examples of this later, so don't worry if you're confused.

Thus, if you have a very long lever arm (perpendicular distance between the force and the axis of rotation), then the torque will be greater. The idea behind torque has influenced many tools such as wrenches.

You may have noticed that a very small wrench can struggle to tighten or loosen a bolt. The reason is that the lever arm is extremely small. The axis you rotate around will be close to the force you apply. This will cause the torque to be less. However, a large wrench will make it much easier to tighten or loosen that same bolt. The reason is that the lever arm is now longer, and this leads to a greater torque.

Another common example of this is a door. Try to open a door by pushing the hinge. It will be EXTREMELY hard. The reason is that the force you apply is super close to the hinge, causing the lever arm to be small. That is why a door knob is placed far from the hinge. It maximizes the lever arm allowing torque to be greater. Thus, you don't need to apply as great of a force due to the long lever arm.

We already covered kinematics in Unit 1. However, our unit of rotation also has kinematics in it. It's known as rotational kinematics. It's extremely similar to the kinematics we have learned. The formulas will be almost the same, but the variables will be different. Make analogies with unit 1 if you want to have an easy time with rotational kinematics.

Note 5.0.2 — Rotational Kinematics
The angular displacement θ is defined as the radians an object has rotated.

The angular velocity ω is defined as the rate of change of θ, or $\frac{\Delta\theta}{\Delta t}$.

The angular acceleration α is defined as the rate of change of ω, or $\frac{\Delta\omega}{\Delta t}$

Similar to regular kinematics, the formulas still hold, where the linear variables are just replaced with the rotational variables. For example, θ replaces d or Δx, ω replaces v, and α replaces a.

Similar to regular kinematics, subscripts will still be used in rotational kinematics. For example, you still want to use ω_o or ω_i to denote the initial angular velocity and ω_f to denote the final angular velocity.

$$\omega_f = \omega_i + \alpha t \qquad (1.1)$$

$$\Delta\theta = \frac{\omega_i + \omega_f}{2} \cdot t \qquad (1.2)$$

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2 \qquad (1.3)$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta \qquad (1.4)$$

All the rotational kinematic equation above are true if the angular acceleration is constant. Out of the 5 variables $t, \omega_i, \omega_f, \alpha, \Delta\theta$, you must know 3 values to find the rest using the equations above.

Note that $\Delta\omega$ represents the angular displacement.

Now, sometimes the angular acceleration might not be constant. That's when you should know the calculus forms for some of this information.

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

The derivative of angular position with respect to time is angular velocity, and the derivative of angular velocity with respect to time is angular acceleration.

In Unit 2, we learned about Newton's Laws and the idea behind inertia. For example, an object with a larger mass would take a lot more force to move. Something similar occurs for this unit.

Note 5.0.3 — Rotational Inertia
In rotational motion, rotational inertia is analogous to mass in linear motion. It is defined as

$$I = MR^2$$

for a particle, where R is the distance of the particle from the pivot. Note that I denotes rotational inertia.

The rotational inertia for multiple objects is $I = \sum m_i r_i^2$. You basically add up the rotational inertias of each object.

The rotational inertias for common rigid bodies are $\frac{2}{5}MR^2$ for a solid sphere, $\frac{2}{3}MR^2$ for a hollow sphere, $\frac{1}{12}ML^2$ for a rod rotated around the center, $\frac{1}{3}ML^2$ for a rod rotated around its end, MR^2 for a ring, MR^2 for a hollow cylinder, $\frac{1}{2}MR^2$ for a disk, and $\frac{1}{2}MR^2$ for a solid cylinder.

Now, you must know another way to write inertia.

$$I = \int r^2 dm$$

dm is a differential that can be found by using the linear mass density. You will see examples of this later.

You should also know how to compare the rotational inertias without actually computing it. For example, let's compare a hoop and a disk. For a hoop and disk that have the same mass radius, the rotational inertia will be larger for the hoop. The reason is that the mass is concentrated towards the ends. However, a disk has mass throughout the entire area unlike a hoop (which only has mass near the circumference).

If you can compare the rotational inertias of certain objects by analyzing how the mass is distributed in them, then you will have a very strong conceptual understanding to approach multiple choice questions.

Now, there is something known as equilibrium. When an object is at complete equilibrium, then $F_{net} = 0$ and $\tau_{net} = 0$. The net force and net torque will both be 0. This is specifically known as **static equilibrium**.

Often for such problems, you will write two equations. One will be for the net torque and one will be for the net force.

Note 5.0.4 — Parallel Axis Theorem
Given that the rotational inertia of an object about an axis going through its center of mass is I_{cm}, the rotational inertia of the object about another axis which is parallel to this axis, and is a distance d from it, is

$$I = I_{\text{cm}} + Md^2,$$

where M is the mass of the object.

This formula is useful when the axis of rotation is in an odd spot. You can then find the rotational inertia of the object with the axis going through the center of mass. Then, you can account for the odd location by adding Md^2 to the rotational inertia around the center of mass.

Note 5.0.5 — Newton's 2nd Law of Rotation is an analog to Newton's second law.

$$\tau_{net} = I\alpha$$

This means that the net torque will cause angular acceleration.
When an object is at rotational equilibrium, we can say that the net torque on it must be 0.

Now, we must learn all about another specific type of energy
The rotational kinetic energy is $\frac{1}{2}I\omega^2$.

If a problem asks you to find the total kinetic energy, then you must add up the rotational kinetic and translational kinetic energies.

$$\text{The total kinetic energy is } \frac{1}{2}mv^2 + \frac{1}{2}Iw^2$$

Note 5.0.6 — Rotational Work Similar to work done by a force, when a torque τ is applied for an angular distance of θ, the rotational work done is

$$W = \tau\theta$$

In terms of calculus, the work done can be found by the formula below.

$$W = \int \tau d\theta$$

This means that work is the integral of torque with respect to the angular position.

Now, we will learn the tricks to solve a popular problem that always shows up on the AP exam.

When a problem describes the motion as rolling without slipping, then our object is both rotating and translating without sliding. In addition, in this scenario, static friction will act on the object, but it will do no work on that object.

In such situations, $v = \omega r$. This means that the angular velocity times the radius will be the velocity.

Note, that if the object is **slipping**, then the relationship $v = wr$ won't be true.

Now, instead of linear momentum, we will look at angular momentum. You will be surprised by how similar both topics are.

Note 5.0.7 — Angular Momentum
Similar to linear momentum, angular momentum is defined as

$$L = mvr\sin(\theta)$$

This is the formula that you should use for a point mass, where θ is the angle between the radius vector and velocity vector.

For a rotating body around a specific origin with angular velocity ω and inertia I, the angular momentum should be represented as

$$L = I\omega$$

Traditionally, you should use $L = mvr\sin(\theta)$ for a point mass and $L = I\omega$ for rigid bodies such as a hoop.

Let's discuss impulse now. Remember how the impulse was equivalent to the change in linear momentum.
Something similar occurs in this unit. **Angular impulse** is the change in **angular momentum**.

Note 5.0.8 —
ΔL represents the change in angular momentum. It's also equivalent to the angular impulse which is $\tau\Delta t$.

$$\Delta L = I\Delta\omega = \tau\Delta t$$

With calculus, this can be written as $\Delta L = \int \tau dt$. This means that the change angular momentum or the angular impulse is the integral of torque with respect to time.

$$\text{Similarly, } \tau = \frac{dL}{dt}$$

Now, we can use this information to figure out what conservation of angular momentum is.
We should know by now that linear momentum is conserved when there are no external forces.
For **angular momentum to be conserved**, there should be no **external torques** on our system.

Conservation of angular momentum gives that $I_1 w_{1i} + I_2 w_{2i} = I_1 w_{1f} + I_2 w_{2f}$

Of course knowing this formula isn't enough to do well on the AP exam. You must practice many problems to be able to do well. There are many different scenarios that you might encounter on the AP exam. You should check out the AP Physics 1 Unit 7 Rapid Review video on the TMAS Academy youtube channel to learn about common examples, such as a ballerina spinning in a circle and pulling her arms inwards.

Now get ready for a big set of problems that will help you process this entire unit.

Problem 5.0.9 — A 15-kg box sits on a lever arm at a distance of 5 meters from the axis of rotation. What distance must a second 10-kg box sit to create a clockwise moment that will result in a net torque of zero?

Solution: Since we want a net torque of zero, both boxes should apply the same torque. They are both on the opposite sides of the axis of rotation, so if they both have an equal magnitude of torque, then the net torque will end up being 0.

The 15-kg block applies a torque of $\tau = Fr$.
Now, we must figure out what force is applied on the lever arm. Since the 15-kg box is sitting on the lever arm, the force will simply be equal to the magnitude of the weight which is $m_1 g$ (let's give the weight of the heavier box the variable m_1).

$$\tau = Fr = m_1 g \cdot r = 5m_1 g$$

This means that the 10-kg block must also apply that same torque. Let's denote the mass of this block as m_2. Let's say that x is the distance of the lighter box from the axis of rotation.
The torque: $\tau = Fr = m_2 g x$

Since the net torque is 0, we know that $5m_1 g = m_2 g x$
We can cancel out the g and rearrange to get $x = \frac{5m_1}{m_2}$

$$\text{We can plug in the masses to get } x = \frac{5m_1}{m_2} = \frac{5 \cdot 15}{10} = 7.5 \text{ m}$$

Problem 5.0.10 — The iron door of a building is x meters wide. It can be opened by applying a force of F_1 newtons at the middle of the door. Calculate the least force which can open the door in terms of F_1.

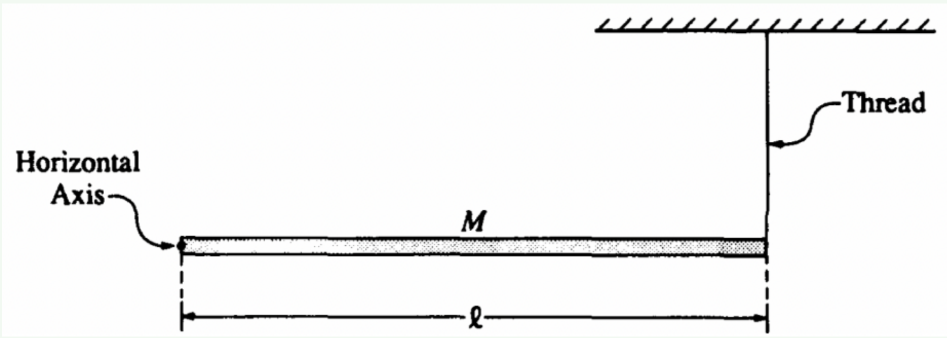
Solution: Since the door is x meters wide, the middle of the door will be $x/2$ meters from the hinge.
That means the door can be opened if a torque of $\tau = F_1 \cdot \frac{x}{2} = \frac{F_1 x}{2}$ is applied.

This required torque will remain constant despite where the force is applied. However, it's the force and distance from the hinge that can vary. The force to open the door will be minimized when it's applied as far as possible from the hinge. The reason is that this will maximize the distance from the hinge, causing the required force to be less.

Thus, the force should be applied at the edge of the door so that the lever arm is x meters long.
Thus, if the force applied is F_a, then the torque will be $F_a x$. We can set this equal to $\frac{F_1 x}{2}$ to find F_a (least force to open the door).

$$\text{We can divide both sides by } x \text{ to find that } F_a = \frac{F_1}{2}$$

Problem 5.0.11 — 1993 AP Physics C: Mechanics FRQ



A long, uniform rod of mass M and length l is supported at the left end by a horizontal axis into the page and perpendicular to the rod, as shown above. The right end is connected to the ceiling by a thin vertical thread so that the rod is horizontal. Express the answers to all parts of this question in terms of M, L, and g.

(a) Determine the magnitude and direction of the force exerted on the rod by the axis.

(b) If the breaking strength of the thread is $2Mg$, determine the maximum distance, r, measured from the hinge axis, that a box of mass $4M$ could be placed without breaking the thread.

Solution to part a: Let's say that force from the horizontal axis will be denoted as F and it will point upwards.
Then, our summation of forces is $F + T = Mg$ (T is the tension force from the thread)
Now, we will apply the concept of torque about the horizontal axis.
We know that $\tau_{net} = \sum F_i r_i$.

About the horizontal axis, the tension force is a distance L away while the gravitational force is $\frac{L}{2}$ away from the pivot(since gravity acts at the center)

Clearly, $\tau_{net} = T \cdot L - Mg \cdot \frac{L}{2}$
We know that our net torque must be 0 since this system is static (there is no motion going on). Since there is no rotational acceleration, net torque must be 0.
This means that $T \cdot L - Mg \cdot \frac{L}{2} = 0$
We can rearrange this and solve to find that $T = \frac{Mg}{2}$

Now, we can plug this into the equation $F + T = Mg$.
Doing so gives that $\boxed{F = \frac{Mg}{2}}$. Since our force is positive, the direction we assumed

initially for this force is the correct direction.

Solution to part b: We are given the maximum possible tension force T to be $2Mg$.

We can use our horizontal axis as the pivot point. This means that

$$\tau_{net} = T \cdot L - Mg \cdot \frac{L}{2} - 4Mg \cdot r$$

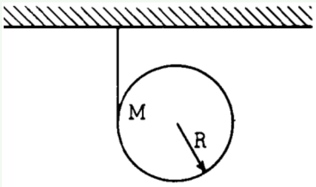
We know that $\tau_{net} = 0$ since the system is in static equilibrium. We can plug in our maximum possible value of tension which is $2Mg$
Doing so simplifies the equation to

$$2MgL - \frac{MgL}{2} - 4Mgr = 0$$

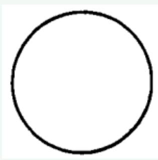
We can further simplify this to $\frac{3MgL}{2} = 4Mgr$

Now, we can divide both sides by $4Mg$ to get that $r = \frac{3L}{8}$

Problem 5.0.12 — 1976 AP Physics FRQ



A cloth tape is wound around the outside of a uniform solid cylinder (mass M, radius R) and fastened to the ceiling as shown in the diagram above. The cylinder is held with the tape vertical and then released from rest. As the cylinder descends, it unwinds from the tape without slipping. The moment of inertia of a uniform solid cylinder about its center is $1/2MR^2$.



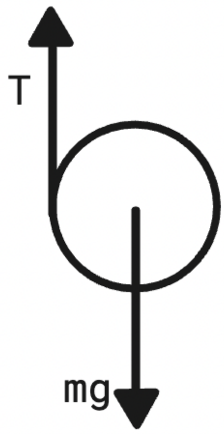
(a) On the circle above draw vectors showing all the forces acting on the cylinder after it is released. Label each force clearly.

(b) In terms of g, find the downward acceleration of the center of the cylinder as it unrolls from the tape.

(c) While descending, does the center of the cylinder move toward the left, toward the right, or straight down? Explain.

Solution to part a: The only forces on the cylinder are tension force and gravitational force.
Sometimes the problem will say to draw the forces at the center. In that case, no matter

from what point the force stems from, you should still draw all forces pointing from the center. In this case, the problem doesn't say list such a condition which is why I'll draw tension force from the side (since that's where the rope is connected to the cylinder at).



Solution to part b: We will write an equation for net force and net torque.

$$F_{net} = Mg - T = Ma$$

We will write an equation for net torque around the center of the cylinder.

$$\tau_{net} = T \cdot R = I\alpha$$

Since the cylinder rolls without slipping, we know that $\alpha = \frac{a}{R}$. This relates acceleration to angular acceleration.
We can plug that into our equation for net torque to get $T \cdot R = I \cdot \frac{a}{R}$

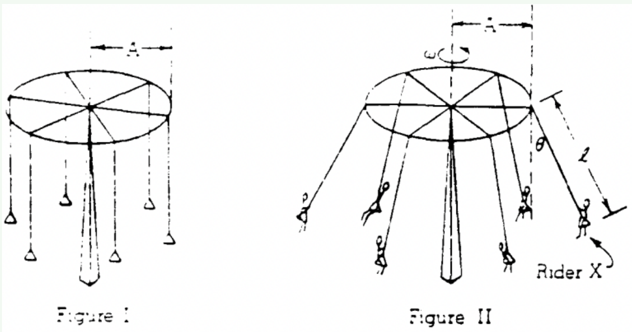
Since we know that $I = \frac{1}{2}MR^2$, we can plug that in to get $T \cdot R = \frac{1}{2}MR^2 \cdot \frac{a}{R} = \frac{MRa}{2}$

We can divide both sides by R to find that $T = \frac{Ma}{2}$

Now, we can plug this into the equation for net force which is $Mg - T = Ma$
Doing so gives $Mg - \frac{Ma}{2} = Ma$
We can rearrange this equation and solve it to find that $\boxed{a = \dfrac{2g}{3}}$

Solution to part c: Clearly, there are no forces in the horizontal direction. It means that the cylinder can't move to the left or to the right. It must move $\boxed{\text{straight down}}$.

Problem 5.0.13 — 1978 AP Physics FRQ



An amusement park ride consists of a ring of radius A from which hang ropes of length l with seats for the riders as shown in Figure I. When the ring is rotating at a constant angular velocity ω, each rope forms a constant angle θ with the vertical as shown in Figure II. Let the mass of each rider be m and neglect friction, air resistance, and the mass of the ring, ropes, and seats.

(a) Draw and label all the forces acting on rider X under the constant rotating condition of Figure II. Clearly define any symbols you introduce.

(b) Derive an expression for ω in terms of A, l, θ, and the acceleration of gravity g.

(c) Determine the minimum work that the motor that powers the ride would have to perform to bring the system from rest to the constant rotating condition of Figure II. Express your answer in terms of m, g, l, θ, and the speed v of each rider.

Solution to part a: The two forces on the rider will be tension force and gravitational force. T is used to denote tension force while mg is the gravitational force.

Solution to part b: We know that a centripetal force points towards the center. The forces in the vertical direction will balance out. However, there will be a net horizontal force and that is what leads to centripetal acceleration.

In the vertical direction, we know that $T\cos(\theta) - mg = 0$

In the horizontal direction, $T\sin(\theta) = ma = mw^2r$

It is the horizontal component of tension that leads to centripetal acceleration. Also, mw^2r is the same thing as $\frac{mv^2}{r}$. The only difference is that one uses angular velocity while the other uses velocity itself in the expression.

Now, we must find an expression for our radius which currently is written as r. Clearly, the rider is a distance A (radius of the ring) summed up with the horizontal length of the rope. This means that $r = A + l\sin(\theta)$

We can plug this into the equation for net force in x-direction to get

$$T\sin(\theta) = mw^2(A + l\sin(\theta))$$

Using the equation in the vertical direction, we can find that $T = \frac{mg}{\cos(\theta)}$
We can plug our expression for tension into $T\sin(\theta) = mw^2(A + l\sin(\theta))$

$$\frac{mg}{\cos(\theta)} \cdot \sin(\theta) = mw^2(A + l\sin(\theta))$$

We can simplify the equation to $w^2 = \frac{g\tan(\theta)}{A+l\sin(\theta)}$
Now, we can take the square root of both sides to get $w = \sqrt{\frac{g\tan(\theta)}{A+l\sin(\theta)}}$

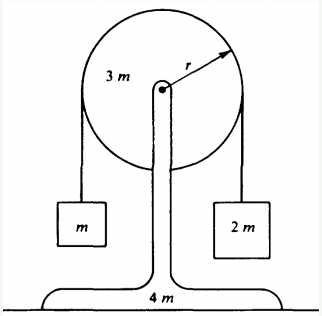
Solution to part c: We know that the work done is change in energy.
$W = \Delta K + \Delta U$

Clearly, $\Delta K = \frac{1}{2}mv^2$
Also, $\Delta U = U_f - U_i = mgh - 0 = mgl(1 - \cos(\theta))$

This means that the work done for one rider is $W = \frac{1}{2}mv^2 + mgl(1 - \cos(\theta))$
However, since we have 6 riders, we must multiply that expression by 6.

Doing so gives that the total work done is $3mv^2 + 6mgl(1 - \cos(\theta))$

Problem 5.0.14 — 1985 AP Physics FRQ



A pulley of mass $3m$ and radius r is mounted on frictionless bearings and supported by a stand of mass $4m$ at rest on a table as shown above. The moment of inertia of this pulley about its axis is $1.5mr^2$. Passing over the pulley is a massless cord supporting a block of mass m on the left and a block of mass $2m$ on the right. The cord does not slip on the pulley, so after the block-pulley system is released from rest, the pulley begins to rotate.

(a) On the diagrams below, draw and label all the forces acting on each block.

(b) Use the symbols identified in part a. to write each of the following.

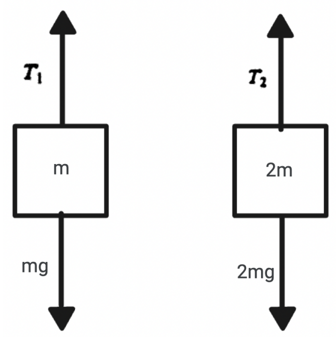
i. The equations of translational motion (Newton's second law) for each of the two blocks

ii. The analogous equation for the rotational motion of the pulley

(c) Solve the equations in part b. for the acceleration of the two blocks.

(d) Determine the tension in the segment of the cord attached to the block of mass m.

Solution to part a: The only forces on each block are tension force and gravitational force. The length of the force vectors don't matter as long as all the forces are labelled. The reason is that the problem doesn't explicitly say to make the lengths of the force vectors comparable by magnitude.



Solution to part b i: We will use Newton's Second Law for both blocks.
For block of mass $2m$, we get $2mg - T_2 = 2ma$
For block of mass m, we get $T_1 - mg = ma$

Solution to part b ii: We can use Newton's Second Law for rotation which says $\tau_{net} = I\alpha$
Clearly, $\tau_{net} = T_2 r - T_1 r = r(T_2 - T_1)$
This means that $r(T_2 - T_1) = I\alpha$

Solution to part c: Since the cord does not slip on the pulley, we know that $\alpha = \frac{a}{r}$
This relates our angular acceleration to acceleration itself.

We can plug that into $r(T_2 - T_1) = I\alpha$ to get $r(T_2 - T_1) = \frac{Ia}{r}$

Now, we can substitute our expression for I which is $1.5mr^2$

Plugging it in turns the equation into $r(T_2 - T_1) = 1.5mra$

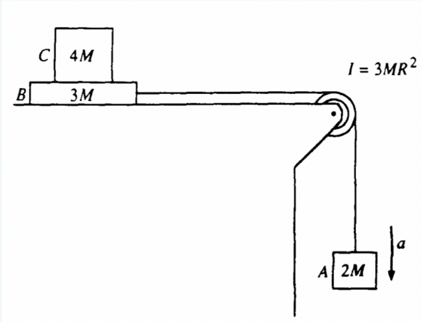
We can divide r from both sides to get $T_2 - T_1 = 1.5ma$

Now, we can solve the two equations we got using Newton's Second Law in part b i. The two equations that we got were $2mg - T_2 = 2ma$ and $T_1 - mg = ma$
We can add both of these equations to get $mg - (T_2 - T_1) = 3ma$
Now, we can substitute the expression we found previously which was $T_2 - T_1 = 1.5ma$

Doing so turns the equation into $mg - 1.5ma = 3ma$
After adding $1.5ma$ to both sides, we can find that $\boxed{a = \frac{2g}{9}}$

Solution to part d: We can find the tension in the cord for the block of mass m by using the equation we found for that block using Newton's Second Law.
We found that $T_1 - mg = ma$
We can rearrange the equation to get $T_1 = m(a + g)$
We can plug in $a = \frac{2g}{9}$ to get $T_1 = m(\frac{2g}{9} + g) = \boxed{\frac{11mg}{9}}$

Problem 5.0.15 — 1989 AP Physics FRQ



Block A of mass $2M$ hangs from a cord that passes over a pulley and is connected to block B of mass $3M$ that is free to move on a frictionless horizontal surface, as shown above. The pulley is a disk with frictionless bearings, having a radius R and moment of inertia $3MR^2$. Block C of mass $4M$ is on top of block B. The surface between blocks B and C is NOT frictionless. Shortly after the system is released from rest, block A moves with a downward acceleration a, and the two blocks on the table move relative to each other.

In terms of M, g, and a, determine the
(a) tension T_v in the vertical section of the cord

(b) tension T_h in the horizontal section of the cord

If $a = 2$ meters per second squared, determine the
(c) coefficient of kinetic friction between blocks B and C

(d) acceleration of block C

Solution to part a: We will find the net force on the block of mass $2M$. The forces on it are the gravitational force and tension in the vertical section (T_v)

The equation for Newton's Second Law on that block is $2Mg - T_v = 2Ma$
We can rearrange the equation to find that $\boxed{T_v = 2M(g - a)}$

Solution to part b: Some people might use Newton's Second Law on blocks B and C to find T_h.
However, this won't work since friction force also exists and we don't know it's magnitude. However, we can still find T_h by using Newton's Second Law for Rotation.

We know that $\tau_{net} = I\alpha$. We can find the net torque on the pulley.

$$\tau_{net} = T_v \cdot R - T_h \cdot R = I\alpha$$

We can plug in $I = 3MR^2$ into the equation for net torque to get

$$T_v \cdot R - T_h \cdot R = 3MR^2 \cdot \alpha$$

We can divide R from both sides to get $T_v - T_h = 3MR\alpha$
We will also use the fact that $\alpha = \frac{a}{R}$. We can plug this in to get $T_v - T_h = 3Ma$

Since we know that $T_v = 2M(g-a)$ from the first part, we can plug that in to find T_h. After we make our substitution, we find that $\boxed{T_h = 2Mg - 5Ma}$

Solution to part c: Now, we will use Newton's Second Law on the block B.
Clearly, the only forces in the horizontal direction on block B are tension force and friction force.
Using Newton's Second Law, we know that $F_{net} = T_h - F_f = m_b a = 3Ma$

We also know that $F_f = \mu N$. From forces in the y-direction on block C, it's obvious that $N = 4Mg$ (this same friction force is also applied on block C but in the opposite direction as of block B)
This means that $F_f = 4\mu Mg$

We can plug this into our equation for Newton's Second Law to get $T_h - 4\mu Mg = 3Ma$
We can plug in $T_h = 2Mg - 5Ma$ to get $2Mg - 5Ma - 4\mu Mg = 3Ma$

We can rearrange the equation to get $2Mg - 8Ma = 4\mu Mg$
We can divide both sides by M to get $2g - 8a = 4\mu g$

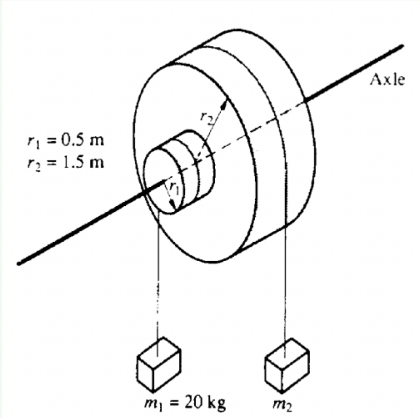
Now, we can plug in $g = 9.8$ and $a = 2$ to get $2 \cdot 9.8 - 8 \cdot 2 = 4 \cdot 9.8 \cdot \mu$
We can solve this equation to find that $\boxed{\mu = 0.092}$

Solution to part d: The acceleration of block C can be found using Newton's Second Law.
We know that $F_f = m_c a$
Clearly, $F_f = \mu N = \mu m_c g$

We can plug that in to get $\mu m_c g = m_c a$
After dividing both sides by m_c, we find that $a = \mu g$

Since $\mu = 0.092$, we know that $a = 0.092 \cdot 9.8 = \boxed{0.9016 \frac{m}{s^2}}$

Problem 5.0.16 — 1991 AP Physics FRQ



Two masses, m_1 and m_2, are connected by light cables to the perimeters of two cylinders of radii r_1 and r_2, respectively, as shown in the diagram above. The cylinders are rigidly connected to each other but are free to rotate without friction on a common axle. The moment of inertia of the pair of cylinders is $I = 45\,\text{kg}\cdot\text{m}^2$. Also, $r_1 = 0.5\,\text{m}$, $r_2 = 1.5\,\text{m}$, and $m_1 = 20\,\text{kg}$.

(a) Determine m_2 such that the system will remain in equilibrium.

The mass m_2 is removed, and the system is released from rest.
(b) Determine the angular acceleration of the cylinders.

(c) Determine the tension in the cable supporting m_1.

(d) Determine the linear speed of m_1 at the time it has descended 1.0 m.

Solution to part a: For the system to remain in equilibrium, the net torque should be 0. On top of that, the net force should also be 0.

We can write the summation of forces for both block of masses m_1 and m_2

Assuming that the force of tension of block of mass m_1 is T_1, we know that by Newton's Second Law, $T_1 - m_1 g = 0$. This means that $T_1 = m_1 g$

We can do something similar for the block of mass m_2. If tension force is T_2, then Newton's Second Law tells us that $T_2 - m_2 g = 0$ which means that $T_2 = m_2 g$

We also know that net torque (τ_{net}) must be 0 on the cylinder.
The net torque about the axle can be written as $\tau_{net} = T_1 \cdot r_1 - T_2 \cdot r_2$

Since $\tau_{net} = 0$, we know that $T_1 \cdot r_1 = T_2 \cdot r_2$
We can plug in $T_1 = m_1 g$ and $T_2 = m_2 g$ to get $m_1 g \cdot r_1 = m_2 g \cdot r_2$

$$\text{We can isolate } m_2 \text{ to get } m_2 = \frac{m_1 r_1}{r_2} = \frac{20 \cdot 0.5}{1.5}$$

This means that $\boxed{m_2 = 6.67 \text{ kg}}$

Solution to part b: To find the angular acceleration, we use the equation $\tau_{net} = I\alpha$
After m_2 is removed, the only force that applies a torque on the cylinder is the tension force from mass m_1.

We can find that $\tau_{net} = T_1 \cdot r_1$ and we know that this equals to $I\alpha$

Now, we can write another equation using Newton's Second Law on block of mass m_1.
Since the only forces on it are tension and gravity, we know that $m_1 g - T_1 = m_1 a$

Now, we can use the relation $\alpha = \frac{a}{r_1}$.
We can plug that into our first equation to get $T_1 \cdot r_1 = \frac{Ia}{r_1}$

We can rewrite this equation as

$$T_1 = \frac{Ia}{r_1^2}$$

Now, we can plug that into $m_1 g - T_1 = m_1 a$ to get

$$m_1 g - \frac{Ia}{r_1^2} = m_1 a$$

Since $I = 45$, $m_1 = 20$, and $r_1 = 0.5$, we can plug all of that into the equation above and solve it to get $a = 0.98 \frac{m}{s^2}$

Now, we can plug this back into $\alpha = \frac{a}{r_1}$ to find angular acceleration.
We can plug in $a = 0.98$ and $r_1 = 0.5$ to get $\boxed{\alpha = 1.96 \text{ rad}/s^2}$

Solution to part c: To find the tension force, we can plug in our value of angular acceleration.
We can plug it into $T_1 \cdot r_1 = I\alpha$
Since $r_1 = 0.5$, $I = 45$, and $\alpha = 1.96$, we can find that $\boxed{T_1 = 176.4 \text{ N}}$

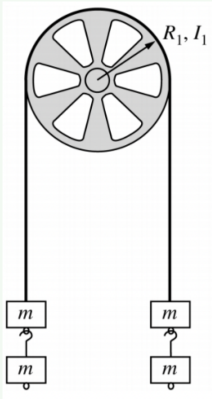
Solution to part d: To find the linear speed, we will use our acceleration (not angular acceleration).

We already found that $a = 0.98 \frac{m}{s^2}$. We also know that $v_i = 0$. On top of that, we know that $\Delta y = 1$ m.
We can use the kinematics equation $v^2 = v_i^2 + 2a\Delta y$
Plugging in our values gives

$$v^2 = 0^2 + 2 \cdot 0.98 \cdot 1 = 1.96$$

After we take the square root of both sides, we find that $\boxed{v = 1.4 \text{ m/s}}$

Problem 5.0.17 — 2000 AP Physics FRQ

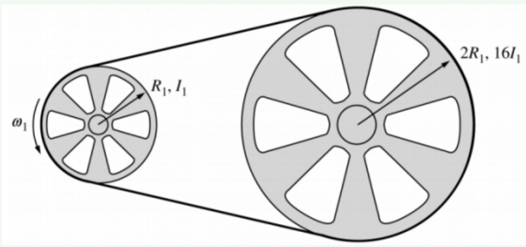


A pulley of radius R_1 and rotational inertia I_1 is mounted on an axle with negligible friction. A light cord passing over the pulley has two blocks of mass m attached to either end, as shown above. Assume that the cord does not slip on the pulley.
(a) Determine the tension T in the cord.
(b) One block is now removed from the right and hung on the left. When the system is released from rest, the three blocks on the left accelerate downward with an acceleration $g/3$. Determine the following.

i. The tension T_3 in the section of cord supporting the three blocks on the left

ii. The tension T_1 in the section of cord supporting the single block on the right

iii. The rotational inertia I_1 of the pulley



(c) The blocks are now removed, and the cord is tied into a loop, which is passed around the original pulley and a second pulley of radius $2R_1$ and rotational inertia $16I_1$. The axis of the original pulley is attached to a motor that rotates it at angular speed ω_1, which in turn causes the larger pulley to rotate. The loop does not slip on the pulleys. Determine the following in terms of I_1, R_1, and ω_1.

i. The angular speed ω_2 of the larger pulley

ii. The angular momentum L_2 of the larger pulley

iii. The total kinetic energy of the system

Solution to part a: Let's consider the two blocks on the left to be part of one system. The only two forces on the two block system are tension force and gravity.

Since it doesn't accelerate, we can use Newton's Second Law to write $T - 2mg = 0$
This means that $\boxed{T = 2mg}$

Solution to part b i: On the side with three blocks, we can consider all three of those blocks as a system. The only forces on that system are tension force and gravity. By using Newton's Second Law we know that $3mg - T_3 = 3ma$
Since $a = \frac{g}{3}$, we can plug that in above to get $\boxed{T_3 = 2mg}$

Solution to part b ii: We can again use Newton's Second Law. On the right side, we will only consider the 1 block alone as its own system. The only forces on it are tension force and gravity. It accelerates upwards, so we will consider that direction to be positive.

By using Newton's Second Law we know that $T_1 - mg = ma$
This means that $T_1 = m(a + g)$
We can plug in $a = \frac{g}{3}$ to get $\boxed{T_1 = \frac{4mg}{3}}$

Solution to part b iii: We can find rotational inertia by using Newton's Second Law for rotation.
We know that $\tau_{net} = I\alpha$. Now, let's write an equation using that for the torque on the pulley.

$$\tau_{net} = T_3 \cdot R_1 - T_1 \cdot R_1 = R_1(T_3 - T_1)$$

On top of that, we can use the relation $\alpha = \frac{a}{r}$

We can plug in $a = \frac{g}{3}$ and $r = R_1$ into $\alpha = \frac{a}{r}$ to find that $\alpha = \frac{g}{3R_1}$

Now we can plug that expression in to find that

$$R_1(T_3 - T_1) = \frac{Ig}{3R_1}$$

We can solve the equation for I to find that

$$I = \frac{3R_1^{22}(T_3 - T_1)}{g}$$

We can plug in $T_3 = 2mg$ and $T_1 = \frac{4mg}{3}$ to find that $\boxed{I = 2mR_1^2}$

Solution to part c i: Even though the angular speeds of the pulley don't have to be equal, the tangential speed of the cord around each pulley must be equal. The tangential speeds are equal because the cord around this system is the same. Thus, it must be moving with the same tangential speed regardless of the radius of any pulley.

The tangential speed around the original pulley is $v_1 = w_1 R_1$
Similarly, the tangential speed around the new pulley is $v_2 = w_2 \cdot 2R_1 = 2w_2 R_1$

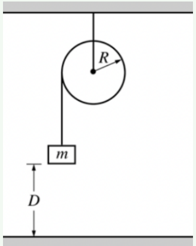
We can set v_1 and v_2 equal to each other to find that $\boxed{w_2 = \frac{w_1}{2}}$

Solution to part c ii: We know that angular momentum is $L = Iw$
For the larger pulley, $I = 16I_1$ and $w = w_2 = \frac{w_1}{2}$. We can plug that in to find that $L_2 = 16I_1 \cdot \frac{w_1}{2} = \boxed{8I_1 w_1}$

Solution to part c iii: The sum of the kinetic energy of the entire system is the sum of the kinetic energies for each pulley.
Since we know rotational inertia and angular velocity for each pulley, we will use $K = \frac{1}{2}Iw^2$

$$\text{The total kinetic energy is } \frac{1}{2}I_1w_1^2 + \frac{1}{2} \cdot 16I_1 \cdot (\frac{w_1}{2})^2 = \frac{5}{2}I_1w_1^2$$

Problem 5.0.18 — 2004 AP Physics FRQ



A solid disk of unknown mass and known radius R is used as a pulley in a lab experiment, as shown above. A small block of mass m is attached to a string, the other end of which is attached to the pulley and wrapped around it several times. The block of mass m is released from rest and takes a time t to fall the distance D to the floor.

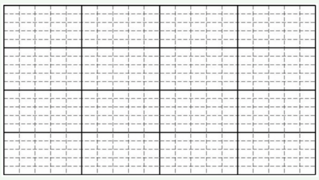
(a) Calculate the linear acceleration a of the falling block in terms of the given quantities.

(b) The time t is measured for various heights D and the data are recorded in the following table.

D (m)	t (s)
0.5	0.68
1	1.02
1.5	1.19
2	1.38

i. What quantities should be graphed in order to best determine the acceleration of the block? Explain your reasoning.

ii. On the grid below, plot the quantities determined in **b. i.**, label the axes, and draw the best-fit line to the data.



iii. Use your graph to calculate the magnitude of the acceleration.

(c) Calculate the rotational inertia of the pulley in terms of m, R, a, and fundamental constants.

(d) The value of acceleration found in b. iii, along with numerical values for the given quantities and your answer to c., can be used to determine the rotational inertia of the pulley. The pulley is removed from its support and its rotational inertia is found to be greater than this value. Give one explanation for this discrepancy.

Solution to part a: We can use kinematics to find an expression for acceleration. We know that vertical displacement Δy is D, $v_i = 0$, and we are given the various times.

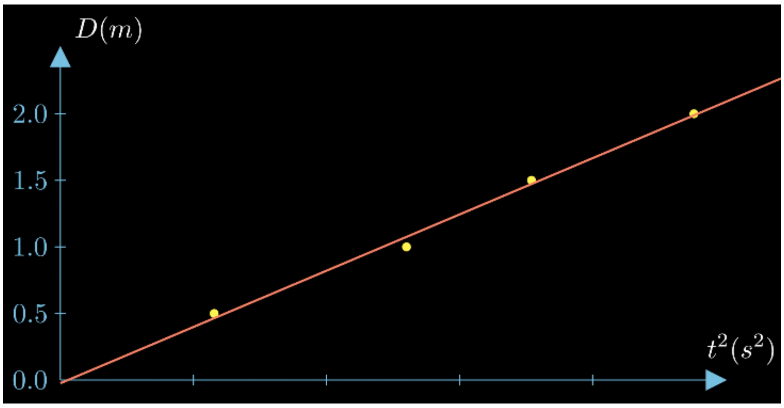
We can plug this into $\Delta y = v_i t + \frac{1}{2}at^2$

We can plug in our values to get $D = 0 + \frac{1}{2}at^2$

We can rearrange the equation to find that $\boxed{a = \dfrac{2D}{t^2}}$

Solution to part b i: We should graph D on the y-axis and t^2 on the x-axis.
The reason is that it causes our graph to be linear. Then, to find acceleration, we will simply only need to find the slope.

Solution to part b ii: The graph is shown below



Solution to part b iii: By observing the graph, it is evident that the slope is 1.
some people will immediately say that the acceleration is 1. However, they are forgetting one step.
The slope of 1 represents $\frac{D}{t^2}$. However, we want to find $\frac{2D}{t^2}$
That means we must multiply our slope by 2 to find acceleration.
$\boxed{a = 2 \text{ m}/s^2}$

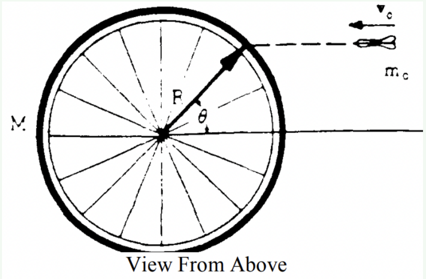
Solution to part c: We will use net torque and net force to find the rotational inertia.
Assuming that downwards is positive, using Newton's Second Law we can write $mg - T = ma$
Using net torque τ_{net}, we know that $\tau_{net} = TR = I\alpha$
Now, for the equation that used net torque, we can divide both sides by R to get $T = \frac{I\alpha}{R}$
We can plug this into $mg - T = ma$ to get $mg - \frac{I\alpha}{R} = ma$

Since $\alpha = \frac{a}{R}$, we can plug that into the equation above to get $mg - \frac{Ia}{R^2} = ma$
We can rearrange the equation to get $mg - ma = \frac{Ia}{R^2}$

After multiplying both sides by $\frac{R^2}{a}$, we get that $\boxed{I = \dfrac{mR^2(g-a)}{a}}$

Solution to part d: One possible discrepancy occurs if the string is wrapped around the pulley multiple times. The reason is that this will cause the radius of the pulley to increase. However, we used the radius of the pulley alone instead of accounting for the extra radius that came from the string being wrapped around many times.

Problem 5.0.19 — 1975 AP Physics FRQ



A bicycle wheel of mass M (assumed to be concentrated at its rim) and radius R is mounted horizontally so it may turn without friction on a vertical axle. A dart of mass m_0 is thrown with velocity v_o as shown above and sticks in the tire.

(a) If the wheel is initially at rest, find its angular velocity ω after the dart strikes.

(b) In terms of the given quantities, determine the ratio:

$$\frac{\text{final kinetic energy of the system}}{\text{initial kinetic energy of the system}}$$

Solution to part a: We can use angular momentum conservation in this problem. Conservation says that $L_i = L_f$

The initial momentum can be found using the formula $L = mvr$. We know that r represents the perpendicular distance to the axis. Clearly, in this problem, the bullet is a distance $R\sin(\theta)$ away. This means that $L_i = mv_oR\sin(\theta)$

We know that after both collide, they will rotate the vertical axle. Since the bicycle wheel has all of its mass concentrated at the rim, the rotational inertia for the wheel alone is MR^2.

After collision, the rotational inertia of the dart will be mR^2 since it spins around with the bicycle wheel with a radius R.

The total rotational inertia can be found by summing both which gives $(m + M)R^2$

We know that $L = Iw$, so we can find that $L_f = (m + M)R^2 w_f$

We can equate this to L_i which is $mv_oR\sin(\theta)$

$$mv_oR\sin(\theta) = (m + M)R^2 w_f$$

Now, we can divide both sides by $(m + M)R^2$ to get $\boxed{w_f = \dfrac{mv_o\sin(\theta)}{(m + M)R}}$

Solution to part b: We know that the initial kinetic energy can simply be found by using $K = \frac{1}{2}mv^2$ due to the translational motion. This means that $K_i = \frac{1}{2}mv_o^2$

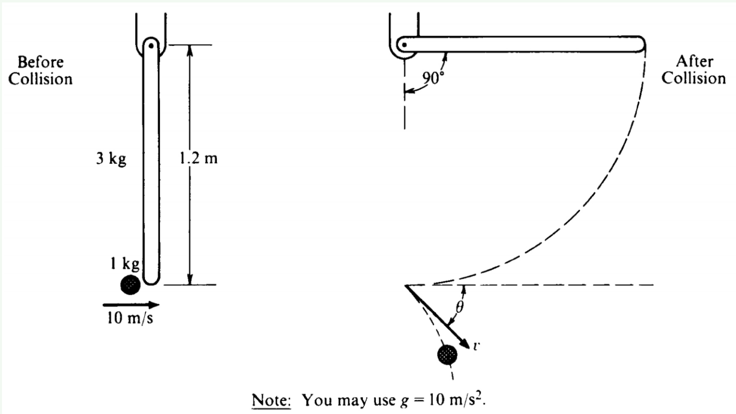
However, after collision, the wheel and bullet both spin around a common axle. The rotational kinetic energy can be found using $\frac{1}{2}Iw_f^2$

We already know that their combined rotational inertia is $(m+M)R^2$. On top of that, we know that $w_f = \frac{mv_o \sin(\theta)}{(m+M)R}$

$$\text{We can plug these expressions into } K_f = \frac{1}{2}Iw^2 \text{ to find that } K_f = \frac{m^2 v_o^2 \sin^2(\theta)}{m+M}$$

When we take the ratio $\frac{K_f}{K_i}$, most variables cancel out.
We are left with $\boxed{\dfrac{m\sin^2(\theta)}{m+M}}$ which is the ratio.

Problem 5.0.20 — 1987 AP Physics FRQ



A 1.0-kilogram object is moving horizontally with a velocity of 10 meters per second, as shown above, when it makes a glancing collision with the lower end of a bar that was hanging vertically at rest before the collision. For the system consisting of the object and bar, linear momentum is not conserved in this collision, but kinetic energy is conserved. The bar, which has a length l of 1.2 meters and a mass m of 3.0 kilograms, is pivoted about the upper end. Immediately after the collision, the object moves with speed v at an angle θ relative to its original direction. The bar swings freely, and after the collision reaches a maximum angle of 90 with respect to the vertical. The moment of inertia of the bar about the pivot is $I_{\text{bar}} = \frac{ml^2}{3}$. Ignore all friction.

(a) Determine the angular velocity of the bar immediately after the collision.

(b) Determine the speed v of the 1-kilogram object immediately after the collision.

(c) Determine the magnitude of the angular momentum of the object about the pivot just before the collision.

Solution to part a: For the bar, its rotational kinetic energy will convert to potential energy.
Its rotational kinetic energy is $\frac{1}{2}Iw^2$

To find the change in potential energy, we will set the reference level as the center of the bar. The reason is that we only need to consider the center of mass to find the change of potential in this case. The center of mass simply moves a distance $\frac{l}{2}$ where l is the length of the bar.

This means that the change in potential energy is $mg \cdot \frac{l}{2}$ which is $\frac{mgl}{2}$
Since $-\Delta K = \Delta U$, we know that $\frac{1}{2}Iw^2 = \frac{mgl}{2}$
We can solve this for w to get $w = \sqrt{\frac{mgl}{I}}$
Now, we can plug in $I = \frac{ml^2}{3}$

Doing so gives that $w = \sqrt{\frac{3g}{l}}$
Since $l = 1.2$ m, we can plug that in to get

$$w = \sqrt{\frac{3 \cdot 9.8}{1.2}} = \boxed{4.95 \text{ rad/s}}$$

Solution to part b: We can find the initial speed of the object by conserving kinetic energy. The problem explicitly says that kinetic energy is conserved.
The initial kinetic energy can be found using the formula $K = \frac{1}{2}mv^2$. Using it gives that it is $\frac{1}{2} \cdot 1 \cdot 10^2 = 50$ J

Now, we will find the final kinetic energy. The 1 kg object moves at a speed of v after collision. This means that it's kinetic energy is simply $\frac{1}{2} \cdot 1 \cdot v^2 = \frac{v^2}{2}$

The kinetic energy of the bar can be found using $K = \frac{1}{2}Iw^2$
Since $I = \frac{ml^2}{3}$, we can plug in $m = 3$ and $l = 1.2$ to find that $I = \frac{3 \cdot 1.2^2}{3} = 1.44$
Now, we can plug $I = 1.44$ and $w = 4.95$ (from part a) into the formula $K = \frac{1}{2}Iw^2$. This gives us that the rotational kinetic energy of the bar is $\frac{1}{2} \cdot 1.44 \cdot 4.95$ which evaluates to 17.642 J.

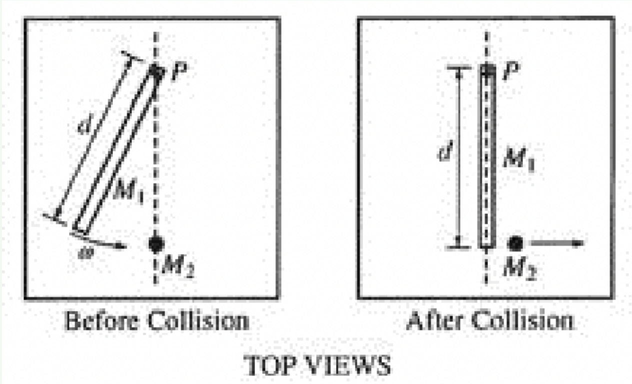
Now, since kinetic energy is conserved, we know that $K_i = K_f$
Our initial kinetic energy was found to be 50. The final kinetic energy is $\frac{v^2}{2} + 17.642$

We can set both expressions equal to each other to get $50 = \frac{v^2}{2} + 17.642$
We can solve the equation to find that $\boxed{v = 8.045 \text{ m/s}}$

Solution to part c: The initial momentum can be found using the equation $L = mvr$. Note that this equation is used often when we are finding the angular momentum for some point object. In this problem, it works because the initial angular momentum only involves the small 1-kg object.

r is the length of the bar which is 1.2 m. We also know that $m = 1$ kg while $v = 10$ m/s. We can plug these values in to find that $L_i = 1 \cdot 10 \cdot 1.2 = \boxed{12 \text{ kg} \cdot m^2/\text{s}}$

Problem 5.0.21 — 2005 AP Physics FRQ



A system consists of a ball of mass M_2 and a uniform rod of mass M_1 and length d. The rod is attached to a horizontal frictionless table by a pivot at point P and initially rotates at an angular speed ω, as shown above left. The rotational inertia of the rod about point P is $\frac{1}{3}M_1d^2$. The rod strikes the ball, which is initially at rest. As a result of this collision, the rod is stopped and the ball moves in the direction shown above right. Express all answers in terms of M_1, M_2, ω, d, and fundamental constants.

(a) Derive an expression for the angular momentum of the rod about point P before the collision.

(b) Derive an expression for the speed v of the ball after the collision.

(c) Assuming that this collision is elastic, calculate the numerical value of the ratio $\frac{M_1}{M_2}$.

Solution to part a: We know that angular momentum for the rod can be written as $L_i = Iw$

The rotational inertia about point P is $\frac{1}{3}M_1d^2$ (the formula for rotational inertia of a rod about its end)

Since the initial angular velocity is w, we multiply both expressions together to get $L_i = \frac{M_1d^2w}{3}$

Solution to part b: We know that angular momentum will be conserved. Since the rod stops rotating after colliding, it means that its final angular momentum is 0. However, the ball moves at a speed of v after.

This means that the final angular momentum is $L_f = M_2vd$ (from the formula $L = mvr$)

We must equate both our expressions: $L_i = L_f$

We plug in $L_i = \frac{M_1d^2w}{3}$ and $L_f = M_2vd$ to get

$$\frac{M_1d^2w}{3} = M_2vd$$

We can solve this for v to find that $\boxed{v = \frac{M_1dw}{3M_2}}$

Solution to part c: If the collision is elastic, then kinetic energy is conserved.

The initial kinetic energy $K_i = \frac{1}{2}Iw^2$ (only the rod moves)
We can plug in $I = \frac{1}{3}M_1 d^2$ and w to find that $K_i = \frac{1}{6}M_1 d^2 w^2$

The final kinetic energy only comes from the ball. We will use the formula for kinetic energy $K = \frac{1}{2}mv^2$ to find that $K_f = \frac{1}{2}M_2 v^2$
We already know that $v = \frac{M_1 dw}{3M_2}$ from part b. We can plug that in to find that

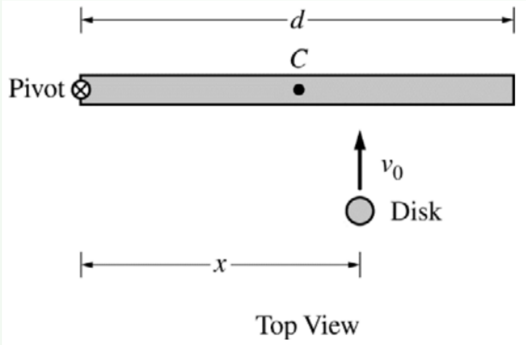
$$K_f = \frac{1}{2} \cdot M_2 \cdot (\frac{M_1 dw}{3M_2})^2 = \frac{M_1^2 d^2 w^2}{18M_2}$$

Now, we can equate K_i and K_f.

$$\frac{1}{6}M_1 d^2 w^2 = \frac{M_1^2 d^2 w^2}{18M_2}$$

We can cancel out many variables to find that $\frac{M_1}{M_2} = 3$

Problem 5.0.22 — 2017 AP Physics 1 FRQ



The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0 perpendicular to the rod, and the disk will stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(a) Suppose the rod is much more massive than the disk. To give the rod as much angular speed as possible, should the student make the disk hit the rod to the left of point C, at point C, or to the right of point C?
_____ To the left of C _____ At C _____ To the right of C
Briefly explain your reasoning without manipulating equations.

(b) On the Internet, a student finds the following equation for the post-collision angular speed ω of the rod in this situation: $\omega = \frac{m_{\text{disk}} x v_0}{I}$. Regardless of whether this equation for angular speed is correct, does it agree with your qualitative reasoning in part (a)? In other words, does this equation for ω have the expected dependence as reasoned in part (a)?
_____ Yes _____ No
Briefly explain your reasoning without deriving an equation for ω.

(c) Another student deriving an equation for the post-collision angular speed ω of the rod makes a mistake and comes up with $\omega = \frac{I x v_0}{m_{\text{disk}} d^4}$. Without deriving the correct equation, how can you tell that this equation is not plausible—in other words, that it does not make physical sense? Briefly explain your reasoning.

Solution to part a: We know that angular momentum will be conserved. We want to try to maximize the initial angular momentum of the disk. Since we know the angular momentum of the disk is mvx, we want it to be as far as possible from the pivot.
The distance x from the pivot point will be maximized when when the disk hits the rod to $\boxed{\text{the right of C}}$.
You could also say that to the right of point C, torque applied on the rod will be maximized due to the large distance x from the pivot.

Solution to part b: Yes. This equation matches with our reasoning in part a. The reason is that as x goes up (meaning distance from pivot increases), then the angular velocity goes up. Clearly this matches with our reasoning in part a.

Solution to part c: We know that the larger the mass of the disk is, then the higher the angular momentum will be. The reason is that the formula for angular momentum of a point object is $L = mvr$.

Since the angular momentum goes up as m_{disk} goes up, the angular velocity must also go up.
However, in the given equation for angular velocity, that isn't the case. In fact, m_{disk} is in the denominator which means as the mass increases, the angular velocity goes down. This contradicts the statement we previously made, proving that the equation is not plausible.

Problem 5.0.23 — Continuation of previous 2017 AP Physics FRQ
For parts (d) and (e), do NOT assume that the rod is much more massive than the disk.
(d) Immediately before colliding with the rod, the disk's rotational inertia about the pivot is $m_{\text{disk}}x^2$ and its angular momentum with respect to the pivot is $m_{\text{disk}}v_0x$. Derive an equation for the post-collision angular speed ω of the rod. Express your answer in terms of $d, m_{\text{disk}}, I, x, v_0$, and physical constants, as appropriate.

(e) Consider the collision for which your equation in part (d) was derived, except now suppose the disk bounces backward off the rod instead of sticking to the rod. Is the post-collision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the post-collision angular speed of the rod when the disk sticks to it?
_____ Greater than _____ Less than _____ Equal to
Briefly explain your reasoning.

Solution to part d: In this problem, we will apply conservation of angular momentum which states that $L_i = L_f$

We are already given that $L_i = m_{\text{disk}}v_0x$
Now, we simply find L_f and equate it to our expression above.
After collision, both the disk and rod will rotate with a common angular velocity.

This means that $L_f = (I_{\text{rod}} + I_{\text{disk}})\omega$
In this problem, it is already given that $I_{\text{rod}} = I$ and $I_{\text{disk}} = m_{\text{disk}}x^2$
We can plug that into our expression for L_f to find that $L_f = (I + m_{\text{disk}}x^2)\omega$

Now, we can equate L_i and L_f to set up the expression $m_{\text{disk}}v_0x = (I + m_{\text{disk}}x^2)\omega$

We can divide both sides by $I + m_{\text{disk}}x^2$ to isolate ω.
Doing so gives that $\boxed{\omega = \dfrac{m_{\text{disk}}v_0x}{I + m_{\text{disk}}x^2}}$

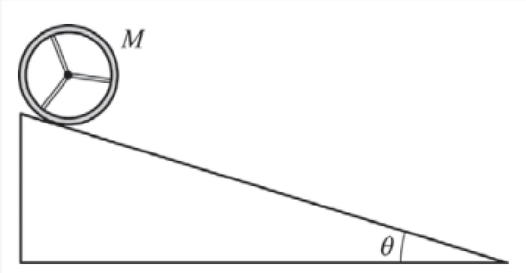
Solution to part e: We will apply the concept of angular momentum being conserved in this problem.
When the disk sticks to the rod, the disk will have an angular momentum in the same direction as the rod. This means that the rod won't receive a high amount of that

momentum since it has to share it with the disk.

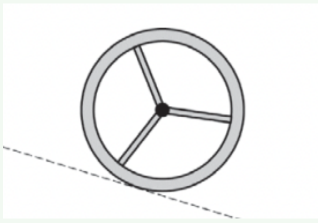
However, when the disk bounces back, it will have an angular momentum in the opposite direction after collision. To counteract this angular momentum, the rod will need to have a significantly greater angular momentum.

Thus, when the disk bounces back, the post-collision angular speed will be $\boxed{\text{greater}}$.

Problem 5.0.24 — 2016 AP Physics 1



A wooden wheel of mass M, consisting of a rim with spokes, rolls down a ramp that makes an angle θ with the horizontal, as shown above. The ramp exerts a force of static friction on the wheel so that the wheel rolls without slipping.
(a)
i. On the diagram below, draw and label the forces (not components) that act on the wheel as it rolls down the ramp, which is indicated by the dashed line. To clearly indicate at which point on the wheel each force is exerted, draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted. The lengths of the arrows need not indicate the relative magnitudes of the forces.



ii. As the wheel rolls down the ramp, which force causes a change in the angular velocity of the wheel with respect to its center of mass.
Briefly explain your reasoning.

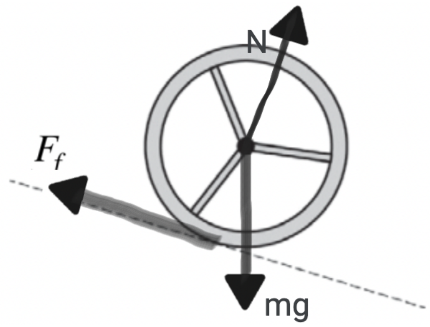
(b) For this ramp angle, the force of friction exerted on the wheel is less than the maximum possible static friction force. Instead, the magnitude of the force of static friction exerted on the wheel is 40 percent of the magnitude of the force or force component directed opposite to the force of friction. Derive an expression for the linear acceleration of the wheel's center of mass of M, theta, and physical constants, as appropriate.

(c) In a second experiment on the same ramp, a block of ice, also with mass M, is released from rest at the same instant the wheel is released from rest, and from the same height. The block slides down the ramp with negligible friction.

i. Which object, if either, reaches the bottom of the ramp with the greatest speed?
_____ Wheel _____ Block _____ Neither; both reach the bottom with the same speed.
Briefly explain your reasoning in terms of forces.

ii. Briefly explain your answer again, now reasoning in terms of energy.

Solution to part a i: The only forces on the wheel are normal force, gravitational force, and frictional force. Note, that the vector lengths of the force arrows don't matter.



Solution to part a ii: Only the friction force can change angular velocity. The reason is that you need torque to change angular velocity. However, the normal force and gravitational force apply no torque. The reason is that the length of the "lever arm" for the two forces is 0 (since the forces pass through the center).

Solution to part b: Along the ramp, there are only two forces. One of them is friction while the other is one component of gravity.
The force of gravity along the ramp is $Mg\sin(\theta)$

We know that the equation for Newton's Second Law along the ramp is

$$F_{net} = Mg\sin(\theta) - F_f = Ma$$

Since the problem says that the force of friction is 40 percent of the force directed opposite to it, we can figure out that $F_f = 0.4Mg\sin(\theta)$
We can plug this into our equation for Newton's Second Law to get

$$Mg\sin(\theta) - 0.4Mg\sin(\theta) = Ma$$

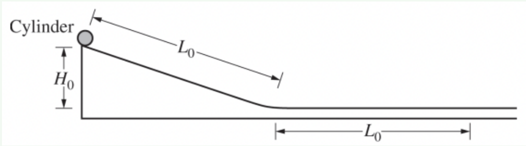
After dividing both sides by M, we find that $\boxed{a = 0.6g\sin(\theta)}$

Solution to c i: The $\boxed{\text{block}}$ reaches the bottom with the greatest speed. The reason is that the wheel has a friction force on it. However, the block doesn't. This causes the net force along the ramp to be larger for the block leading to greater acceleration.

Solution to c ii: We know that both objects have the same potential energy at the top of the ramp (assuming the reference level is the bottom of the ramp)
Once they reach the bottom of the ramp, all the potential energy will turn into kinetic energy.

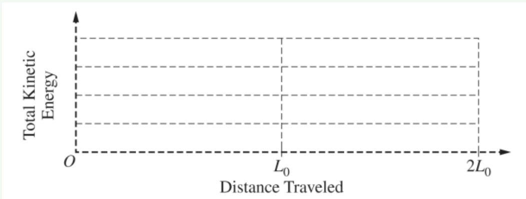
For the wheel, the kinetic energy will be in the forms of translational kinetic energy and rotational kinetic energy. However, for the ice block, there will only be translational kinetic energy.
This means that the ice block will have a greater speed due to all of its kinetic energy being in the form of translational kinetic energy.

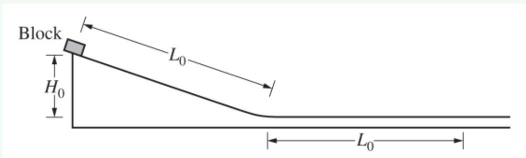
Problem 5.0.25 — 2021 AP Physics 1



A cylinder of mass m_0 is placed at the top of an incline of length L_0 and height H_0, as shown above, and released from rest. The cylinder rolls without slipping down the incline and then continues rolling along a horizontal surface.

(a) On the grid below, sketch a graph that represents the total kinetic energy of the cylinder as a function of the distance traveled by the cylinder as it rolls down the incline and continues to roll across the horizontal surface.



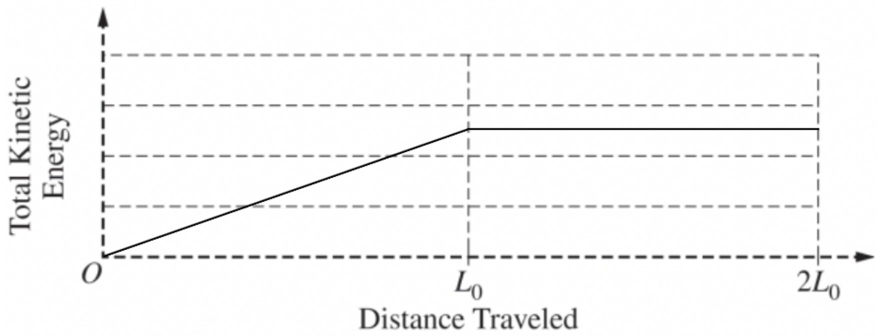


The cylinder is again placed at the top of the incline. A block, also of mass m_0, is placed at the top of a separate rough incline of length L_0 and height H_0, as shown above. When the cylinder and block are released at the same instant, the cylinder begins to roll without slipping while the block begins to accelerate uniformly. The cylinder and the block reach the bottoms of their respective inclines with the same translational speed.

(b) In terms of energy, explain why the two objects reach the bottom of their respective inclines with the same translational speed. Provide your answer in a clear, coherent paragraph-length response that may also contain figures and/or equations.

Solution to part a: For each unit of distance travelled while going down the ramp, the change in potential energy will be the same. This is due to the ramp having a constant slope.
This means that the potential energy decreases linearly. Since the sum of potential energy and kinetic energy are constant, kinetic energy must increase linearly.

When the distance travelled is between 0 and L_0 (meaning the cylinder is on the ramp), the kinetic energy will increase linearly. However, once it reaches the bottom of the ramp, the kinetic energy will remain constant since the cylinder continues to roll in the horizontal section.



Solution to part b: At the top, both the cylinder and block will have the same gravitational potential energy.
Whenever something rolls without slipping, friction doesn't dissipate its energy. The reason is that friction does no work on that object.
Thus, the cylinder's potential energy entirely converts to kinetic energy when it reaches the bottom of the ramp. Some of that kinetic energy will be in the form of translational kinetic energy while the rest will be in the form of rotational kinetic energy.
We can assume that the potential energy at the top of the ramp is U while the translational kinetic energy and rotational kinetic energy at the bottom of the ramp are K_{trans} and K_{rot} respectively.
From conservation of energy, we know that $U = K_{\text{trans}} + K_{\text{rot}}$

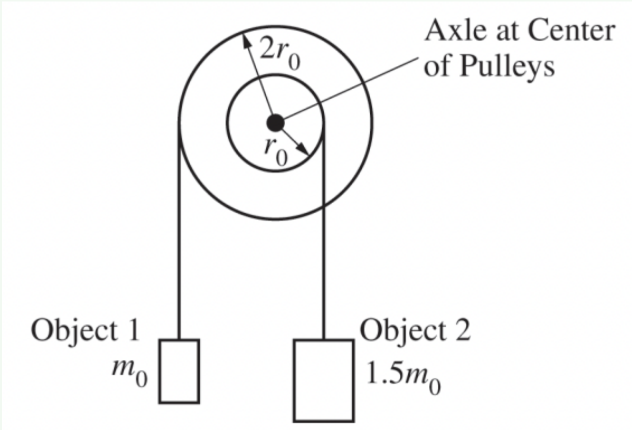
On the other hand, for the block, some energy will indeed be lost as friction. The remaining energy that is left at the bottom of the ramp will be in the form of translational kinetic energy.

Assuming that the energy lost by friction is represented as E_f, then by conservation of mechanical energy we know that $U = E_f + K_{\text{trans}}$

We can write both expressions for K_{trans}. Doing this for the cylinder gives $U - K_{\text{rot}}$ while for the block we get $U - E_f$

We know that both of these expressions equate, and that is only possible when $K_{\text{rot}} = E_f$. This means that the rotational kinetic energy must equate to the energy lost by friction for the block.
On the real AP exam, you don't need to show all the steps above to show that the rotational kinetic energy of the cylinder must equate to the energy lost by friction for the block. I only showed all that work to highlight the intuition behind that idea.

Problem 5.0.26 — 2021 AP Physics 1 FRQ



Two pulleys with different radii are attached to each other so that they rotate together about a horizontal axle through their common center. There is negligible friction in the axle. Object 1 hangs from a light string wrapped around the larger pulley, while object 2 hangs from another light string wrapped around the smaller pulley, as shown in the figure above.

m_0 is the mass of object 1.
$1.5m_0$ is the mass of object 2.
r_0 is the radius of the smaller pulley.
$2r_0$ is the radius of the larger pulley.
(a) At time $t = 0$, the pulleys are released from rest and the objects begin to accelerate.

i. Derive an expression for the magnitude of the net torque exerted on the objects-pulleys system about the axle after the pulleys are released. Express your answer in terms of m_0, r_0, and physical constants, as appropriate.

ii. Object 1 accelerates downward after the pulleys are released. Briefly explain why.

(b) At a later time $t = t_C$, the string of object 1 is cut while the objects are still moving and the pulley is still rotating. Immediately after the string is cut, how do the directions of the angular velocity and angular acceleration of the pulley compare to each other?
☐ Same direction ☐ Opposite directions
Briefly explain your reasoning.

Solution to part a i: We know that $\tau_{net} = \sum F_i \cdot r_i$
This means that we must sum up the product of the forces times distance from pivot.

The forces on the pulley are the two tension forces. We can say that T_1 is the tension force on the left side while T_2 is the tension force on the right side.

$$\tau_{net} = T_1 \cdot 2r_0 - T_2 \cdot r_0$$

Now, we can write equations using Newton's Second Law for each block.

On object 1, the equation is $T_1 - m_0 g = 0$. This means that $T_1 = m_0 g$

Similarly, on object 2, the equation is $T_2 - 1.5m_0 g = 0$. This means that $T_2 = 1.5m_0 g$

We can plug in these values of tension into our equation for τ_{net}.

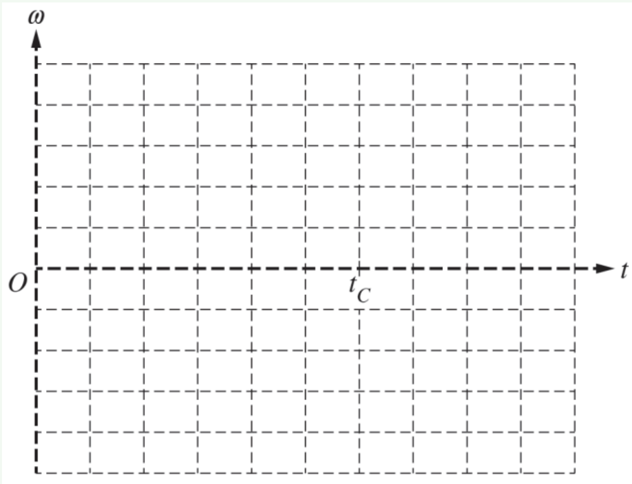
$$\tau_{net} = m_0 g \cdot 2r_0 - 1.5m_0 g \cdot r_0 = \boxed{0.5m_0 r_0 g}$$

Solution to part a ii: Object 1 accelerates downward because it exerts a larger torque on the pulley. It is also double the distance away from the axle compared to object 2 while object 2 only has a mass that is 1.5 times as great as object 1.

Solution to part b: They will move in $\boxed{\text{opposite directions}}$. The reason is that now the torque will switch directions and become clockwise causing angular acceleration to be clockwise. However, the angular velocity will still be counterclockwise temporarily. It won't change directions immediately since it was already spinning in a counterclockwise direction.

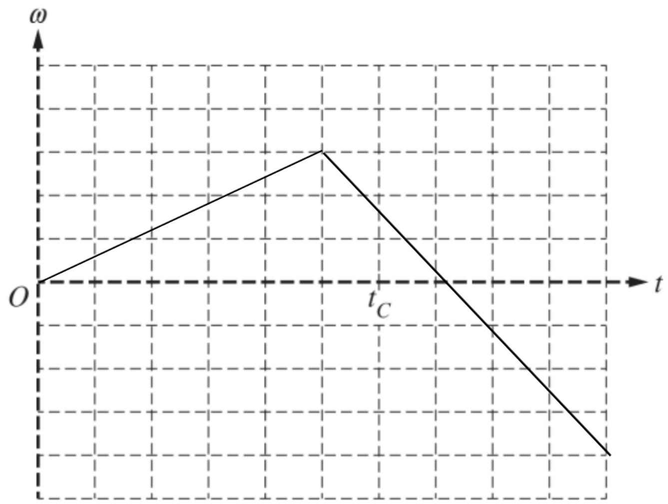
Problem 5.0.27 — Continuation of the FRQ above

(c) On the axes below, sketch a graph of the angular velocity ω of the system consisting of the two pulleys as a function of time t. Include the entire time interval shown. The pulleys are released at $t = 0$, and the string is cut at $t = t_C$.

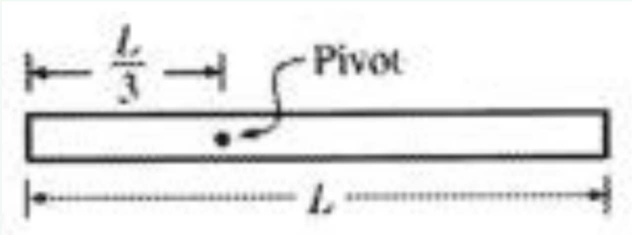


Before the string is cut, we know that the system will accelerate in the counterclockwise direction. This will cause the angular velocity to increase linearly until time t_c.

However, after t_c, we no longer have block of mass m_1 in this system. The angular acceleration is now in the clockwise direction which will cause the angular velocity to fall linearly.


ω
O
t
t_C

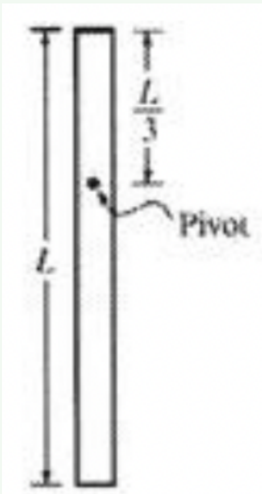
Problem 5.0.28 — 2004 AP Physics FRQ



A uniform rod of mass M and length L is attached to a pivot of negligible friction as shown above. The pivot is located at a distance $L/3$ from the left end of the rod. Express all answers in terms of the given quantities and fundamental constants.

(a) Calculate the rotational inertia of the rod about the pivot.

The rod is then released from rest from the horizontal position shown above.
(b) Calculate the linear speed of the bottom end of the rod when the rod passes through the vertical.



The rod is brought to rest in the vertical position shown above and hangs freely. It is then displaced slightly from this position by a small angle θ. Write the differential equation that governs the motion of the rod as it swings.

(c) Calculate the period of oscillation as it swings, assuming that the angle of oscillation is small.

Solution to part a: The rotational inertia about the pivot can be found by integrating $\int r^2 dm$
Our bounds will be from $\frac{-L}{3}$ to $\frac{2L}{3}$ since that is the length to the left of the pivot and to the right of the pivot respectively.

Due to linear mass density, we know that $dm = \frac{M}{L}dr$
We can plug that into our integral to find that

$$I = \int_{\frac{-L}{3}}^{\frac{2L}{3}} r^2 \cdot \frac{M}{L} dr$$

Integrating gives $I = \frac{ML^2}{9}$

Solution to part b: To find the linear speed at the bottom, we must conserve energy. The initial gravitational potential energy will convert to rotational kinetic energy.

The change in gravitational potential energy can be found using the center of the mass. We must figure out the height by which the center of mass displaces. Clearly, the center of mass only moves down a distance $\frac{L}{6}$

This means that the change in gravitational potential energy is $\frac{MgL}{6}$

All of this will convert to rotational kinetic energy which is $\frac{1}{2}Iw^2$

We can plug in our rotational inertia $I = \frac{ML^2}{9}$ to find that

$$\frac{MgL}{6} = \frac{ML^2w^2}{18}$$

We can solve for w to find that $w = \sqrt{\frac{3g}{L}}$
Since $v = wr$ and our distance r of the bottom from the pivot is $\frac{2L}{3}$, we can find that

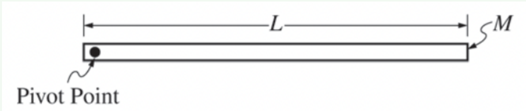
$$v = \sqrt{\frac{3g}{L}} \cdot \frac{2L}{3} = 2\sqrt{\frac{gL}{3}}$$

Solution to part c: This problem require a topic from simple harmonic motion. Please skip it if you haven't done simple harmonic motion. Make sure to come back to it later.

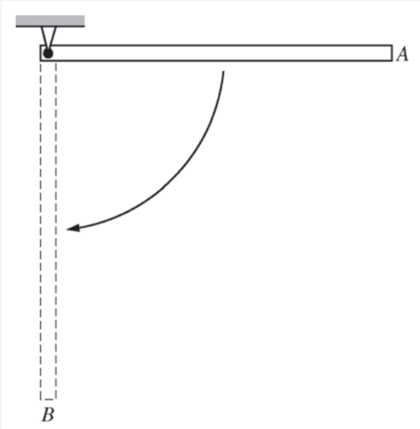
Period can be found using the formula $T = 2\pi\sqrt{\frac{I}{mgd}}$
Since the center of mass is a distance $d = \frac{L}{6}$ from the pivot, we can plug that in along with our expression for rotational inertia.

Doing so gives that $T = 2\pi\sqrt{\frac{2L}{3g}}$

Problem 5.0.29 — 2015 AP Physics C Mechanics FRQ



A uniform, thin rod of length L and mass M is allowed to pivot about its end, as shown in the figure above.
(a) Using integral calculus, derive the rotational inertia for the rod around its end to show that it is $ML^2/3$.



The rod is fixed at one end and allowed to fall from the horizontal position A through the vertical position B.
(b) Derive an expression for the velocity of the free end of the rod at position B. Express your answer in terms of M, L, and physical constants, as appropriate.

Solution to part a: We know that rotational inertia is $\int r^2 dm$
In this problem, our linear mass density is $\lambda = \frac{M}{L}$.
For an infinitesimally small part of the rod (with a length of dr), the mass will be dm. We can relate the length to the mass using our linear mass density. We know that $dm = \lambda dr$

We can plug that into our integral for rotational inertia which will give us $I = \int \lambda r^2 dr$

We also need to integrate from 0 to L (since all the points are on the same side of the pivot).

$$I = \int_0^L \lambda r^2 dr$$

$$\text{Integrating gives that } I = \frac{\lambda r^3}{3}\Big|_0^L = \frac{\lambda L^3}{3}$$

We can plug in $\lambda = \frac{M}{L}$ to get that $I = \frac{ML^2}{3}$

Solution to part b: We must conserve energy to first find the angular velocity of the rod as it swings.

We must first find the gravitational potential energy that is lost when the rod swings down. We observe how much the center of mass goes down by.

Clearly, the center of mass only moves down by half the length of the rod which is $\frac{L}{2}$. This means that the rod loses $\frac{MgL}{2}$ joules of potential energy when it swings down. This energy becomes rotational kinetic energy.

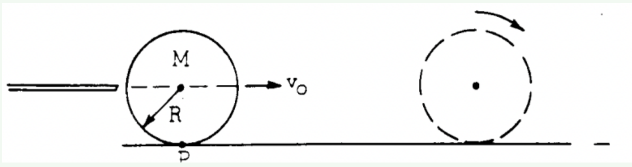
Assuming that the angular velocity is w, we can write the rotational kinetic energy as $\frac{1}{2}Iw^2$ which is

$$\frac{1}{2} \cdot \frac{ML^2}{3} \cdot w^2 = \frac{ML^2w^2}{6}$$

This means that $\frac{MgL}{2} = \frac{ML^2w^2}{6}$
We can isolate w to find that $w = \sqrt{\frac{3g}{L}}$

Since $v = wr$, we can find the velocity of the bottom end of the rod using this. Since the bottom end is a distance L from the pivot, the velocity v is $\sqrt{\frac{3g}{L}} \cdot L$ which is $\sqrt{3gL}$

Problem 5.0.30 — 1980 AP Physics FRQ



A billiard ball has mass M, radius R, and moment of inertia about the center of mass $I_c == \frac{2}{5}MR^2$. The ball is struck by a cue stick along a horizontal line through the ball's center of mass so that the ball initially slides with a velocity v_0 as shown above. As the ball moves across the rough billiard table (coefficient of sliding friction μ_k), its motion gradually changes from pure translation through rolling with slipping to rolling without slipping.

(a) Develop an expression for the linear velocity v of the center of the ball as a function of time while it is rolling with slipping.

(b) Develop an expression for the angular velocity ω of the ball as a function of time while it is rolling with slipping.

(c) Determine the time at which the ball begins to roll without slipping.

(d) When the ball is struck, it acquires an angular momentum about the fixed point P on the surface of the table. During the subsequent motion, the angular momentum about point P remains constant despite the frictional force. Explain why this is so.

Solution to part a: The linear velocity will change overtime due to the frictional force. The friction force opposes the ball's linear velocity.
the friction force is $-\mu N$. We also know that $N = Mg$ which means that the friction force can be simplified to $-\mu Mg$. We can divide this by M (the mass) to find the acceleration a to be $-\mu g$

Since the initial velocity is v_o and the acceleration is $-\mu g$, an expression for linear velocity after time t is $v = v_o - \mu gt$

Solution to part b: To find an expression for angular velocity, we will use Newton's Second Law for rotation. We must first find the net torque.

The torque by friction force is μMgR since μMg is the frictional force while R is the distance of the force from the pivot. Now we can use the equation $\tau_{net} = I\alpha$
Since we know τ_{net} and I, we can plug in those values.

$$\mu MgR = \frac{2MR^2}{5}\alpha$$

Isolating α gives that $\alpha = \frac{5\mu g}{2R}$

We can now use the equation $w = w_o + \alpha t$ to find an expression for angular velocity.
Since the initial angular velocity is 0, we can find that $w = \frac{5\mu gt}{2R}$

Solution to part c: Rolling without slipping occurs as soon as $wr = v$
We know that $w = \frac{5\mu gt}{2R}$. We also know that $v = v_o - \mu gt$.

We can plug in both expressions into $wR = v$ (we use R instead of r since that's the variable that denotes our radius) to find that

$$\frac{5\mu gt}{2R} \cdot R = v_o - \mu gt$$

Simplifying the equation gives $\frac{7\mu gt}{2} = v_o$

We can isolate t to find that

$$t = \frac{2v_o}{7\mu g}$$

Solution to part d: Although friction force occurs at point P, it has 0 torque relative to point P. Since it applies no torque relative to that point (due to the force passing through the pivot itself), the angular momentum relative to point P is constant.

Problem 5.0.31 — 2002 AP Physics C: Mechanics FRQ

The cart shown above is made of a block of mass m and four solid rubber tires each of mass $m/4$ and radius r. Each tire may be considered to be a disk. (A disk has rotational inertia $\frac{1}{2}ML^2$, where M is the mass and L is the radius of the disk.) The cart is released from rest and rolls without slipping from the top of an inclined plane of height h. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the total rotational inertia of all four tires.

(b) Determine the speed of the cart when it reaches the bottom of the incline.

(c) After rolling down the incline and across the horizontal surface, the cart collides with a bumper of negligible mass attached to an ideal spring, which has a spring constant k. Determine the distance x_m the spring is compressed before the cart and bumper come to rest.

(d) Now assume that the bumper has a non-negligible mass. After the collision with the bumper, the spring is compressed to a maximum distance of about 90

Solution to part a: The total rotational inertia for all 4 tires can be add by adding up each individual rotational inertia.
The rotational inertia for each tire can be represented as $I = \frac{1}{2}ML^2$. The mass of each tire is $\frac{m}{4}$. We must plug that in to find that the rotational inertia I for each tire is $\frac{mr^2}{8}$. We multiply this by 4 to account for all tires. The rotational inertia for all tires is

$$\frac{mr^2}{2}$$

Solution to part b: We can find the speed of the cart at the bottom by conserving energy, We know that the initial gravitational potential energy will convert to kinetic energy.
Just remember that there will be both rotational and translational kinetic energy. The rotational kinetic energy will come from the tires while the translational kinetic energy will come from the entire car (block and all tires).

$$U_i = K_{rot} + K_t$$

We know that the rotational kinetic energy can be represented as $\frac{1}{2}Iw^2$ while the translational kinetic energy can be represented as $\frac{1}{2}mv^2$
We know that the combined rotational inertia is $\frac{mr^2}{2}$. This means that the rotational kinetic energy is $\frac{mr^2w^2}{4}$.

We also know that the total mass of the entire car is $2m$. The reason is that we must add the mass of the block and all 4 tires.

This means that the final translational kinetic energy is $\frac{1}{2} \cdot 2m \cdot v^2 = mv^2$

In addition to this, the initial gravitational potential energy can be found using the formula mgh. We must use $2m$ since that's the total mass of the entire cart. This means that the gravitational potential energy is $2mgh$.

We can plug all of this into $U_i = K_{rot} + K_t$ to find that

$$2mgh = \frac{mr^2w^2}{4} + mv^2$$

However, there is one thing that is still missing. We will need to use the relation $v = wr$. Instead of r^2w^2 in $\frac{mr^2w^2}{4}$, we can substitute $v^2 = r^2w^2$ (after squaring out relation) to get $\frac{mv^2}{4}$.

$$\text{This means that } 2mgh = \frac{mv^2}{4} + mv^2 = \frac{5mv^2}{4}$$

$$\text{We can solve for } v \text{ to find that } v = \sqrt{\frac{8gh}{5}}$$

Solution to part c: The initial gravitational potential energy will all convert to spring potential energy.

We know that the initial gravitational potential energy is $2mgh$

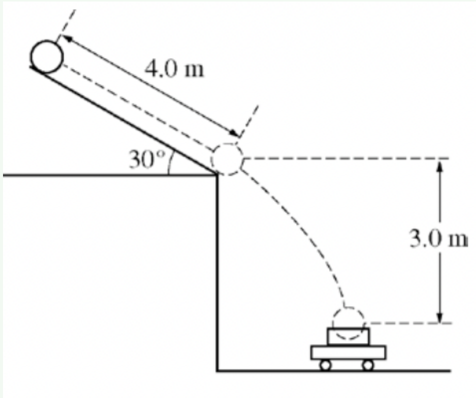
The spring potential energy can be represented as $\frac{1}{2}kx_m^2$
We can solve for x_m by setting both energies equal to each other.

$$x_m = 2\sqrt{\frac{mgh}{k}}$$

Solution to part d: The cart and the bumper (attached to the spring) will collide inelastically. Previously, the cart would continue to move with the same velocity despite colliding with the bumper. Kinetic energy before colliding with the bumper and after would be constant. However, now since there's an inelastic collision, kinetic energy will not be conserved!

Since kinetic energy will now decrease due to the inelastic collision, there will be a little less spring potential energy after all the kinetic energy is converted. The lower spring potential energy will cause the extension to be a little less.

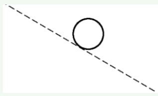
Problem 5.0.32 — 2010 AP Physics C: Mechanics FRQ



Note: Figure not drawn to scale.

A bowling ball of mass 6.0 kg is released from rest from the top of a slanted roof that is 4.0 m long and angled at 30°, as shown above. The ball rolls along the roof without slipping. The rotational inertia of a sphere of mass M and radius R about its center of mass is $\frac{2}{5}MR^2$.

(a) On the figure below, draw and label the forces (not components) acting on the ball at their points of application as it rolls along the roof.

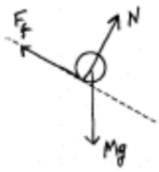


(b) Calculate the force due to friction acting on the ball as it rolls along the roof. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

(c) Calculate the linear speed of the center of mass of the ball when it reaches the bottom edge of the roof.

(d) A wagon containing a box is at rest on the ground below the roof so that the ball falls a vertical distance of 3.0 m and lands and sticks in the center of the box. The total mass of the wagon and the box is 12 kg. Calculate the horizontal speed of the wagon immediately after the ball lands in it.

Solution to part a: The only forces will be the normal force, frictional force, and gravitational force. Friction force points along the plane, gravitational force points straight down, and normal force points perpendicular to the surface.



Solution to part b: Since the ball rolls without slipping, we know that $a = \alpha R$
We must first find the acceleration and angular acceleration due to them being related by the radius.

We must first find the net torque to be able to find α (angular acceleration). The net torque only comes from the frictional force since it's the only force applying a torque relative to the center. The torque that it applies is $F_f R$. Since $\tau_{net} = I\alpha$, we can rewrite it as $\alpha = \frac{\tau_{net}}{I}$. We plug in the net torque and rotational inertia $I = \frac{2MR^2}{5}$ to find that $\alpha = \frac{5F_f}{2MR}$

Now, we must find the net force. The net force only comes from the component of gravitational force along the ramp and the frictional force. The net force is $Mg\sin(\theta) - F_f$. We use $F = ma$ to find that acceleration is $g\sin(\theta) - \frac{F_f}{M}$

Now, we can plug in both expressions into $a = \alpha R$.

$$g\sin(\theta) - \frac{F_f}{M} = \frac{5F_f}{2MR} \cdot R = \frac{5F_f}{2M}$$

We can bring F_f (force of friction) to one side to find that $\frac{7F_f}{2M} = g\sin(\theta)$.
Isolating F_f gives that $F_f = \frac{2Mg\sin(\theta)}{7}$

Solution to part c: We must conserve energy to find the linear speed of the ball at the edge of the roof. Even though there is a friction force, no energy is lost by friction since the ball rolls without slipping.

We know that the initial gravitational potential energy will convert to rotational and translational kinetic energy,

The gravitational potential energy can be written as Mgh
The rotational kinetic energy can be written as $\frac{1}{2}Iw^2$ while translational kinetic energy is $\frac{1}{2}Mv^2$

$$Mgh = \frac{1}{2}Iw^2 + \frac{1}{2}Mv^2$$

Now, we must plug in $I = \frac{2MR^2}{5}$ and $w = v/R$ to get that

$$Mgh = \frac{7Mv^2}{10}$$

We can cancel out M to get $gh = \frac{7v^2}{10}$

Now, we must find the value of h (the height above the bottom edge of the roof). Clearly, $h = 4\sin(30) = 2$

We can plug this in to get that $9.8 \cdot 2 = \frac{7v^2}{10}$
We can isolate v to find that $v = 5.3$ m/s.

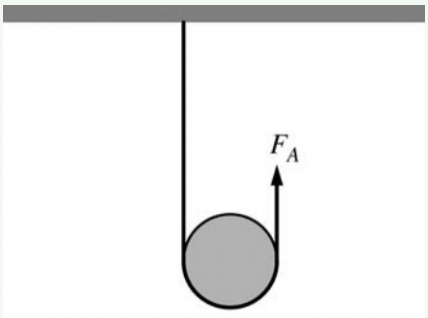
Solution to part d: This problem involves conserving momentum. We must conserve momentum in the x-direction. The wagon's horizontal speed is caused by the box's horizontal velocity. In part c, we already found the box's velocity to be 5.3 at the end of the roof. We must use this velocity to find the horizontal component. By using some basic trigonometry, the horizontal velocity is $5.3\cos(30)$

The initial momentum p_i is $mv = 6(5.3\cos 30)$ (for the bowling ball)

Since we have an inelastic collision, both objects will move together after colliding. The combined mass of the bowing ball, wagon, and box is 18. Assuming that the final velocity for the combined system is v_f after collision, we know that the final momentum p_f is $18v_f$

We can equate p_i and p_f to find that $v_f = 1.5$ m/s.

Problem 5.0.33 — 2013 AP Physics C Mechanics FRQ



(a) Calculate the magnitude of the force F_A necessary to hold the disk at rest.

At time $t = 0$, the force F_A is increased to 12 N, causing the disk to accelerate upward. The rope does not slip on the disk as the disk rotates.
(b) Calculate the linear acceleration of the disk.

(c) Calculate the angular speed of the disk at $t = 3.0$ s.

(d) Calculate the increase in total mechanical energy of the disk from $t = 0$ to $t = 3.0$ s.

(e) The disk is replaced by a hoop of the same mass and radius. Indicate whether the linear acceleration of the hoop is greater than, less than, or the same as the linear acceleration of the disk.

Solution to part a: Many people will say that the net force $F_{net} = F_A - Mg = 0$. Although the net force is indeed 0, our expression for net force is $F_{net} = 2F_A - Mg$ The reason is that the rope "wraps" around the disk so the tension force is applied from both sides of the disk.
Since $2F_A - Mg = 0$, we know that $F_A = \frac{Mg}{2}$

Solution to part b: We will again use Newton's Second Law.
We know that $F_{net} = Ma$
The net force $F_{net} = F_A + T - Mg$. The applied force and the tension force will be pointing upwards. The tension force will come from the left side of the rope while the applied force is on the right side.

We know that $F_A + T - Mg = Ma$

Now, we write an equation using Newton's Second Law for Rotation.

We must find the net torque τ_{net}. The pivot that we will use will be the disk's center. Clearly, gravitational force applies no torque since the force goes through the pivot. The net torque $\tau_{net} = F_A R - TR = I\alpha$

We can substitute $I = \frac{MR^2}{2}$ to get that $F_A - T = \frac{MR\alpha}{2}$

Since the rope does not slip, we know that $a = \alpha \cdot R$. This allows us to relate our acceleration to angular acceleration.

We can plug in $\alpha = \frac{a}{R}$ into $F_A - T = \frac{MR\alpha}{2}$. Doing so gives $F_A - T = \frac{Ma}{2}$

We can add this to our other equation which is $F_A + T - Mg = Ma$.

$$\text{Doing so gives } 2F_A - Mg = \frac{3Ma}{2}$$

We can plug in $F_A = 12$ and $M = 2$ to get that acceleration a is 1.47

Solution to part c: Since we know acceleration a, we can find the angular acceleration using the equation $a = \alpha R$
We can manipulate the equation to find that $\alpha = \frac{a}{R} = \frac{1.47}{0.1} = 14.7 \text{ rad/s}^2$

Now, we can use the equation $w = w_i + \alpha t$
The initial angular velocity is 0. This means that the final angular velocity is

$$w = 14.7 \cdot 3 = 44.1 \text{ rad/s}$$

Solution to part d: The increase in total mechanical energy can be found by summing the change in potential and kinetic energy.

$$\Delta ME = \Delta K + \Delta U$$

The change in kinetic energy must incorporate the change in kinetic and rotational kinetic energy. Both of the quantities are initially 0. However, later there's both types of kinetic energies.

$$\Delta K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

The gravitational potential energy will change since the pulley goes up. Since the pulley has a linear acceleration of 1.47, we can use that to find the height it gains.

$$\Delta y = v_0 t + \frac{1}{2}at^2$$

The above equation can be used to find how much it goes up by. Since the initial velocity is 0, we know that

$$\Delta y = \frac{1}{2}at^2$$

The change in potential energy can be represented as

$$\Delta U = Mg\Delta y = \frac{1}{2}Mgat^2$$

This means the total change in mechanical energy is

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}Mgat^2$$

We plug in $v = \omega R$ and $I = \frac{1}{2}MR^2$ to get

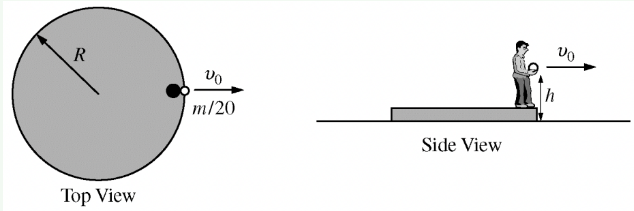
$$\frac{1}{2}M\omega^2 R^2 + \frac{1}{4}MR^2\omega^2 + \frac{1}{2}Mgat^2 = \frac{3}{4}MR^2\omega^2 + \frac{1}{2}Mgat^2$$

We can plug in the values to get $\frac{3}{4} \cdot 2 \cdot 0.1^2 \cdot 44.1^2 + \frac{1}{2} \cdot 2 \cdot 9.8 \cdot 1.47 \cdot 3^2 = 158.8$ J

Solution to part e: Although both the disk and hoop have the same mass and radius, the rotational inertia will differ.

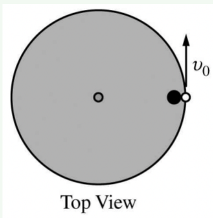
The hoop will have all of its mass concentrated in the rim. Thus, its rotational inertia will be greater. The higher rotational inertia will cause the angular acceleration to decrease due to the relationship $\tau = I\alpha$. A lower angular acceleration will correspond to a lower linear acceleration.

Problem 5.0.34 — 2014 AP Physics C Mechanics FRQ



A large circular disk of mass m and radius R is initially stationary on a horizontal icy surface. A person of mass $\frac{m}{2}$ stands on the edge of the disk. Without slipping on the disk, the person throws a large stone of mass $\frac{m}{20}$ horizontally at initial speed v_0 from a height h above the ice in a radial direction, as shown in the figures above. The coefficient of friction between the disk and the ice is μ. All velocities are measured relative to the ground. The time it takes to throw the stone is negligible. Express all algebraic answers in terms of m, R, v_0, h, μ, and fundamental constants, as appropriate.

(a) Derive an expression for the length of time it will take the stone to strike the ice.
(b) Assuming that the disk is free to slide on the ice, derive an expression for the speed of the disk and person immediately after the stone is thrown.
(c) Derive an expression for the time it will take the disk to stop sliding.



(d) Derive an expression for the angular speed ω of the disk immediately after the stone is thrown.

(e) The person now stands on the disk at rest $\frac{R}{2}$ from the center of the disk. The person now throws the stone horizontally with a speed v_0 in the same direction as in Part D. Is the angular speed of the disk immediately after throwing the stone from this new position greater than, less than, or equal to the angular speed found in Part D? Justify your answer.

Solution to part a: Instead of observing the stone's motion in the x-direction, we need to consider the motion in the y-direction.
In the y-direction, the initial velocity of the stone is 0. The acceleration is g and the vertical displacement is h.

We can use the kinematics equation $\Delta y = v_i t + \frac{1}{2}at^2$ to model this.
Using it for this problem gives $h = 0 + \frac{gt^2}{2}$
Solving for t gives that $t = \sqrt{\frac{2h}{g}}$

Solution to part b: Initially, everything is at rest. Thus, the initial momentum of the disk + person and the stone is 0.

Using this, we can apply conservation of momentum to the disk, person, and stone system in this problem.

Let's assume that that the final speed of the disk and person is v_f. This means that their combined final momentum is $(m + \frac{m}{2})v_f$ (we must account for both masses which is why we need to add them both)

$$\text{The stone's final momentum is } \frac{mv_0}{20}$$

$$\text{From conservation of momentum, } 0 = \frac{mv_0}{20} + (m + \frac{m}{2})v_f$$

We can simplify to find that $v_f = -\frac{v_0}{30}$ which is the speed of the disk and person after the stone is thrown.

Solution to part c: The disk will eventually stop moving due to friction. We know that the friction force can be represented as $F_f = -\mu N$

In this case, the normal force will occur due to the disk and person. This means we must account for the person's mass also. That means the normal force is $(m+\frac{m}{2})g$ which is $\frac{3mg}{2}$

This means that the friction force on the disk and person system is $-\frac{3\mu mg}{2}$. Now, we must find the acceleration that this friction force causes. Since the combined mass of the disk and person is $\frac{3mg}{2}$, we can use Newton's Second Law to find that the acceleration is $-\mu g$.

Now, we must write an expression for velocity due to the acceleration. We will use the kinematics equation $v_f = v_0 + at$
We want the final velocity to be 0, and we know that $a = -\mu g$. We also know that the disk and person's initial speed (after collision) is $-\frac{v_0}{30}$.

Just remember to be consistent with signs. Since the velocity is in the negative direction, our friction force and the acceleration must be in the positive direction (to opposite the initial velocity). This means that the acceleration in our equation should be written as μg.

We can plug this in to write the equation: $0 = -\frac{v_0}{30} + \mu gt$

$$\text{We can solve for } t \text{ to find that } t = \frac{v_0}{30\mu g}$$

Solution to part d: This problem speaks the idea of conservation of angular momentum. Initially, the angular momentum of the person + disk and stone is 0.

However, after the stone is thrown, it's angular momentum can be found using the formula $L = mvr$ (since it can be treated like a point mass). Since the mass is $\frac{m}{20}$, velocity is v_0, and radius is R, the angular momentum is $\frac{mv_0R}{20}$

The final angular momentum of the disk must be represented as $L = Iw$. It is not a point mass which is why we can't use $L = mvr$.

We are already given that the rotational inertia of the disk is $\frac{mR^2}{2}$. On top of that, since the person stands on the circumference of the disk, we must account for the rotational inertia of the person. Since they stand a distance of R (the radius) from the pivot (the center), their rotational inertia can be found using the mR^2. Since the person's mass is $\frac{m}{2}$, their rotational inertia is $\frac{mR^2}{2}$.

The combined rotational inertia of the disk and person can be found by adding up their individual inertias. That means their combined rotational inertia is

$$\frac{mR^2}{2} + \frac{mR^2}{2} = mR^2$$

Now, we can use $L = Iw$. The final angular momentum of the disk and person is mR^2w.

$$\text{Using conservation of angular momentum, } \frac{mv_0R}{20} + mR^2w = 0$$

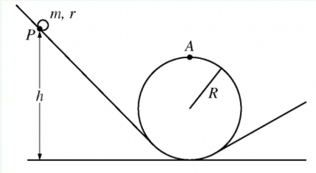
$$\text{We can isolate } w \text{ to find that } w = -\frac{v_0}{20R}$$

Solution to part e: If the person stands a distance of $\frac{R}{2}$ from the center rather than R, then the person's rotational inertia will go down. A smaller distance from the center will lead to a lower rotational inertia. This means that the combined rotational inertia of the disk and person will also go down.

On top of that, the stone's angular momentum will decrease. We found the stone's angular momentum through the formula $L = mvr$. Although the mass and velocity are still the same, the distance from the pivot decreases; this causes the angular momentum to decrease.

We can account for all of this in our calculations from part d.
To guarantee the point for this problem, you should write the equation from part d again by using $\frac{R}{2}$ as the distance from center rather than R. That part will be left to you as an exercise.

Problem 5.0.35 — 2019 AP Physics C Mechanics FRQ



Note: Figure not drawn to scale.

The rotational inertia of a rolling object may be written in terms of its mass m and radius r as $I = bmr^2$, where b is a numerical value based on the distribution of mass within the rolling object. Students wish to conduct an experiment to determine the value of b for a partially hollowed sphere. The students use a looped track of radius $R >> r$, as shown in the figure above. The sphere is released from rest a height h above the floor and rolls around the loop.

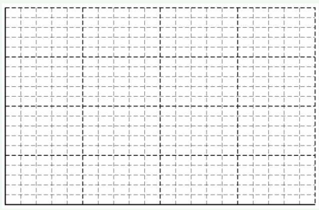
(a) Derive an expression for the minimum speed of the sphere's center of mass that will allow the sphere to just pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

(b) Suppose the sphere is released from rest at some point P and rolls without slipping. Derive an equation for the minimum release height h that will allow the sphere to pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

The students perform an experiment by determining the minimum release height h for various other objects of radius r and known values of b. They collect the following data.

Object	b	h (m)
Solid sphere	0.40	1.08
Hollow sphere	0.67	1.13
Solid cylinder	0.50	1.10
Hollow cylinder	1.0	1.20

(c) On the grid below, plot the release height h as a function of b. Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



(d) The students repeat the experiment with the partially hollowed sphere and determine the minimum release height to be 1.16 m. Using the straight line from part (c), determine the value of b for the partially hollowed sphere.

(e) Calculate R, the radius of the loop.

(f) In part (b), the radius r of the rolling sphere was assumed to be much smaller than the radius R of the loop. If the radius r of the rolling sphere was not negligible, would the value of the minimum release height h be greater, less, or the same? Justify your answer.

Solution to part a: We will first figure out the forces on the object at point A. We know that gravitational and normal force will both point downwards. In addition, the object will be undergoing centripetal motion.

Thus, $F_{net} = N + mg = \frac{mv^2}{R}$.
We know that the normal force must be 0 to find the minimum possible speed.

Plugging that in gives $mg = \frac{mv^2}{R}$. We can simplify this to find that $v = \sqrt{gR}$

Solution to part b: At the object's initial point, it is at rest and only has gravitational potential energy.
That energy will convert to kinetic energy, both translational and rotational kinetic energy.

We can write the equation $\Delta U = K_{tot} = K_{rot} + K_{trans}$
We know that $\Delta U = mg\Delta h = mg(h - 2r)$ (since the object goes down a distance $h - 2r$).

$$\text{Thus, we know that } mg(h-2r) = \frac{1}{2}Iw^2 + \frac{1}{2}mv^2$$

Since the object rolls without slipping, we know that $v = wr$. This means that $w = \frac{v}{r}$. We can plug this into our equation.

$$mg(h-2r) = \frac{1}{2}I(\frac{v}{r})^2 + \frac{1}{2}mv^2$$

Now, we can plug in $I = bmr^2$ and simplify.

$$\text{Doing so gives } mg(h-2r) = \frac{bmv^2}{2} + \frac{mv^2}{2}$$

We can cancel m from both sides to get $g(h-2r) = \frac{bv^2}{2} + \frac{v^2}{2} = \frac{v^2}{2}(b+1)$

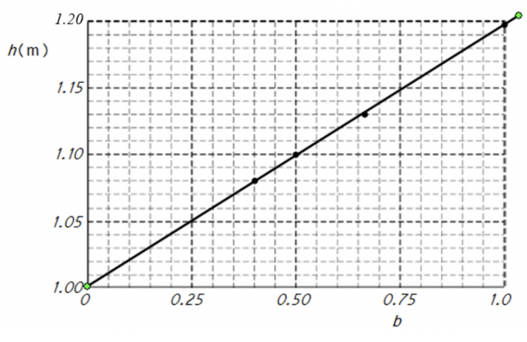
Now, we can plug in the minimum possible value of velocity we found that would occur at point A. We know that it is $\sqrt{gR}$.

$$\text{Plugging this in gives that } g(h-2r) = \frac{bgR}{2} + \frac{gR}{2}$$

Now, we can divide both sides by g to get $h - 2R = \frac{bR}{2} + \frac{R}{2}$
Adding $2R$ to both sides gives that

$$h = \frac{bR}{2} + \frac{5R}{2}$$

Solution to part c: We plot the points in the table.



Credits: The graph is from the College Board website The graph can be a little different for everyone. Try to make your graph so that it takes up a large part of the grid. Don't try to fit the graph into a small place. Use the entire grid!

Solution to part d: We should find an equation to relate h and b.
We can use two of the points from our table to approximate the slope.

$$\text{Slope } \approx \frac{1.2 - 1.08}{1.0 - 0.4} = 0.2$$

The equation of a line in general is $y = mx + b$ where m represents the slope.
Also, we plotted h on the y-axis and b on the x-axis. Thus, we should use those variables in our equation. b will take the place of x and h will take the place of y.
We can also find our y-intercept from using our points. Clearly, the intercept is 1.

We can find that our equation is $h = 0.2b + 1$

Now, we can plug in $h = 1.16$ to find the value of b.

Doing so gives $1.16 = 0.2b + 1$. We can simplify this to find that $b = 0.8$

Solution to part e: In part b, we found that $h = \frac{bR}{2} + \frac{5R}{2}$.
In part d, we found that $b = 0.8$

We can plug in one of the points (b, h) to find the radius R.

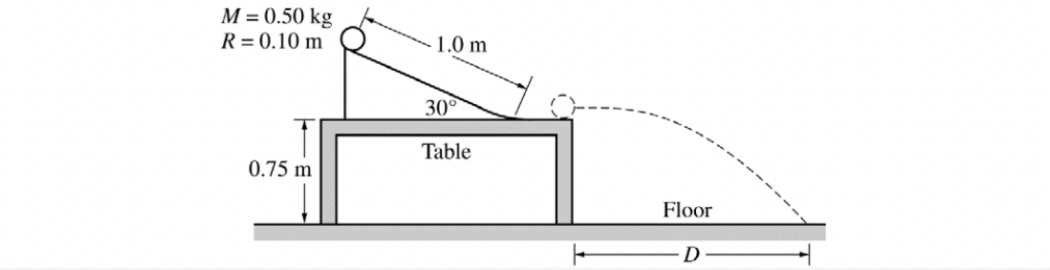
We will plug in the point $b = 1.0$ and $h = 1.20$

Doing so gives that $1.2 = \frac{0.8R}{2} + \frac{5R}{2} = 2.9R$
We can divide both sides by 2.9 to find that $R = \frac{1.2}{2.9} = 0.414$ m.

Solution to part f: If the radius of the object is not negligible, then the change in gravitational potential energy would decrease compared to before. The reason is that the center of mass would move a smaller distance downwards. Since the change in gravitational potential energy would decrease, there would be less kinetic energy. The object would need a lower speed to pass the loop.

Thus, the object can be released from a lower height h and still have the right amount of energy needed to make it through the loop. The answer is $\boxed{\text{less}}$.

Problem 5.0.36 — 2017 AP Physics C Mechanics FRQ



A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m. The inclined plane makes an angle of 30° with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition from the inclined plane to the horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $\frac{MR^2}{2}$

(a) Calculate the total kinetic energy of the cylinder as it reaches the horizontal table.

(b) Calculate the angular velocity of the cylinder around its axis at the moment it reaches the floor.

(c) Calculate the ratio of the rotational kinetic energy to the total kinetic energy for the cylinder at the moment it reaches the floor.

(d) Calculate the horizontal distance D.

A sphere of the same mass and radius is now rolled down the same inclined plane. The rotational inertia of a sphere around its center is $\frac{2MR^2}{5}$

(e)

i. Is the total kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the total kinetic energy of the cylinder at the moment it reaches the floor?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

ii. Is the rotational kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the rotational kinetic energy of the cylinder at the moment it reaches the floor?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

iii. Is the horizontal distance the sphere travels from the table to where it hits the floor greater than, less than, or equal to the horizontal distance the cylinder travels from the table to where it hits the floor?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Solution to part a: All of the gravitational potential energy at the top of the inclined plane will convert to kinetic energy. Relative to the top of the table, the cylinder is $1 \cdot \sin(30) = \frac{1}{2}$ meters above the top. This means that we can use $h = \frac{1}{2}$ to find the gravitational potential through the formula mgh.

The gravitational potential energy at the top is $Mgh = 0.5 \cdot 9.8 \cdot \frac{1}{2} = 2.45$ J.
All of the gravitational potential energy converts to kinetic energy. That means that the kinetic energy is 2.45 J.

Solution to part b: The angular velocity on the table and on the floor will remain the same. The reason is that once it leaves the table, there is no force that applies torque. Thus, the angular acceleration is 0 causing the angular velocity to remain constant. That means we can just find the angular velocity at the moment it reaches the table.

We know that kinetic energy on the table is $K = \frac{1}{2}Mv^2 + \frac{1}{2}Iw^2$ since we must account for both translational kinetic energy and rotational kinetic energy.

Since it rolls without slipping, we know that $v = wr$. We can plug that in to simplify our expression. For this problem, we should be consistent with the fact that the radius is R, not r (since in the diagram it uses uppercase R). That means we use $v = wR$.
We get $K = \frac{1}{2}Mw^2R^2 + \frac{1}{2}Iw^2$
Now, we can also plug in our rotational inertia which is $\frac{MR^2}{2}$ to get

$$K = \frac{1}{2}Mw^2R^2 + \frac{1}{4}Mw^2 = \frac{3Mw^2R^2}{4}$$

We also already know that the total kinetic energy is 2.45
This means that $2.45 = \frac{3Mw^2R^2}{4}$
We can plug in our known values and simplify to find that $w = 25.56$ rad/s.

Solution to part c: The total kinetic energy on the floor comes from the potential energy. This time, the cylinder doesn't fall a distance of $1 \cdot \sin(30)$. We must also account for the height of the table. This means that it falls $0.75 + 1 \cdot \sin(30) = 1.25$ m. Thus, the gravitational potential energy at the top is $Mgh = 0.5 \cdot 9.8 \cdot 1.25 = 6.125$ J. By the time it reaches the floor, all of this potential energy will convert to kinetic energy.

Thus, the total kinetic energy is 6.125 J. The rotational kinetic energy can be found by using the angular velocity we found in part b.
$K_{rot} = \frac{1}{2}Iw^2 = \frac{MR^2w^2}{4}$. We can plug in our values to find that $K_{rot} = 0.82$ J.

$$\text{This means that } \frac{K_{rot}}{K_{tot}} = \frac{0.82}{6.125} = 0.134$$

Solution to part d: The horizontal distance d can be found using the cylinder's velocity at the end of the table. We know that the velocity at the end of the table can be found using $v = wR$. We already found that the angular velocity is 25.56 and we know that the radius is 0.1

$$\text{This means that } v_x = 2.556 \text{ m/s}$$

Now, we need to find the time it takes for this motion to occur. We apply kinematics in the y-direction.
We know that the acceleration in the y-direction will be g. The vertical displacement is $\Delta y = 0.75$ while the initial vertical velocity is 0.

We can use the kinematics equation $\Delta y = v_{0y}t + \frac{1}{2}at^2$. Plugging in our variables gives $0.75 = 0 + \frac{gt^2}{2}$

We can solve for t to find that $t = 0.39$ s.

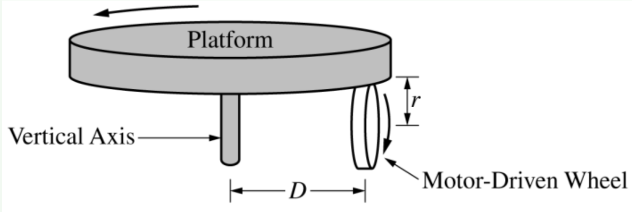
Since we know v_x and t, we can simply use the formula $D = v_x t$ (since there is no acceleration in the x-direction). This means that the distance D is $2.556 \cdot 0.39$ which is around 1 m.

Solution to part e i: The total kinetic energy will be the same when it reaches the floor. The reason is that the gravitational potential energy at the top of the inclined plane will still remain the same. The reason is that the cylinder still has the same mass and is the same distance above the floor. Thus, $U = mgh$ will remain constant causing the total kinetic energy to remain constant.

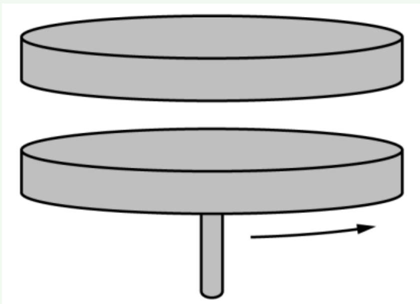
Solution to part e ii: The rotational inertia of the sphere is less than the cylinder. Thus, the sphere will have a greater linear speed and it will rotate faster. Since the sphere and the cylinder have the same mass, the sphere will have a greater translational kinetic energy. Since their total energy is the same, the sphere must have a smaller rotational kinetic energy due to it's large translational kinetic energy. Thus, the answer is $\boxed{\text{Less than}}$

Solution to part e iii: We already showed that the sphere will have a greater linear speed due to it's smaller rotational inertia. This greater linear speed will lead to a larger distance travelled when it hits the floor. Although the time taken to hit the floor is the same for both, the larger linear velocity will cause the sphere to travel a greater distance. Thus, the answer is $\boxed{\text{Greater than}}$

Problem 5.0.37 — 2019 AP Physics C Mechanics FRQ



A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the wheel until the platform reaches an angular speed w_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.
(a) Derive an expression for the angular speed w_P of the platform. Express your answer in terms of I_P , r, D, F, Δt, and physical constants, as appropriate.
(b) Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed w_P. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.
(c) Derive an expression for the angular speed of the wheel w_W when the platform has reached angular speed w_P. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.



When the platform is spinning at angular speed w_P, the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_P as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed w_f.
(d) Derive an expression for w_f. Express your answer in terms of w_P, I_P, and physical constants, as appropriate.

Solution to part a: Since we are given the force in the problem, we are indirectly given the torque since we also have the radius of the platform.
We know that $\tau = F \cdot D = FD$ (D is the radius of the platform)
We can now use the equation $\Delta L = \tau \cdot \Delta t$

ΔL represents the change in angular momentum. The initial angular momentum $L_i = 0$

since there is no angular velocity. The final angular velocity can be represented as $L = I_P \cdot w_P$
We can substitute this along with τ to get $I_P w_P = FD\Delta t$
We can isolate w_P to find that $w_P = \frac{FD\Delta t}{I_P}$

Solution to part b: We know that rotational kinetic energy exists since the platform spins.
We know that $K_{rot} = \frac{1}{2}Iw^2$
Since our rotational inertia is I_P and angular velocity is w_P, $K_{rot} = \frac{1}{2}I_P w_P^2$ (we write it in this form to use the right variables that are being used throughout this problem).

$$\text{We can plug in } w_P = \frac{FD\Delta t}{I_P} \text{ to find that } K_{rot} = \frac{1}{2}I_P(\frac{FD\Delta t}{I_P})^2 = \frac{(FD\Delta t)^2}{2I_P}$$

Solution to part c: The platform and wheel must move with the same linear speed due to their contact point.
This means that $v_P = v_W$
We can now also relate their individual velocities to their angular velocity.
We know that in general, $v = wr$.

This means that $v_P = w_P D$ and $v_W = w_W r$ (we multiply their individual angular velocities to their radius)

We can set v_P and v_W equal to each other: $w_P D = w_W r$

$$\text{We can isolate } w_W \text{ to find that } w_W = \frac{w_P D}{r}$$

Solution to part d: No external torque is applied to the system. Thus, we can conserve angular momentum.
The initial angular momentum comes from the platform itself. We know that $L_i = I_P w_P$

After the disk falls on the platform, both the disk and platform will move with the same common angular speed of w_f.

The final angular momentum will be $L_f = I_P w_f + I_P w_f$ (since the disk that is added has the same rotational inertia). Their rotational inertias simply add up leaving us with $L_f = 2I_P w_f$

We can set L_i and L_f equal to each other to get $I_P w_P = 2I_P w_f$

$$\text{We can isolate } w_f \text{ to find that } w_f = \frac{w_P}{2}$$

Unit 6 Oscillations

Simple Harmonic Motion is a new type of motion that combines all of the concepts we've learned until now. This motion is the name for repetitive back and forth motion, such the grandfather clock. In AP Physics C: Mechanics, we will cover 2 types of SHM: the spring and the pendulum.

Note 6.0.1 — Spring Review
Although we all have a general idea of what a spring is, the three main characteristics of it is that the force applied by the spring is proportional to the distance compressed/stretched *from the equilibrium position.* That is,

$$F_{\text{spring}} = -kx$$

where k is the spring constant, and x is the distance it's compressed/stretched.

The other characteristic is that the potential energy of the spring at any given point is

$$U_{\text{spring}} = \frac{1}{2}kx^2$$

where x is the distance it is stretched/compressed from the equilibrium position.

Note that the potential energy is 0 when the spring is at equilibrium, and is maximized when the spring is maximally stretched (or compressed).

The period and frequency of the spring is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

and

$$f = \frac{1}{T} = 2\pi\sqrt{\frac{k}{m}}$$

Let's derive something that is extremely important to be able to understand simple harmonic motion.

Let's say that the spring force is the only force on an object in the horizontal direction. From Newton's Second Law, we know that $F_{net} = -kx = ma$
Now, we will use the other form of acceleration. We know that $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$. We will use the fact that $a = \frac{d^2x}{dt^2}$.

Plugging that in gives that

$$-kx = m\frac{d^2x}{dt^2}$$

Integrating this is way above the scope of the AP Physics C: Mechanics curriculum. However, remember that $x(t) = x_{max}\cos(wt + \theta)$.

The formula above for x is rarely used. However, it is used to find something extremely important.

Instead of using x_{max}, we will use A since they both represent amplitude.
We can plug in our expression $x(t) = A\cos(wt+\theta)$ into $m\frac{d^2x}{dt^2}$. We find the second derivative of $x(t)$ and multiply it to m. Doing so gives

$$m\frac{d^2x}{dt^2} = -mAw^2\cos(wt+\theta)$$

Now, we can plug in $x(t) = A\cos(wt+\theta)$ into $-kx$ to get $-kA\cos(wt+\theta)$

We can set our expressions for $-kx$ and $m\frac{d^2x}{dt^2}$ equal to each other. Doing so gives

$$-kA\cos(wt+\theta) = -mAw^2\cos(wt+\theta)$$

We can cancel like terms from both sides to find that $k = mw^2$. This is the key relationship that we just found. We can rearrange that equation to find that $w = \sqrt{\frac{k}{m}}$.

Remember that $\frac{d^2x}{dt^2} = -w^2x$

Thus, since $T = \frac{2\pi}{w}$, we can find that $T = 2\pi\sqrt{\frac{k}{m}}$

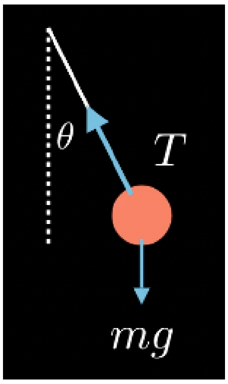
Also, remember that the period T and frequency f are related to each other. $T = \frac{1}{f}$.

Note 6.0.2 — Mechanical energy is always conserved in an oscillating system with a spring and a mass.

The maximum potential energy exists when the spring is stretched to its maximum displacement. At this point, the spring will instantaneously be at rest causing the kinetic energy to be 0.
This means that the total mechanical energy is equivalent to the maximum potential energy which is $\frac{1}{2}kA^2$ (and A is the amplitude). Maximum kinetic energy will occur at the equilibrium point for a spring.

On top of a spring, oscillations can occur for pendulums. Note that the equilibrium point for pendulums occurs when it's hanging straight down.
For a pendulum, we displace it by a small angle θ. Then, the gravitational force will cause it to rotate back and forth.

For a pendulum, the period T is $T = 2\pi\sqrt{\frac{L}{g}}$ where L is the length of the pendulum.



The above image shows a pendulum. There is a ball of mass m attached at the end of the pendulum. If we find the components of the gravitational force, then we can see that $mg\sin(\theta)$ is perpendicular to the pendulum.

Thus, $mg\sin(\theta)$ will apply a torque, and the torque on the pendulum is

$$-mgL\sin\theta$$

We know that $\tau_{net} = I\alpha$ (Newton's Second Law for Rotation).
We also know that $I = mL^2$ because the mass of our ball is M and it is a distance L away from the pivot.

We can plug torque and rotational inertia in to find that

$$\alpha = \frac{\tau_{net}}{I} = \frac{-mgL\sin\theta}{mL^2} = -\frac{g\sin\theta}{L}$$

Since our θ will be a very very small angle to make it an oscillation, we can use the approximation $\sin\theta = \theta$.
This means that $\alpha = -\frac{g\theta}{L}$.

We also know that $\frac{d^2\theta}{dt^2} = \alpha$. This means that

$$\frac{d^2\theta}{dt^2} = -\frac{g\theta}{L}$$

Please remember that $\alpha = \frac{d^2\theta}{dt^2} = -\omega^2\theta$
Using that, we can find out that for a pendulum, $\omega^2 = \frac{g}{L}$. Thus,

$$\omega = \sqrt{\frac{g}{L}}$$

$$\text{This means that } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

For harmonic oscillator problems (both spring and pendulum), find an equation for the acceleration ($\frac{d^2x}{dt^2}$) or angular acceleration ($\frac{d^2\theta}{dt^2}$).

Then use the idea that $a = \frac{d^2x}{dt^2} = -\omega^2 x$ or $\alpha = \frac{d^2\theta}{dt^2} = -\omega^2\theta$ to find an expression for ω. Using that, you can find the period since $T = \frac{2\pi}{\omega}$.

Problem 6.0.3 — A spring with constant k_1 is initially at rest, with a block of mass M attached to it. It is then stretched a distance d and released.

What is the velocity of the block when the spring reaches equilibrium?

Solution: Initially, the energy of the spring is $\frac{1}{2}kd^2$ using the formula $U_s = \frac{1}{2}kx^2$. When the spring reaches the equilibrium point, all the spring potential energy will convert to kinetic energy.

$$\text{Thus, } \frac{1}{2}kd^2 = \frac{1}{2}Mv^2$$

Solving, we get that $v = d\sqrt{\frac{k}{M}}$, at the equilibrium position.

Problem 6.0.4 — A spring with constant k hangs from a ceiling, with a ball of mass m attached. How much does the spring stretch?

Solution: Note that the force applied on the ball by gravity is mg, and the force applied by the spring is kx. Since the ball is in equilibrium, we know that $mg = kx$, which means that $x = \frac{mg}{k}$.

This is the main difference between a **horizontal** and **vertical** spring. In a horizontal spring, gravitational force will not cause the block to accelerate. In a vertical spring, gravitational force plays a role on the other hand. That is why for a horizontal spring, the spring force is 0 at the equilibrium point while for a vertical spring, spring force isn't 0.

Note 6.0.5 — The Pendulum Review

A pendulum oscillates due to the force of gravity. In a way, it performs repeated back-and-forth circular motion.

The period of a pendulum (time it takes to make one "tick" and one "tock", which is one back-and-forth motion) is

$$T = 2\pi\sqrt{\frac{L}{g}}$$

The frequency (number of ticks and tocks per second, which is the number of back-and-forth motions in one second) of it is

$$f = 2\pi\sqrt{\frac{g}{L}}$$

Note that both of these formulas are independent of the original displacement of the pendulum. (Think about it, this is the reason grandfather clocks are still accurate! No matter how much the air resistance reduces the displacement of the pendulum, the time between each tick and each tock is the same.)

Problem 6.0.6 — A pendulum has a period of 5 seconds. If the length of the string of the pendulum is quadrupled, what is the new period of the pendulum?

Solution: Using the formula $T = 2\pi\sqrt{\frac{L}{g}}$, if the length is multiplied by 4, the whole formula is multiplied by 2. Hence the new period is $5 \cdot 2 = 10$ seconds.

Problem 6.0.7 — A ball of mass 2 kg is attached to a string of length 4m, forming a pendulum. If the string is raised to have an angle of 30 degrees above the vertical and released, what is the velocity of the ball as it passes through its lowest point?

Solution: The change in potential energy from the initial position to the final position is

$$U_f - U_i = mg(4 - 4\cos(30^\circ)) \approx 0.54mg$$

So,

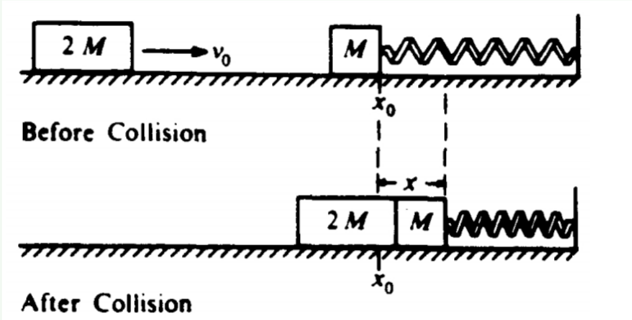
$$\frac{1}{2}mv^2 = 0.54mg$$
$$\implies v = \boxed{3.3}\,\text{m/s}$$

Problem 6.0.8 — Find the total mechanical energy of a spring given a spring constant of 5 N/m and an amplitude of 2 meters.

Solution: The total mechanical energy is the sum of the potential and kinetic energies. When the spring is stretched to its amplitude, the kinetic energy will be 0. All of the energy will be in the form of spring potential energy. Thus, we can simply find the spring potential energy at the amplitude to find the total mechanical energy.

$$\frac{1}{2}kx^2 = \frac{1}{2} \cdot 5 \cdot 2^2 = 10 \text{ J}$$

Problem 6.0.9 — 1983 AP Physics B FRQ



A block of mass M is resting on a horizontal, frictionless table and is attached as shown above to a relaxed spring of spring constant k. A second block of mass $2M$ and initial speed v_0 collides with and sticks to the first block. Develop expressions for the following quantities in terms of M, k, and v_0:

(a) v, the speed of the blocks immediately after impact

(b) x, the maximum distance the spring is compressed

(c) T, the period of the subsequent simple harmonic motion

Solution to part a: The speed of the blocks after impact can be found by conserving momentum.
$p_i = p_f$
The initial momentum simply comes from the block of mass $2M$. Since it moves at a speed v_o, the initial momentum is $2Mv_o$

After collision, both of the blocks move together since it is an inelastic collision. This means they will move with the same speed, which we can assume to be v_f.
The combined mass of both blocks is $2M + M$ which is $3M$. This means that the final momentum is $3Mv_f$

After equating initial and final momentum, we can write the equation $2Mv_o = 3Mv_f$
We can solve this to find that $v_f = \frac{2v_o}{3}$

Solution to part b: We can find the maximum distance the spring is compressed by conserving energy.
The kinetic energy of both blocks after collision will convert to spring potential energy.

$$K = U_s$$

U_s (spring potential energy) can be represented as $\frac{1}{2}kx^2$
We can find kinetic energy by using the equation $\frac{1}{2}mv^2$. The combined mass is $3M$ and velocity is $\frac{2v_o}{3}$. We can plug this into the equation to find that $K = \frac{2Mv_o^2}{3}$

Now, we can equate our expressions for spring potential energy and kinetic energy.

$$\frac{2Mv_o^2}{3} = \frac{1}{2}kx^2$$

We can multiply both sides by $\dfrac{2}{k}$ to find that $x^2 = \dfrac{4Mv_o^2}{3k}$

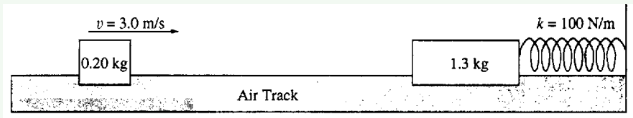
We can take the square root of both sides to find that $x = \sqrt{\frac{4Mv_o^2}{3k}}$ which can be further simplified to

$$x = 2v_o\sqrt{\frac{M}{3k}}$$

Solution to part c: Period is simply found through the equation $T = 2\pi\sqrt{\frac{m}{k}}$ Since our combined mass is $3M$, we can plug that in to find that the period T is

$$2\pi\sqrt{\frac{3M}{k}}$$

Problem 6.0.10 — 1995 AP Physics B FRQ



As shown above, a 0.20-kilogram mass is sliding on a horizontal, frictionless air track with a speed of 3.0 meters per second when it instantaneously hits and sticks to a 1.3-kilogram mass initially at rest on the track. The 1.3-kilogram mass is connected to one end of a massless spring, which has a spring constant of 100 newtons per meter. The other end of the spring is fixed.

(a) Determine the following for the 0.20-kilogram mass immediately before the impact.

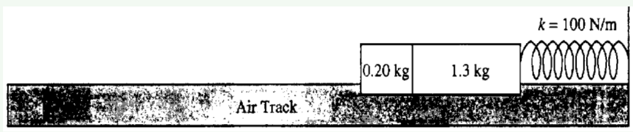
i. Its linear momentum

ii. Its kinetic energy

(b) Determine the following for the combined masses immediately after the impact.

i. The linear momentum

ii. The kinetic energy



(c) After the collision, the two masses undergo simple harmonic motion about their position at impact.

Determine the amplitude of the harmonic motion.

(d) After the collision, the two masses undergo simple harmonic motion about their position at impact.

Determine the period of the harmonic motion.

Solution to part a: The initial linear momentum can be found from $p = mv$ (the formula for momentum).

Since the mass is 0.20 kg and velocity is $3.0\frac{m}{s}$, we find that the initial momentum is $0.20 \cdot 3.0$ which is $0.6\frac{\text{m}\cdot\text{kg}}{s}$
On top of that, the initial kinetic energy can be found from $\frac{1}{2}mv^2$.

$$\text{This means that } K_i = \frac{1}{2} \cdot 0.20 \cdot 3.0^2 = 0.9 \text{ J}$$

Solution to part b: Momentum is conserved in a collision. This means that we can just use our formula for conservation of momentum which says $p_i = p_f$
We already found in part a that $p_i = 0.6$. In this collision, the linear momentum will be conserved so $p_f = 0.6$ $0.6\frac{\text{m}\cdot\text{kg}}{s}$

However, kinetic energy is not conserved in an inelastic collision. We must find the velocity of the two blocks right after collision.
We know that the two blocks move with a common velocity after collision. We can denote it with the variable v_f
This means that the final momentum is sum of the two masses times v_f (since the two blocks move together).

$$p_f = (0.2 + 1.3)v_f = 1.5v_f$$

We also already know that $p_f = 0.6$.
We can set both expressions to each other to get $1.5v_f = 0.6$
Solving it gives that $v_f = 0.4$ m/s.

Now, we can find the final kinetic energy. It can be found using the equation $K = \frac{1}{2}mv^2$
In this case, the two masses move together so we sum up the masses to get 1.5 kg. We also know that $v_f = 0.4$

$$\text{We can plug this in to find } K_f = \frac{1}{2} \cdot 1.5 \cdot 0.4^2 = \boxed{0.12 \text{ J}}$$

Solution to part c: To find the amplitude, we simply apply conservation of energy to find the maximum compression.
We equate the kinetic energy right after the two blocks collide and equate it to spring potential energy.

$$K = U_s = \frac{1}{2}kx^2$$

From part b, we already know that the kinetic energy is 0.12

We can substitute that in to get $0.12 = \frac{1}{2}kx^2$

We also know that the spring constant $k = 100$. We can plug that into the equation.

Plugging the value of k in gives $0.12 = \frac{1}{2} \cdot 100 \cdot x^2$

We can simplify the equation to find that $x^2 = 0.0024$

Now, we can take the square root of both sides to find that the amplitude $\boxed{A = 0.049 \text{ m}}$

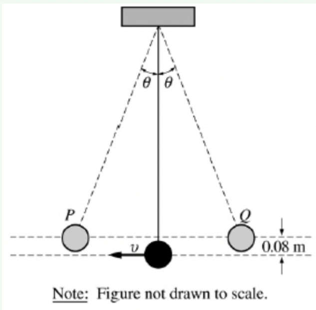
Solution to part d: To find the period, we simply need to apply our formula.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

The above is the formula to find period. For m, we will use the sum of the masses which is 1.5 kg.
We also know that the spring constant is 100.

$$\text{We can plug these values in to get } T = 2\pi\sqrt{\frac{1.5}{100}} = \boxed{0.77 \text{ s}}$$

Problem 6.0.11 — 2005 AP Physics B FRQ
A simple pendulum consists of a bob of mass 0.085 kg attached to a string of length 1.5 m. The pendulum is raised to point Q, which is 0.08 m above its lowest position, and released so that it oscillates with small amplitude θ between the points P and Q as shown below.



(a) On the figures below, draw free-body diagrams showing and labeling the forces acting on the bob in each of the situations described.

i. When it is at point P

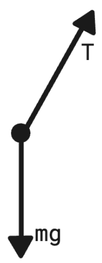
ii. When it is in motion at its lowest position

(b) Calculate the speed v of the bob at its lowest position.

(c) Calculate the tension in the string when the bob is passing through its lowest position.

(d) Describe one modification that could be made to double the period of oscillation.

Solution to part a i: The only forces on the bob are tension and gravitational force. Tension force always points along the string/rope.



Solution to part a ii: The only forces on the bob are again tension and gravitational force. However, this time tension force will point directly upwards instead of at an angle. Also, the size of the arrows don't need to represent which one's larger by magnitude since the problem doesn't explicitly say so. However, for those that are curious, tension force will be larger in magnitude. The reason is that there is an acceleration vector (centripetal acceleration) that points upwards. Thus, the net force must also be upwards which means tension must be larger.

Solution to part b: The speed can be found using conservation of energy.
We will apply conservation of energy by using point P as the initial position and the bottom point as the final position.
We know that $K_i + U_i = K_f + U_f$

To swing down from point P to the lowest point, the bob moves down a height h. This means that it gravitational potential energy decreases by mgh. This energy will convert to kinetic energy.

$$\text{This means } mgh = K_f$$

Kinetic energy can be represented as $\frac{1}{2}mv^2$

We can plug that in for K_f to get $mgh = \frac{1}{2}mv^2$

We can cancel out m and rearrange the equation to get $v = \sqrt{2gh}$

$$\text{Since } h = 0.08 \text{ m, we can find that } v = \sqrt{2 \cdot 9.8 \cdot 0.08} = \boxed{1.252 \text{ m/s}}$$

Solution to part c: We can find the tension at the bottom point by using Newton's Second Law along with some concepts from our centripetal motion chapter.

We can use Newton's Second Law to write $T - mg = ma$
We know that $a = \frac{v^2}{r}$.

$$\text{We can plug that in to get } T - mg = \frac{mv^2}{r}$$

$$\text{We can add } mg \text{ to both sides to get } T = m(g + \frac{v^2}{r})$$

We know that $m = 0.085$ kg, v at the bottom point is 1.252, and $r = 1.5$ m (since that's the length of the string).

$$\text{We can plug those values in to get } T = 0.085(9.8 + \frac{1.252^2}{1.5}) = \boxed{0.922 \text{ N}}$$

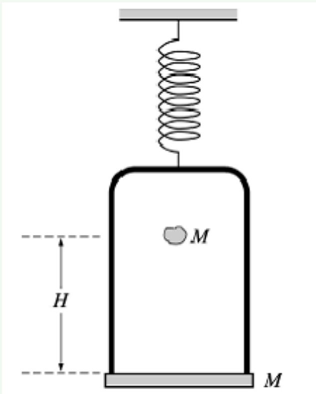
Solution to part d: We know that the period of oscillation for a pendulum can be represented using the formula below.

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Clearly, L (length of string) and g are the two variables that affect period.

We can simply quadruple the length of the string to double the period of oscillation.

Problem 6.0.12 — 2003 AP Physics C: Mechanics FRQ



An ideal massless spring is hung from the ceiling and a pan suspension of total mass M is suspended from the end of the spring. A piece of clay, also of mass M, is then dropped from a height H onto the pan and sticks to it. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the speed of the clay at the instant it hits the pan.

(b) Determine the speed of the clay and pan just after the clay strikes it.

(c) After the collision, the apparatus comes to rest at a distance H/2 below the current position. Determine the spring constant of the attached spring.

(d) Determine the resulting period of oscillation.

Solution to part a: The speed of the clay when it hits the pan can be found using conservation of energy.

Since the clay descends a height H, it's gravitational potential energy will decrease. It will convert to kinetic energy.

On top of that, we know that the initial kinetic energy K_i must be 0 since the clay is dropped from rest so it's initial speed is 0.

This means we can simplify the conservation of energy equation to $U_i = K_f$ (this means that the gravitational potential energy converts to kinetic energy)

We can plug in our formulas for U and K to get $mgh = \frac{1}{2}mv^2$

Cancelling m and simplifying gives that $v = \sqrt{2gh}$

We know that the height where the clay is dropped from is denoted as H in this problem.

We can plug H in replacement of h to get $\boxed{v = \sqrt{2gH}}$

Solution to part b: The clay will collide with the pan. We will have an inelastic collision.
We know that momentum is conserved. This means $p_i = p_f$ from the conservation of momentum formula.

We know that $p_i = mv = M\sqrt{2gH}$
To write an expression for p_f, we can denote the common velocity of both masses as v_f.
We know that $p_f = mv = (M + M)v_f = 2Mv_f$

Setting both expressions equal to each other gives $M\sqrt{2gH} = 2Mv_f$

Dividing both sides by $2M$ gives $\boxed{v_f = \frac{\sqrt{2gH}}{2}}$

Solution to part c: We can use conservation of energy in this problem.

$$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}$$

The initial point that we consider is the point of collision. The final point will be the point when the apparatus is at rest.

We will make our reference level be the point where the apparatus comes to rest. This causes U_{gf} (final gravitational potential energy) to be 0. Also, $K_f = 0$ since the apparatus comes to rest.

We will set U_{si} to 0. We will set the position immediately after collision to 0 spring potential energy.

We can find U_{sf} by finding spring potential energy for the additional stretch distance that comes after the collision.

We know that $U_{sf} = \frac{1}{2}k(\frac{H}{2})^2 = \frac{kH^2}{8}$

We can plug these values into our conservation of energy formula to get $K_i + U_{gi} = U_{sf}$

We also know that $K = \frac{1}{2}mv^2$. Since our combined mass is $2M$ and velocity is $v_f = \frac{\sqrt{2gH}}{2}$, we can plug that in to get

$$K_i = \frac{1}{2} \cdot 2M \cdot (\frac{\sqrt{2gH}}{2})^2 = \frac{MgH}{2}$$

Similarly, U can be represented as mgh. In our case, the mass is $2M$ and height is $\frac{H}{2}$. This means that $U_i = MgH$

$$\text{We can plug these values in to get } \frac{MgH}{2} + MgH = \frac{kH^2}{8}$$

$$\text{We can divide both sides by } H \text{ to get } \frac{Mg}{2} + Mg = \frac{kH}{8}$$

The left side is $\frac{3Mg}{2}$ which means the equation is $\frac{3Mg}{2} = \frac{kH}{8}$

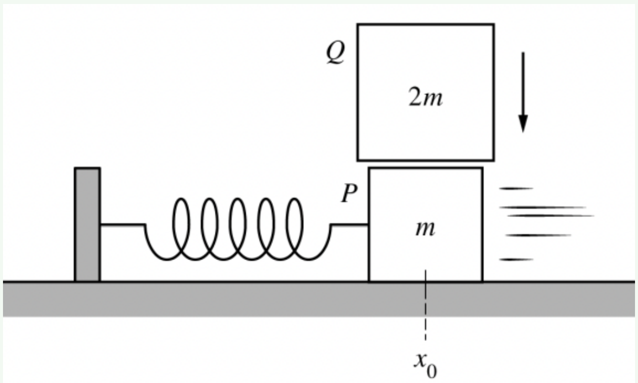
We can multiply both sides by $\frac{8}{H}$ to get $\boxed{k = \frac{12Mg}{H}}$

Solution to part d: We know that period can be represented as $2\pi\sqrt{\frac{m}{k}}$
We can plug in $2M$ for mass and $\frac{12Mg}{H}$ for spring constant.

Doing so gives that $T = 2\pi\sqrt{2M \div \frac{12Mg}{H}} = \boxed{2\pi\sqrt{\frac{H}{6g}}}$

Problem 6.0.13 — 2018 AP Physics 1 FRQ
Block P of mass m is on a horizontal, frictionless surface and is attached to a spring with spring constant k. The block is oscillating with period T_P and amplitude A_P about the spring's equilibrium position x_0. A second block Q of mass $2m$ is then dropped from rest and lands on block P at the instant it passes through the equilibrium position, as shown above. Block Q immediately sticks to the top of block P, and the two-block system oscillates with period T_{PQ} and amplitude A_{PQ}.

(a) Determine the numerical value of the ratio $\frac{T_{PQ}}{T_P}$.



(b) The figure is reproduced above. How does the amplitude of oscillation A_{PQ} of the two-block system compare with the original amplitude A_P of block P alone?
_____ $A_{PQ} < A_P$ _____ $A_{PQ} = A_P$ _____ $A_{PQ} > A_P$
In a clear, coherent paragraph-length response that may also contain diagrams and/or equations, explain your reasoning.

Solution to part a: We know that period can be written as $T = 2\pi\sqrt{\frac{m}{k}}$
T_P only involves block P which has a mass of m. That means our period is simply $T_p = 2\pi\sqrt{\frac{m}{k}}$

However, for T_{PQ}, things are a little different. The reason is that now the combined mass of block P and Q is $3m$. This means we must plug in $3m$ instead of m into our formula for period. This means $T_{PQ} = 2\pi\sqrt{\frac{3m}{k}}$

Clearly in the ratio of $\frac{T_{PQ}}{T_P}$, everything cancels out except for the factor of 3. We can find that

$$\frac{T_{PQ}}{T_P} = \sqrt{3}$$

Solution to part b: We know that the amplitude can be found from conservation of energy.
We will first find the speed after collision. This will be done using conservation of momentum.
We know that $p_i = p_f$

If block P moves with velocity v initially, after block Q is dropped the velocity of both blocks will drop.
Since $p_i = mv$, we know that $p_f = (m + 2m)v_f$
We know that both blocks will move with a common final speed v_f due to this collision being inelastic.
We can solve $mv = (m + 2m)v_f$ to find that $v_f = \frac{v}{3}$

Before collision, we know that $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$ since the kinetic energy will convert to spring potential energy.
We can solve it to find that the amplitude A_P is $v\sqrt{\frac{m}{k}}$

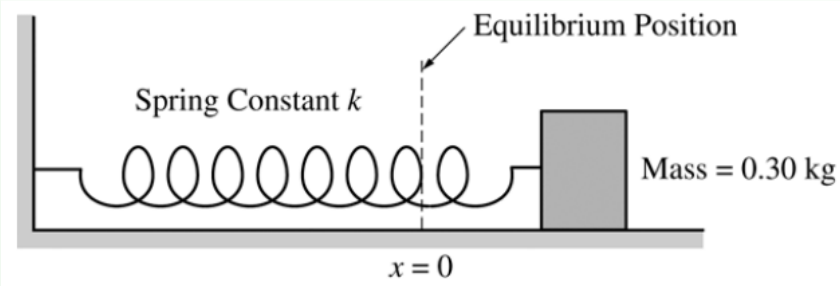
When both block P and Q are attached to the spring, we can apply the same method again.
$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$
However, now there is one difference, Instead of m, we must use $3m$ since that is the combined mass. In addition, our velocity is $\frac{v}{3}$ since that's the new velocity after collision.

We can plug those values in to get that $\frac{1}{2} \cdot 3m \cdot (\frac{v}{3})^2 = \frac{1}{2}kx^2$
We can solve this to find that the amplitude A_{PQ} is $v\sqrt{\frac{m}{3k}}$

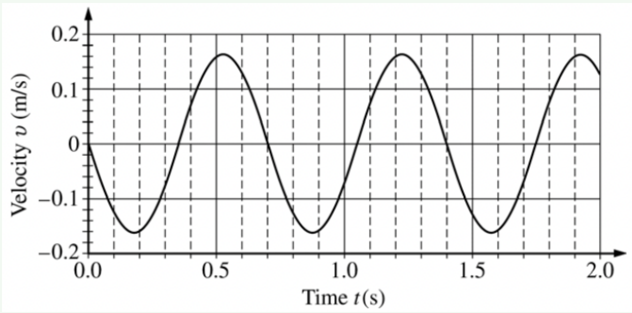
Clearly, the amplitude is lower for blocks P and Q combined. The reason for this is the lower velocity of the two-block system which leads to lower kinetic energy. This causes the amplitude to be lower after we equate kinetic energy to maximum spring potential energy which occurs at the amplitude.

The answer is $\boxed{A_{PQ} < A_P}$

Problem 6.0.14 — 2012 AP Physics C Mechanics FRQ



A block of mass 0.30 kg is placed on a frictionless table and is attached to one end of a horizontal spring of spring constant k, as shown above. The other end of the spring is attached to a fixed wall. The block is set into oscillatory motion by stretching the spring and releasing the block from rest at time $t = 0$. A motion detector is used to record the position of the block as it oscillates. The resulting graph of velocity v versus time t is shown below. The positive direction for all quantities is to the right.



(a) Determine the equation $v(t)$, including numerical values for all constants.

(b) Given that the equilibrium position is at $x = 0$, determine the equation for $x(t)$, including numerical values for all constants.

(c) Calculate the value of k.

I specifically chose this problem for the book since it allows people to find the equation relating velocity and time. They will think about the trigonometric functions required along with the amplitude.

Solution to part a: From our oscillation unit, we should know a general form of the equation for $v(t)$.

$v(t) = -v_{max} \sin(\omega t)$

We know that $\omega = \frac{2\pi}{T}$. In this problem, the period $T = 0.7$ s since that's the amount of time it takes for a full cycle.

This means that $\omega = \frac{2\pi}{0.7} = 8.97$ rad/s.

We can plug this into our equation to get $v(t) = -v_{max} \sin(8.97t)$

Now we must find our amplitude which is v_{max}. The maximum value of velocity is around 0.16, and that is our amplitude. The reason is that the graph peaks at around 0.16.

We can plug that in to simplify our expression: $v(t) = -0.16 \sin(8.97t)$

Solution to part b: Using our expression for $v(t)$, we can find $x(t)$ since displacement is the integral of velocity with respect to time.

$$x(t) = \int v(t)dt$$

Integrating $v(t) = -0.16\sin(8.97t)$ gives that $x(t) = 0.018\cos(8.97t)$

Solution to part c: We can find our spring constant k by conserving energy.
The maximum spring potential energy will convert to maximum kinetic energy at equilibrium.

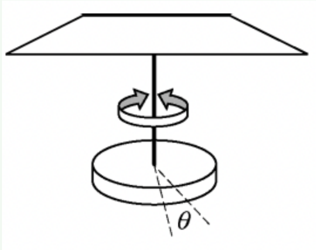
$$\text{We can write this out as } \frac{1}{2}kx_{max}^2 = \frac{1}{2}mv_{max}^2$$

$$\text{We can cancel out } \frac{1}{2} \text{ to get } kx_{max}^2 = mv_{max}^2$$

We know that $x_{max} = 0.018$ since that's the amplitude of the equation $x(t) = 0.018\cos(8.97t)$. Similarly, we know that $v_{max} = 0.16$. We also know the mass m is 0.3 kg.

$$\text{We can plug all of this in to find that } k = \frac{mv_{max}^2}{x_{max}^2} = \frac{0.3 \cdot 0.16^2}{0.018^2} = 23.7 \text{ N/m}$$

Problem 6.0.15 — 2011 AP Physics C Mechanics FRQ



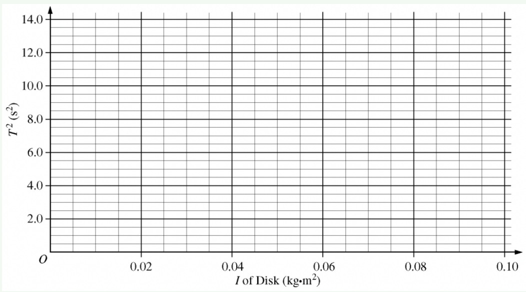
The torsion pendulum shown above consists of a disk of rotational inertia I suspended by a flexible rod attached to a rigid support. When the disk is twisted through a small angle θ, the twisted rod exerts a restoring torque τ that is proportional to the angular displacement: $\tau = -\beta\theta$, where β is a constant. The motion of a torsion pendulum is analogous to the motion of a mass oscillating on a spring.

(a) In terms of the quantities given above, write but do NOT solve the differential equation that could be used to determine the angular displacement θ of the torsion pendulum as a function of time t.

(b) Using the analogy to a mass oscillating on a spring, determine the period of the torsion pendulum in terms of the given quantities and fundamental constants, as appropriate. To determine the torsion constant β of the rod, disks of different, known values of rotational inertia are attached to the rod, and the data below are obtained from the resulting oscillations.

Rotational Inertia I of Disk $(kg \cdot m^2)$	Average Time for Ten Oscillations (s)	Period T (s)	T^2 (s^2)
0.025	22.4	2.24	5.0
0.036	26.8	2.68	7.2
0.049	29.5	2.95	8.7
0.064	33.3	3.33	11.1
0.081	35.9	3.59	12.9

(c) On the graph below, plot the data points. Draw a straight line that best represents the data.



(d) Determine the equation for your line.

(e) Calculate the torsion constant b of the rod from your line.

(f) What is the physical significance of the intercept of your line with the vertical axis?

Solution to part a: We know that $\alpha = \frac{d^2\theta}{dt^2}$. This means that the angular acceleration is the second derivative of the angular position with respect to time.

We also know that $\tau = I\alpha$ which comes from Newton's Second Law for Rotation. We can divide both sides by α to find that $\alpha = \frac{\tau}{I}$.
Since $\alpha = \frac{d^2\theta}{dt^2}$, we know that $\frac{d^2\theta}{dt^2} = \frac{\tau}{I}$.

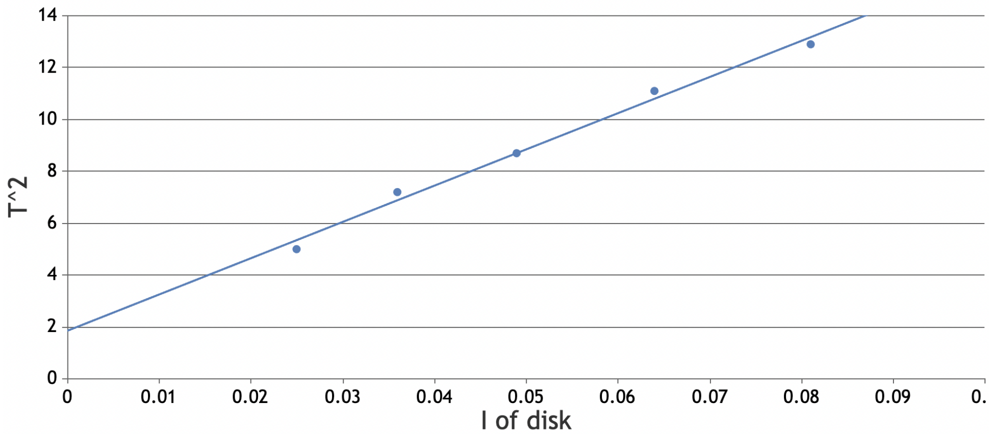
We can plug in $\tau = -\beta\theta$ to get the differential equation $\frac{d^2\theta}{dt^2} = \frac{-\beta\theta}{I}$.

Solution to part b: We already know that pendulums are related to springs when it comes to oscillations.
For a spring, we know that $\frac{d^2x}{dt^2} = -\frac{kx}{m}$ and $w^2 = \frac{k}{m}$.

We can make an analogy between k and β along with x and θ. Similarly, an analogy can be made with mass m and rotational inertia I.
Instead of $w^2 = \frac{k}{m}$ like a spring, we can use the analogous variables to get $w^2 = \frac{\beta}{I}$ for the pendulum. This means that $w = \sqrt{\frac{\beta}{I}}$.

We also know that $T = \frac{2\pi}{w}$. We can plug in w into this to find that $T = 2\pi\sqrt{\frac{I}{\beta}}$

Solution to part c: We simply graph T^2 and I. Make sure to scale the coordinate plane properly so the graph takes majority of the plane.



Don't forget to add the units on the axes. For example, on the axis with T^2, the unit will be s^2.

Solution to part d: We can find the slope of the line of T^2 and I to be around $\frac{12.9-8.7}{0.081-0.049} = 131.25$ (There can multiple possible values of the slope, but it should be around this. You simply use two of the points to find the slope).

For a general line, we know the equation is $y = mx + b$. In our case, we will use T^2 for y and I for x.
That means $T^2 = mI + b$

On top of that we already know that the slope is 131.25
Plugging that in gives that the equation is $T^2 = 131.25I + b$

We know that $I = 0.049$ and $T^2 = 8.7$ is a point on this graph. We can plug that

point in to find the value of b (the y-intercept).
Plugging those values in gives $8.7 = 131.25 \cdot 0.049 + b$. We can solve this to find that $b = 2.27$.
This means that the equation of the line is $T^2 = 131.25I + 2.27$

Solution to part e: In part b we found that $T = 2\pi\sqrt{\frac{I}{\beta}}$.
We can square this equation to get $T^2 = 4\pi^2\frac{I}{\beta}$.

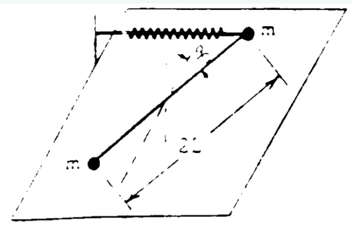
We can divide both sides by I to get $\frac{T^2}{I} = \frac{4\pi^2}{\beta}$.
We already know that the slope of the line of I vs. T^2 was 131.25

We can plug that in to get $131.25 = \frac{4\pi^2}{\beta}$.

$$\text{This means that } \beta = \frac{4\pi^2}{131.25} = 0.3\ \mathrm{kg \cdot m^2/s^2}$$

Solution to part f: When the disk's rotational inertia is 0, the value represented by T^2 is the square of the period of oscillation for the rod alone. The reason is that the disk will have no effect on the period due to a rotational inertia of 0.

Problem 6.0.16 — 1978 AP Physic C: Mechanics FRQ



Note: the diagram shows a total length of 2L and an angle of θ from the vertical

A stick of length $2L$ and negligible mass has a point mass m affixed to each end. The stick is arranged so that it pivots in a horizontal plane about a frictionless vertical axis through its center. A spring of force constant k is connected to one of the masses as shown above. The system is in equilibrium when the spring and stick are perpendicular. The stick is displaced through a small angle θ_0.

(a) Determine the restoring torque when the stick is displaced from equilibrium through the small angle θ_0 as shown and then released from rest at $t = 0$.

(b) Determine the magnitude of the angular acceleration of the stick just after it has been released.

(c) Write the differential equation whose solution gives the behavior of the system after it has been released.

(d) Write the expression for the angular displacement T of the stick as a function of time t after it has been released from rest.

Solution to part a: We know that torque involves force and perpendicular distance from the pivot.

We know that our force from the spring can be found using the formula $F = -kx$. The force by the spring can be found to be $-kL\sin(\theta)$. Since θ is extremely small, we can use our approximation to θ which will cause the force to be $-kL\theta$.

Now, our perpendicular distance from the pivot is $L\cos(\theta)$ which approximates to L.
That means our torque is $-kL\theta \cdot L = -kL^2\theta$

Solution to part b: From Newton's Second Law for rotation, we know that $\tau_{net} = I\alpha$

Our rotational inertia at the center is $mL^2 + mL^2 = 2mL^2$.

$$\text{We can plug that in to find that } \alpha = \frac{-kL^2\theta}{2mL^2} = -\frac{k\theta}{2m}$$

Solution to part c: We know that $\alpha = \frac{d^2\theta}{dt^2}$.

We can plug in our expression for angular acceleration to find that $\frac{d^2\theta}{dt^2} = -\frac{k\theta}{2m}$

Solution to part d: We know that the equation for simple harmonic motion can be written in the form $x = A\cos(w)t$

Since $\alpha = -w^2\theta$ (where w is frequency), we can use the equation $\alpha = -\frac{k\theta}{2m}$

We can set both equal to each other to get $-w^2\theta = -\frac{k\theta}{2m}$

We can now solve for w to find that $w = \sqrt{\frac{k}{2m}}$. We also know that our amplitude (the maximum stretched angle in this case) is θ_0.

$$\text{We can plug all of this in to find that } \theta = \theta_0\cos(\sqrt{\frac{k}{2m}})t$$

Unit 7 Gravitation

By now, we should know that the weight of an object is represented as $F_g = mg$. However, have you ever wondered what causes the moon to rotate around Earth? Why doesn't the moon just "fall off" or go somewhere else?

The answer to this lies in the Universal Law of Gravitation! On a brief note, the Earth exerts a force on the moon which prevents it from going somewhere else. That force is also known as gravity.

Newton discovered that ANY two objects with mass will have a gravitational force between them. In addition to the addiction that many of us have with our phones, there will also be a gravitational force between any person and their phone. However, the force is extremely, extremely, extremely small which is why you don't feel any effects of that force.

However, this force plays a big role in keeping planets in orbit.

Note 7.0.1 — Universal Law of Gravitation
The gravitational force between any two things with mass is defined by

$$F = \frac{Gm_1m_2}{R^2},$$

where m_1, m_2 are the masses of the objects and R is the distance between them.

G is known as The Newton's Law of Gravitation constant, and its value is

$$6.67 \cdot 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{ s}^2}$$

Remember that the gravitational force is always an **attractive** force. It is never repulsive.

Note 7.0.2 — Gravitational Potential Energy
The gravitational potential energy between two masses is

$$-\frac{Gm_1m_2}{R}$$

Note that if the two masses are an infinite distance apart, then the gravitational potential energy is 0.

Now let's figure out how to describe the total mechanical energy on an object in orbit. The total mechanical energy will include U_g (gravitational potential energy) and kinetic energy.

Let's assume that the object has mass m_o and the planet has a mass of m_p, both separated by a distance R. In that case, the force between them can be found using the universal law of gravitation.

$$F = Gm_o m_p R^2$$

Also, gravitational potential energy will be $-\frac{Gm_o m_p}{R}$

Now, let's try to simplify $\frac{1}{2}m_o v^2$ (the kinetic energy). We can do that by finding an expression for the velocity.

The force of $F = \frac{Gm_o m_p}{R^2}$ will be directed towards the center. It will be causing the object to move in a circle. Thus, there will be centripetal acceleration. We can set the force equal to $\frac{mv^2}{R}$

$$F = \frac{Gm_o m_p}{R^2} = \frac{m_o v^2}{R}$$

We can cancel out $\frac{m_o}{R}$ to get

$$\frac{Gm_p}{R} = v^2$$

We can square root both sides to get $v = \sqrt{\frac{Gm_p}{R}}$

Now, we can substitute that expression of velocity into $\frac{1}{2}m_o v^2$ which represents the kinetic energy.

$$\frac{1}{2}m_o(\sqrt{\frac{Gm_p}{R}})^2 = \frac{Gm_o m_p}{2R}$$

We can find that $K + U_g = \frac{Gm_o m_p}{2R} - \frac{Gm_1 m_2}{R} = -\frac{Gm_p m_p}{2R}$

The key takeaway from this is that for an object in circular orbit around a planet, the force found using Universal Law of Gravitation is the cause of the circular motion. Thus, that force is what causes centripetal acceleration, so we can set the force equal to $\frac{mv^2}{R}$ to find the speed

Note 7.0.3 — Escape and Orbital velocities
There are two more important things in gravitation, which is the escape and orbital velocities of an object.

The Orbital velocity is the required velocity of an object in order to stay in the orbit around a planet. This velocity is

$$v_{\text{orbital}} = \sqrt{\frac{GM}{R}},$$

where M is the mass of the planet, not the revolving object.

The Escape velocity is the minimum velocity that will take the object out of its current orbit around a planet. This velocity is

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}},$$

where again, M is the mass of the planet.

In addition to the escape and orbital velocities, you should know how to find the gravitational field which is pretty simple.

By now we should know that the gravitational force between two masses is $\frac{Gm_1m_2}{R^2}$ Let's assume that the mass of m_2 represents the mass of the planet.

Then, to find the gravitational field/acceleration at the location of the other mass, we equate the gravitational force to m_1g where g represents the gravitational field.

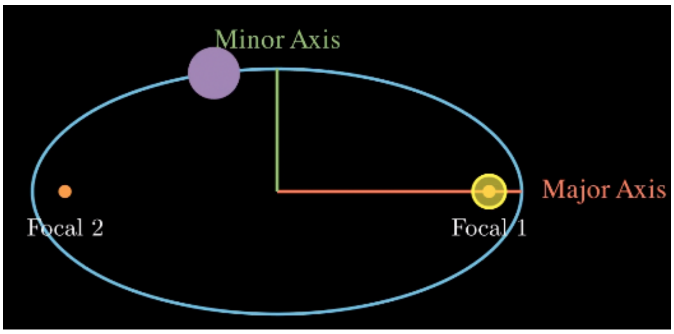
$$\frac{Gm_1m_2}{R^2} = m_1g$$

$$\text{We divide both sides by } m_1 \text{ to get } \frac{Gm_2}{R^2} = g$$

This means that the gravitational field only depends on the mass of the planet and the distance of an object from that planet. It doesn't depend on the mas of an object that is present in that field.

Note 7.0.4 — Kepler's First Law
Kepler's First Law says that all planets move in an elliptical orbit around the Sun. Please note the word elliptical; it's different from a circular orbit!

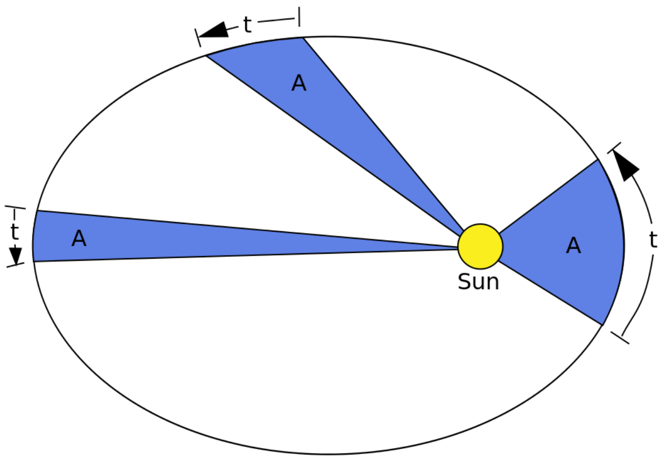
The sun will be located at one of the focal points of the ellipse.



The above represents an elliptical orbit around the sun. The purple circle represents a planet that is moving in an elliptical orbit.
Remember that energy will still be conserved throughout the orbit.

Note 7.0.5 — Kepler's Second Law
The second law describes the changes in velocity as the planet moves around the sun in the elliptical orbit.

As the planet moves closer to the sun, its velocity increases. However, as it moves further away from the sun, its velocity decreases. The reason is that when the planet is closer to the Sun, gravitational potential energy will be less. This causes kinetic energy to be greater; hence the planet is moving faster at points closer to the Sun.



The second law also says that the planet sweeps out equal areas in time intervals of equal length. This idea can be seen in the image above.

Before I jump to the third law, I want to describe something SUPER important that many people ignore.
At all points in an orbit, angular momentum is conserved! This is extremely useful for elliptical orbits since the velocity changes as the planet moves around something such as the Sun. Since angular momentum is conserved, we know that

$$mv_1r_1 = mv_2r_2$$

We can cancel out m to get $v_1r_1 = v_2r_2$

This is extremely useful when you need to find the velocity at a certain point of an elliptical orbit.

Note 7.0.6 — Kepler's Third Law
The third law says that the square of the orbital period of a planet around something like the sun is proportional to the cube of the semi-major axis.
Note that the semi-major axis has half the length of the entire major axis.

$$\frac{T_1^2}{R_1^3} = \frac{T_2^2}{R_2^3}$$

The above shows the relationship that the third law highlights.

It's important to understand how this relationship is derived. By now we know that the gravitational force is what drives such motion.
Let's say that m_1 is the mass of the planet and m_2 is the mass of the Sun. Then, we can find the gravitational force and equate it to $\frac{m_1v^2}{R}$.

$$\frac{Gm_1m_2}{R^2} = \frac{m_1v^2}{R}$$

Now, we can cancel out like terms to get $\frac{Gm_2}{R} = v^2$

Now, we can use the fact that $v = \frac{2\pi R}{T}$. We substitute that to get

$$\frac{Gm_2}{R} = \left(\frac{2\pi R}{T}\right)^2 = \frac{4\pi^2R^2}{T^2}$$

We can rearrange this to get $\dfrac{T^2}{R^3} = \dfrac{4\pi^2}{Gm_2}$

Clearly, we can see that $\frac{T^2}{R^3}$ equals to a constant. The reason is that m_2 is a constant, since it's just the mass of the Sun. Thus, this proves that $\frac{T^2}{R^3}$ is constant.

Also, another key takeaway from this should be the substitution $v = \frac{2\pi R}{T}$. The reason regarding why this is true is that $2\pi R$ represents the circumference, the length of the path. Then, you can divide it by the period to find the velocity.

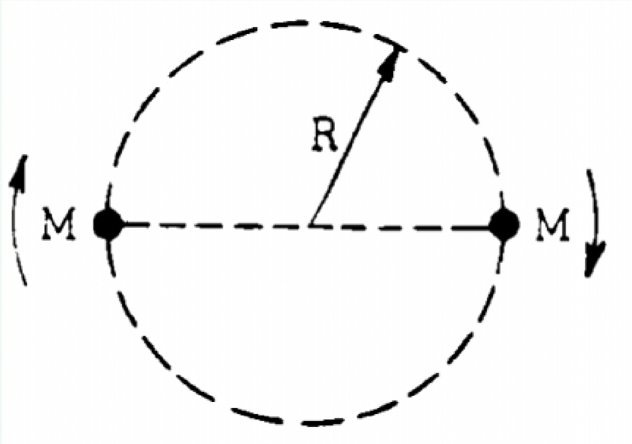
There's one last topic that I would like to discuss. I am surprised by the number of students that aren't taught this topic even though it's important.

In a binary star system (two stars in a system), both of them will orbit around their **center-of-mass**. If the mass of the first star is m_1 and its a distance r_1 from the center-of-mass and the mass of the second star is m_2 with a distance r_2 from the center-of-mass, then the following equation is satisfied.

$$m_1 r_1 = m_2 r_2$$

An example above can be seen. You can see how the product of the mass of the planet and its respective distance from the center of mass is constant.

Problem 7.0.7 — 1977 AP Physics C: Mechanics FRQ

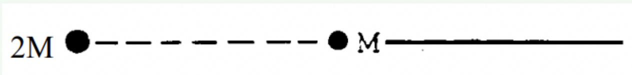


Two stars, each of mass M, form a binary star system in such a way that both stars move in the same circular orbit of radius R. The universal gravitational constant is G.

(a) Use Newton's laws of motion and gravitation to find an expression for the speed v of either star in terms of R, G, and M.

(b) Express the total energy E of the binary star system in terms of R, G, and M.

Suppose instead, one of the stars had a mass $2M$.
(c) On the following diagram, show circular orbits for this star system.



(d) Find the ratio of the speeds, v_{2M}/v_M.

Solution to part a: For such problems, we need to know that the gravitational force is what drives the centripetal motion.

$$\text{This means that } F_g = \frac{Mv^2}{R}$$

We know that $F_g = \frac{G \cdot M \cdot M}{(2R)^2} = \frac{GM^2}{4R^2}$ (don't forget that the distance between the two masses is $2R$, not R since the distance between them is a diameter, not radius).

We can equate $\frac{GM^2}{4R^2}$ to $\frac{Mv^2}{R}$
After cancelling and simplifying, we find that that $v = \frac{1}{2}\sqrt{\frac{GM}{R}}$

Solution to part b: The total energy of the binary-star system will involve computing the kinetic and potential energy of the system.

The potential energy can be represented as $-\frac{GM_1M_2}{r}$. In this case, the PE for us is

$$-\frac{GM^2}{2R}$$

The kinetic energy can be found using the formula $\frac{1}{2}mv^2$. Since we have two masses, we must find the kinetic energy for one of them and then multiply it by 2.

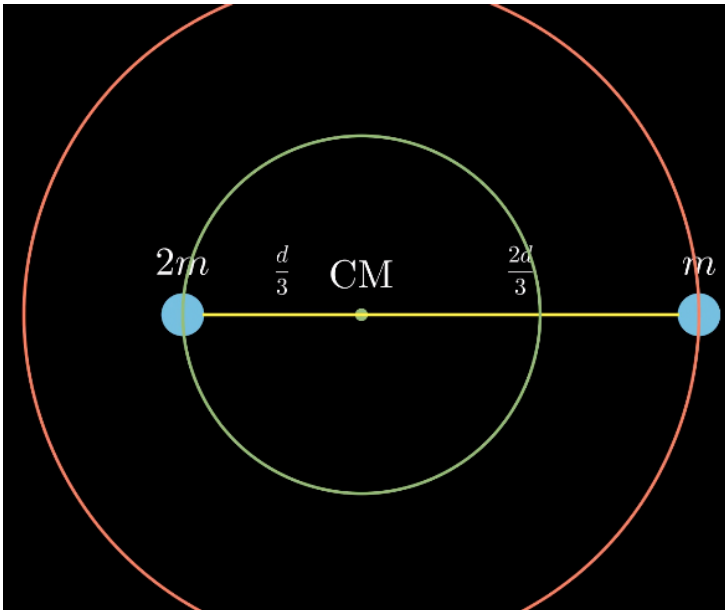
The kinetic energy of one mass is $\frac{1}{2} \cdot M \cdot (\frac{1}{2}\sqrt{\frac{GM}{R}})^2 = \frac{GM^2}{8R}$
That means the kinetic energy of both masses is $2 \cdot \frac{GM^2}{8R}$ which is $\frac{GM^2}{4R}$

We can now add the PE and KE to get

$$-\frac{GM^2}{2R} + \frac{GM^2}{4R} \text{ which simplifies to } -\frac{GM^2}{4R}$$

Solution to part c: We have a binary star system. Thus, both stars will revolve around their center of mass.

Assuming that the distance between the masses $2M$ and M is d, then the center of mass will lie $\frac{d}{3}$ from the left. The reason is that this causes the product of the mass and its respective distance from the center of mass to be constant.



Solution to part d: Both masses will exert equal and opposite forces to each other. We know that the force can be found using the Universal Law of Gravitation. However, if we find the force using that method, then we won't be able to find any relationship involving velocity.

However, we can represent the net force using the mass and the acceleration. Since they're both moving in a circle, centripetal acceleration is present. Be careful about the radius that you use for centripetal acceleration. The radius length used isn't the distance between the two masses. It is the distance between each respective mass to the center-of-mass.

$$\text{The force on the mass } 2m \text{ is } \frac{2mv_2^2}{\frac{d}{3}}$$

$$\text{The force on the mass } m \text{ is } \frac{mv_1^2}{\frac{2d}{3}}$$

$$\text{We can set both forces equal to each other } \frac{2mv_2^2}{\frac{d}{3}} = \frac{mv_1^2}{\frac{2d}{3}}$$

$$\text{Canceling like terms gives } 2v_2^2 = \frac{v_1^2}{2}$$

$$\text{We can square root both sides and rearrange to get } \frac{v_2}{v_1} = \frac{1}{2}$$

Problem 7.0.8 — 1984 AP Physics C: Mechanics FRQ
Two satellites, of masses m and $3m$, respectively, are in the same circular orbit about the Earth's center, as shown in the diagram above. The Earth has mass M_e and radius R_e. In this orbit, which has a radius of $2R_e$, the satellites initially move with the same orbital speed v_o but in opposite directions.

(a) Calculate the orbital speed v_o of the satellites in terms of G, M_e, and R_e.

(b) Assume that the satellites collide head-on and stick together. In terms of v_o, find the speed v of the combination immediately after the collision.

(c) Calculate the total mechanical energy of the system immediately after the collision in terms of G, m, M_e, and R_e. Assume that the gravitational potential energy of an object is defined to be zero at an infinite distance from the Earth.

Solution to part a: We know that the gravitational force between each satellite and earth is the cause of the centripetal motion.
This means that $F_g = \frac{mv^2}{r}$
We must write the expression in terms of the variable that are given in the problem.
We know that $F_g = \frac{GM_1M_2}{R^2}$. We will use m for the mass of the satellite and M_e for the mass of earth. We also know that the distance between the satellites and Earth is $2R_e$.
This means that

$$F_g = \frac{GmM_e}{4R_e^2}$$

We can equate the expression above to $\frac{mv^2}{2R_e}$ (the centripetal force on the satellite of mass m)

$$\text{Doing so gives } \frac{GmM_e}{4R_e^2} = \frac{mv^2}{2R_e}$$

$$\text{After simplifying, we get that } v = \sqrt{\frac{GM_e}{2R_e}}$$

Solution to part b: Since the two masses stick together after collision, we have an inelastic collision. This means that they move with the same velocity after collision. Both masses move at a speed v_o, but they occur in opposite directions. Thus, we must account for that with a negative sign when we find initial momentum.

$$3mv_0 - mv_0 = (3m + m)v_f$$

We can solve for v_f to find that $v_f = \frac{v_0}{2}$

Solution to part c: The mechanical energy is simply the sum of the kinetic and potential energies.

$$\text{E} = \text{KE} + \text{PE}$$

The kinetic energy of the combined mass is $\frac{1}{2} \cdot 4m \cdot v_f^2 = 2mv_f^2$.

The potential energy can be found using the formula $U = -\frac{GM_1M_2}{R}$. We plug in $4m$ and M_e for the masses (to represent Earth's mas and the combined satellite's mass). Our distance R is $2R_e$.

$$\text{This means that } U = -\frac{4GmM_e}{2R_e} = -\frac{2GmM_e}{R_e}$$

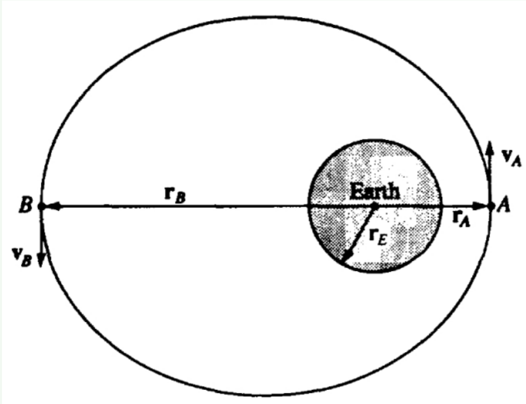
$$\text{Thus, the total energy is } 2mv_f^2 - \frac{2GmM_e}{R_e}$$

However, we are missing one thing. We can't write the energy in terms of v_f. We must substitute what we found in part a and b. We know that $v_f = \frac{v_0}{2}$. We also know that v_0 can be represented as $\sqrt{\frac{GM_e}{2R_e}}$. This means that $v_f = \frac{1}{2}\sqrt{\frac{GM_e}{2R_e}}$

$$\text{We can plug this in to find that the total energy is } \frac{1}{2} \cdot 4m \cdot (\frac{1}{2}\sqrt{\frac{GM_e}{2R_e}})^2 - \frac{2GmM_e}{R_e}$$

$$\text{Simplifying gives } \frac{GmM_e}{4R_e} - \frac{2GmM_e}{R_e} \text{which is } -\frac{7GmM_e}{4R_e}$$

Problem 7.0.9 — 1992 AP Physics C: Mechanics FRQ



A spacecraft of mass $1,000$ kilograms is in an elliptical orbit about the Earth, as shown above. At point A, the spacecraft is at a distance $r_A = 1.2 \times 10^7$ meters from the center of the Earth and its velocity, of magnitude $v_A = 7.1 \times 10^3$ meters per second, is perpendicular to the line connecting the center of the Earth to the spacecraft. The mass and radius of the Earth are $M_E = 6.0 \times 10^{24}$ kilograms and $r_E = 6.4 \times 10^6$ meters, respectively.

Determine each of the following for the spacecraft when it is at point A .
(a) The total mechanical energy of the spacecraft, assuming that the gravitational potential energy is zero at an infinite distance from the Earth.

(b) The magnitude of the angular momentum of the spacecraft about the center of the Earth.

Later the spacecraft is at point B on the exact opposite side of the orbit at a distance $r_B = 3.6 \times 10^7$ meters from the center of the Earth.
(c) Determine the speed v_B of the spacecraft at point B.

Suppose that a different spacecraft is at point A, a distance $r_A = 1.2 \times 10^7$ meters from the center of the Earth.
(d) Determine the speed of the spacecraft if it is in a circular orbit around the Earth.

(e) Determine the minimum speed of the spacecraft at point A if it is to escape completely from the Earth.

Solution to part a: The total mechanical energy will consist of the potential and kinetic energy.

$$\text{E} = \text{KE} + \text{PE}$$

The kinetic energy is $\frac{1}{2} \cdot m \cdot v_A^2$ (where m represents the mass of the satellite)

The potential energy can be found using the formula $-\frac{GM_1M_2}{R}$. R is equivalent to r_A since r_A is the distance between Earth's center and the spacecraft. We can pug in the right variables m and M_E for mass to get

$$U = -\frac{GmM_E}{r_A}$$

$$\text{This means that the total mechanical energy is } \frac{1}{2} \cdot m \cdot v_A^2 - \frac{GmM_E}{r_A}$$

We can plug in our values to get

$$\frac{1}{2} \cdot 1000 \cdot (7.1 \cdot 10^3)^2 - \frac{6.67 \cdot 10^{-11} \cdot 1000 \cdot 6 \cdot 10^{24}}{1.2 \cdot 10^7}$$

We can use our calculator to find that the total energy is $-8.1 \cdot 10^9$ J.

Solution to part b: Angular momentum can be found using the formula $L = mvr$
We can plug in $m = 1000$ (mass of spacecraft), $v = v_A = 7.1 \cdot 10^3$ (velocity of spacecraft), and $r = r_A = 1.2 \cdot 10^7$ (distance between spacecraft and Earth).

$$\text{Doing so gives that } L = 8.52 \cdot 10^{13} \text{ kg } \cdot \text{m}^2/\text{s}$$

Solution to part c: Angular momentum of the spacecraft will be conserved.

$$\text{This means that } mv_Ar_A = mv_Br_B$$

The mass m of the spacecraft cancels out from both sides leaving us with $v_Ar_A = v_Br_B$

$$\text{This means that } v_B = \frac{v_Ar_A}{r_B}$$

We can now plug in $v_A = 7.1 \cdot 10^3$, $r_A = 1.2 \cdot 10^7$, and $r_B = 3.6 \cdot 10^7$ to find that $v_B = 2.4 \cdot 10^3$

Solution to part d: We know that the gravitational force will be causing the centripetal motion.
This means that $F_g = F_c$
We know that $F_g = \frac{GmM_e}{r_A^2}$

We also know that $F_c = \frac{mv^2}{r} = \frac{mv^2}{r_A}$

$$\text{We can set both expressions equal to each other: } \frac{GmM_e}{r_A^2} = \frac{mv^2}{r_A}$$

After cancelling our known variables and isolating v,

$$\text{We can find that } v = \sqrt{\frac{GM_e}{r_A}} = 5.8 \cdot 10^3 \text{ m/s .}$$

Solution to part e: Escape velocity occurs when the potential and kinetic energy sums to 0. Since gravitational potential energy will be negative, the kinetic energy must have that exact same magnitude.

We know that the spacecraft is a distance r_A from Earth. Thus, that is the radius that will be used for all of our calculations since we want the distance between the center of masses.

Since $U = -\frac{GmM_e}{r_A}$, we know that $K = \frac{GmM_e}{r_A}$. The reason is that they both must sum to 0.

Now, we can use $K = \frac{1}{2}mv^2$ (the formula for kinetic energy) and set it equal to $\frac{GmM_e}{r_A}$.

We can cancel out some variables and simplify to find that

$$v = \sqrt{\frac{2GM_e}{r_A}} = \sqrt{\frac{2 \cdot 6.67 \times 10^{-11} \cdot 6 \times 10^{24}}{1.2 \cdot 10^7}} = 8.2 \cdot 10^3 \text{ m/s}$$

Problem 7.0.10 — 2007 AP Physics C: Mechanics FRQ
In March 1999, the Mars Global Surveyor (GS) entered its final orbit about Mars, sending data back to Earth. Assume a circular orbit with a period of 1.18×10^2 minutes $= 7.08 \times 10^3$ seconds and an orbital speed of 3.40×10^3 m/s. The mass of the GS is 930 kg, and the radius of Mars is 3.43×10^6 m.

(a) Calculate the radius of the GS orbit.

(b) Calculate the mass of Mars.

(c) Calculate the total mechanical energy of the GS in this orbit.

(d) If the GS were to be placed in a lower circular orbit (closer to the surface of Mars), would the new orbital period of the GS be greater than or less than the given period?
_____ Greater than _____ Less than

(e) In fact, the orbit the GS entered was slightly elliptical with its closest approach to Mars at 3.71×10^5 m above the surface and its furthest distance at 4.36×10^5 m above the surface. If the speed of the GS at closest approach is 3.40×10^3 m/s, calculate the speed at the furthest point of the orbit.

Solution to part a: We know that $v = \frac{2\pi R}{T}$

We can rearrange the equation to get $R = \frac{vT}{2\pi}$

We know that the orbital speed v is $3.4 \cdot 10^3$. On top of that, the period T is $7.08 \cdot 10^3$

$$\text{Plugging that in gives that } R = \frac{3.4 \cdot 10^3 \cdot 7.08 \cdot 10^3}{2 \cdot 3.14} = 383312 \text{ m.}$$

Solution to part b: We know that the gravitational force between GS and mars will be equivalent to the centripetal force.
The gravitational force is $F_g = \frac{GmM_m}{R^2}$ while the centripetal force, caused by the gravitational force, is $\frac{mv^2}{R}$. Note, that M_m in this represents the mass of Mars.

$$\text{We can equate both expressions: } \frac{GmM_m}{R^2} = \frac{mv^2}{R}$$

$$\text{We can isolate } M_m \text{ to find that } M_m = \frac{v^2 R}{G} = 6.64 \cdot 10^{23}$$

Solution to part c: The total mechanical energy of GS can be found by adding its potential and kinetic energy.

We know that PE is $-\frac{GmM_m}{R}$ while KE is $\frac{1}{2}mv^2$

We can substitute our values and add up both PE and KE to find that the total energy is $-5.38 \cdot 10^9$ J.

Solution to part d: We must use Kepler's Law to solve this problem. Note, on the actual AP exam, Kepler's Law has an extremely low yield. It is unlikely to show up on an FRQ, but it's still important to know just in case.

Now, when GS is placed in a lower orbit, then the semi-major axis of the orbit will decrease. This means that the period must also decrease.
The reason is that $\frac{T^2}{R^3}$ is constant by Kepler's Third Law.

Solution to part e: In an orbit, the angular momentmu will be conserved. This means that at all points of the orbit, angular momentum is the same.

Thus, let's say that r_1 represents the closest distance while v_1 represents the velocity of the GS at the closest distance from Mars. Similarly, r_2 represents the furthest distance and v_2 represents the velocity of the GS at the furthest distance from Mars Mars.

$$\text{Since angular momentum is conserved, } mr_1v_1 = mr_2v_2$$

$$\text{Mass cancels out so } r_1v_1 = r_2v_2$$

$$\text{We can rearrange this to get } v_2 = \frac{r_1v_1}{r_2}$$

We need to be very careful about the values we plug in. r_1 and r_2 are the distance from the center of Mars to the GS. The distances 3.71×10^5 and 4.36×10^5 are the distance from the surface of Mars to the GS. We must also add the radius of Mars to this!

Thus, $r_1 = 3.71 \times 10^5 + 3.43 \times 10^6 = 3.8 \times 10^6$ m
$r_2 = 4.36 \times 10^5 + 3.43 \times 10^6 = 3.866 \times 10^6$ m

$$\text{We can plug in our values to get } v_2 = \frac{3.8 \times 10^6 \cdot 3.4 \times 10^3}{3.866 \times 10^6} = 3341 \text{ m/s}$$

Problem 7.0.11 — 2005 AP Physics C Mechanics FRQ
A student is given the set of orbital data for some of the moons of Saturn shown below and is asked to use the data to determine the mass M_S of Saturn. Assume the orbits of these moons are circular.

Orbital Period, T (seconds)	Orbital Radius, R (meters)		
8.14×10^4	1.85×10^8		
1.18×10^5	2.38×10^8		
1.63×10^5	2.95×10^8		
2.37×10^5	3.77×10^8		

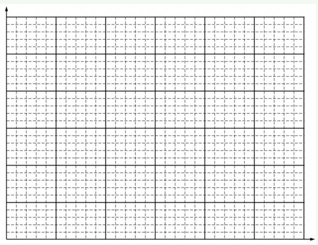
(a) Write an algebraic expression for the gravitational force between Saturn and one of its moons.

(b) Use your expression from part (a) and the assumption of circular orbits to derive an equation for the orbital period T of a moon as a function of its orbital radius R.

(c) Which quantities should be graphed to yield a straight line whose slope could be used to determine Saturn's mass?

(d) Complete the data table by calculating the two quantities to be graphed. Label the top of each column, including units.

(e) Plot the graph on the axes below. Label the axes with the variables used and appropriate numbers to indicate the scale.



(f) Using the graph, calculate a value for the mass of Saturn.

Solution to part a: For two objects in general, the force between them is $\frac{Gm_1m_2}{R^2}$ where m_1 and m_2 represent the two masses.
Assuming the mass of one of the moons is m, the force between Saturn and that moon is $F = \frac{GmM_S}{R^2}$.

Solution to part b: Since we have a circular orbit, we know that centripetal motion is occurring. Thus, we can set the force equal to $\frac{mv^2}{R}$.

That means $F = \frac{GmM_S}{R^2} = \frac{mv^2}{R}$.
From here, we need to find an expression for v. We know that $v = \frac{2\pi R}{T}$. We can plug that into our equation.

$$\frac{GmM_S}{R^2} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2$$

$$\implies \frac{GmM_S}{R^2} = \frac{m}{R} \cdot \frac{4\pi^2 R^2}{T^2}$$

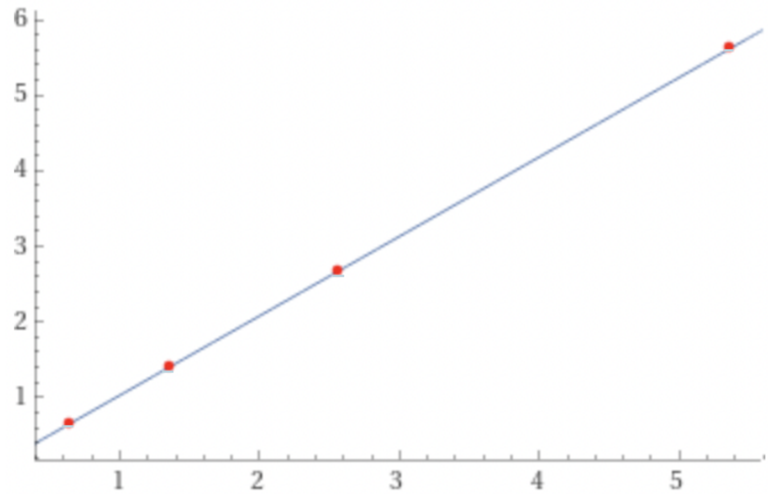
We can cancel out like terms to get $\frac{GM_S}{R} = \frac{4\pi^2 R^2}{T^2}$
Now, we can rearrange this equation to isolate T: $T^2 = \frac{4\pi^2 R^3}{GM_S}$
We can square root both sides to get $T = 2\pi\sqrt{\frac{R^3}{GM_S}}$

Solution to part c: From the equation, we can tell that graphing T^2 and R^3 will give us a linear line. The reason is that T^2 is on the left and R^3 is on the right.

Solution to part d: In the two given columns, we write the values of T^2 and R^3.

Orbital Period, T (seconds)	Orbital Radius, R (meters)	T^2 (s^2)	R^3 (m^3)
8.14×10^4	1.85×10^8	0.663×10^{10}	0.633×10^{25}
1.18×10^5	2.38×10^8	1.39×10^{10}	1.35×10^{25}
1.63×10^5	2.95×10^8	2.66×10^{10}	2.57×10^{25}
2.37×10^5	3.77×10^8	5.62×10^{10}	5.36×10^{25}

Solution to part e: We graph our values for T^2 and R^3 on the grid. Make sure that you scale the graph properly so the graph takes up most of the given grid.



We scaled the graph down to ignore 10^{10} for T^2 and 10^{25} for R^3. Also, the unit for T^2 is s^2 and the unit for R^3 is m^3. Don't forget to label that on the graph on the AP exam.

Solution to part f: We can use two of the data points from our table to find the slope of the graph T^2 vs. R^3.

$$\text{The slope is } \frac{2.66 \cdot 10^{10} - 0.663 \cdot 10^{10}}{2.57 \cdot 10^{25} - 0.633 \cdot 10^{25}} = 1.047 \cdot 10^{-15}$$

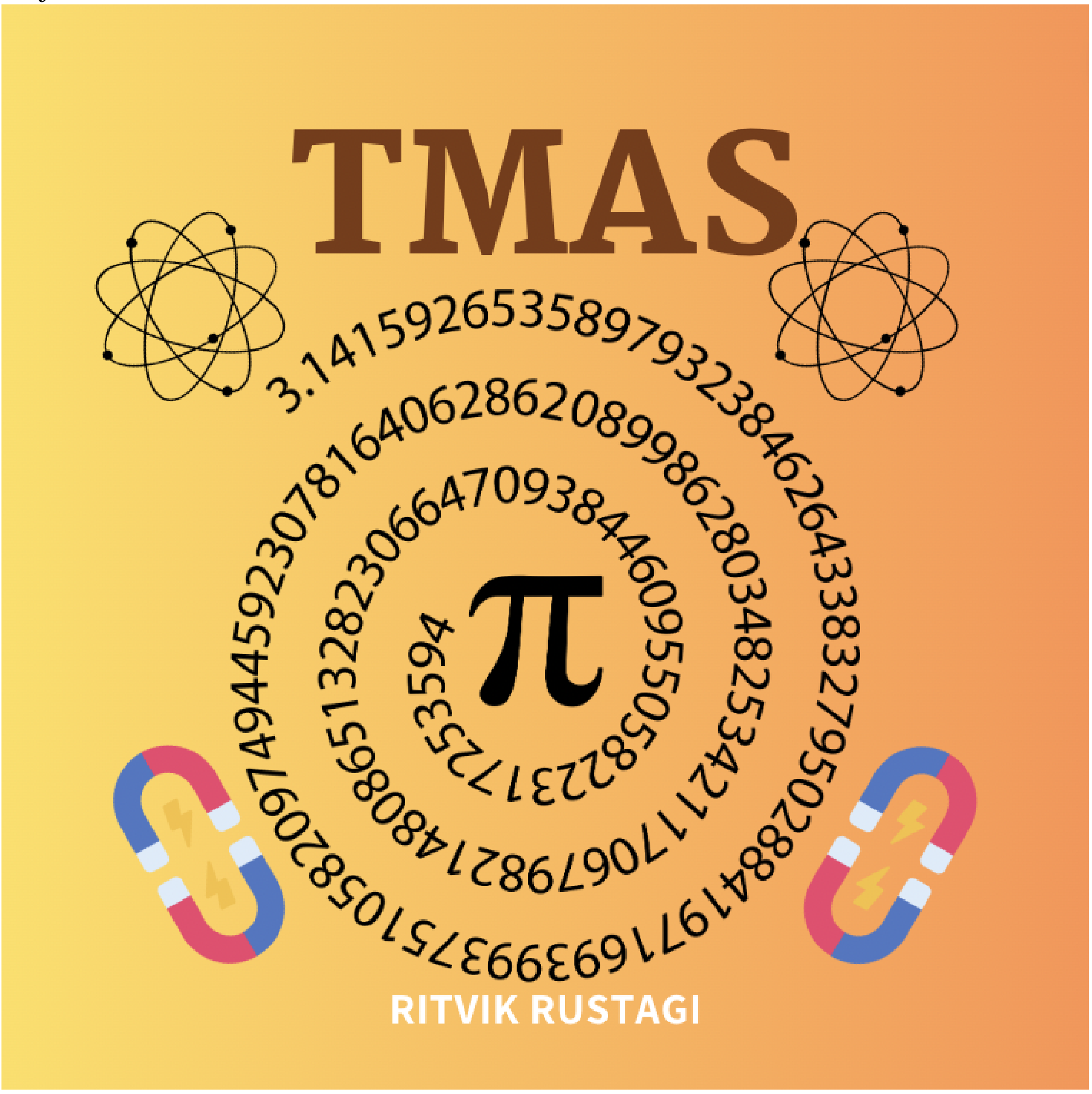
In part b, we were able to find that $T^2 = \frac{4\pi^2 R^3}{GM_S}$.
We can rearrange that equation to get $\frac{T^2}{R^3} = \frac{4\pi}{GM_S}$.

We can plug in the slope $1.047 \cdot 10^{-15}$ for $\frac{T^2}{R^3}$.

$$\text{Doing so gives } 1.047 \cdot 10^{-15} = \frac{4\pi^2}{GM_S}$$

We can isolate M_S to find that $M_S = \dfrac{4\pi^2}{1.047 \cdot 10^{-15} G} = 5.65 \cdot 10^{26}$ kg

Thank you for going through this book!
It is an honor for me to have contributed to your physics journey in some way!



Thanks,

Ritvik Rustagi